ANALYSIS OF COUPLED SHEAR WALL

TALL BUILDINGS

by

AMR E. SALAMA B.Sc., M.Sc.

Thesis submitted for the Degree of Doctor of Philosophy in the Faculty of Engineering at Heriot-Watt University, Edinburgh.

Department of Civil Engineering,
Heriot-Watt University,
EDINBURGH

April, 1979.

CONTENTS

Page

Page

ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to Dr. A.W. Irwin for his helpful advice, discussions, guidance and encouragement. I would also like to thank my family for their encouragement and support.

ABSTRACT

A number of basic systems and concepts for the design, analysis and erection of tall buildings are examined in this thesis, important factors concerning the economic and social impact of such buildings are also discussed.

An experimental investigation was carried out using 1:25 scale complete reinforced concrete models of buildings and a two-storey high model at one-third full size, reinforced in a similar manner to slab and wall structures designed for buildings for construction in an earthquake risk area. The experimental investigation was used to study the overall behaviour of complete shear wall buildings under elastic, elasto-plastic and dynamic loadings, and to study, in detail, the action of the floor slabs coupling shear walls. The results obtained from the experimental investigation are presented.

The theoretical method used to analyse the coupled shear wall structures adopted both the continuous connection technique and the finite element method. The analytical analysis can be used for symmetrical and non-symmetrical coupled shear walls with any number of wall openings when subject to both lateral and vertical loadings. The theoretical results are compared with the results obtained from the experiments to assess the accuracy of the analytical solutions.

Design charts are presented for the rapid evaluation of the effective width and the stiffness of slabs coupling shear walls. The design charts are given for a number of wall/slab configur-:ations. The finite element technique, using a rectangular plate

in bending element, was used to obtain these charts.

The 'initial stress' method was used with the finite element method to assess the performance of the slabs coupling shear walls for the post-elastic situation.

CHAPTER ONE

1.1 INTRODUCTION

The increase in world population, the drift from rural to urban communities and the prosperity which accompanied the recent social and economic developments in most parts of the world have required the utilization of available urban areas to their full potential.

The shortage of skilled labour and the rapid increase in cost of building materials and construction contributed towards the use of prefabricated or factory-produced elements and slipform methods to speed construction and minimise material loss. Mainly for the above reasons, together with the desire for prestige and the availability of new building techniques and machines, a modern trend is the increasing use of various types of tall buildings for office and apartment accommodation and, in some instances, for industrial manufacturing processes.

History shows that for about the past five thousand years many societies have been able to build high; for example, the Egyptian pyramids, the Tower of Babel, the Dravidian Temples of India, the Gothic and Renaissance cathedrals of Europe, the minarets of Muslim mosques, the high Eiffel Tower in Paris and the Chigago Home Insurance building of 1885 which was the first modern high building (skyscraper). Since then many tall building systems which use mainly steel, reinforced concrete, masonry and other high strength materials have been developed to best utilise the materials, to provide the required accommodation and provide efficient vertical and lateral load resisting structures.

1.2 Tall buildings: Systems and Concepts

In planning and designing of tall buildings a number of technical, social, human and aesthetic concepts should be considered. The last few years have seen the development of new concepts of structural systems, construction systems, service systems and environ:mental systems.

1.2a *Structural Systems*

Recently there have been great improvements in the ability of engineers to analyse and design tall buildings using new methods and techniques such as the use of reinforced, precast and prestressed concrete, prefabricated elements, structural steel frames, composite steel and concrete sections and masonry. However, the general choice of structural materials is between reinforced concrete and steel.

The most frequently used structural systems for concrete tall buildings are:

1. The frames system
2. The slab and wall system
3. The complex system
4. The central cores system and the tubed system.

Figure 1.1 shows the early used moment resisting frames system which offers the stiffness and stability required for tall buildings, but at uneconomically high prices. The slab and wall system of Figure 1.2 is frequently used for appartment buildings where the need is for limited storey heights, short spans and numerous partitions. For greater than 20 - 30 storeys, the weight of closely placed walls beside the necessity for ensuring stiffness in all directions reduces the possibility of using such systems in taller buildings. As buildings are constructed taller it becomes increasingly more important

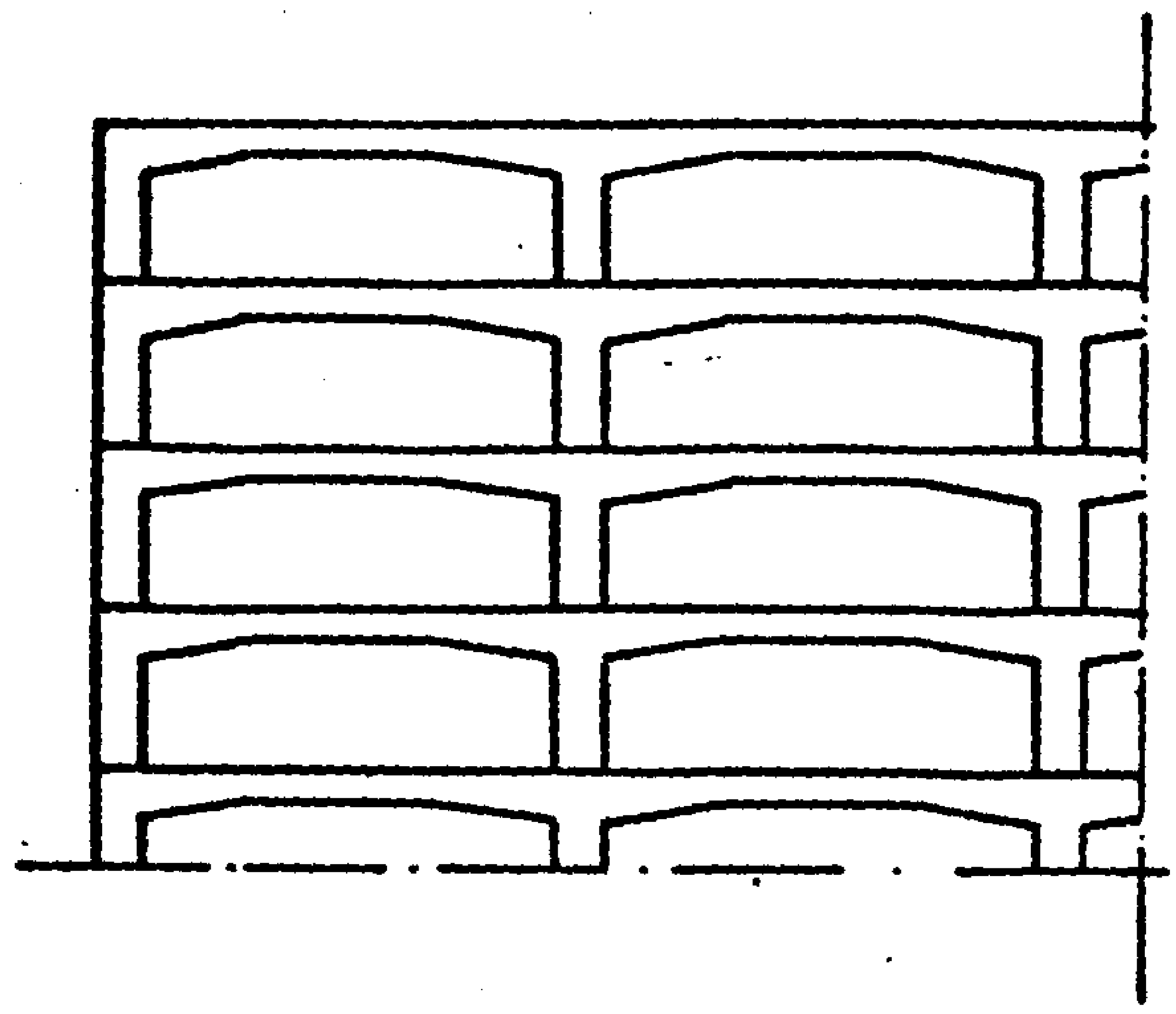


FIGURE 1.1 MOMENT RESISTING FRAMES

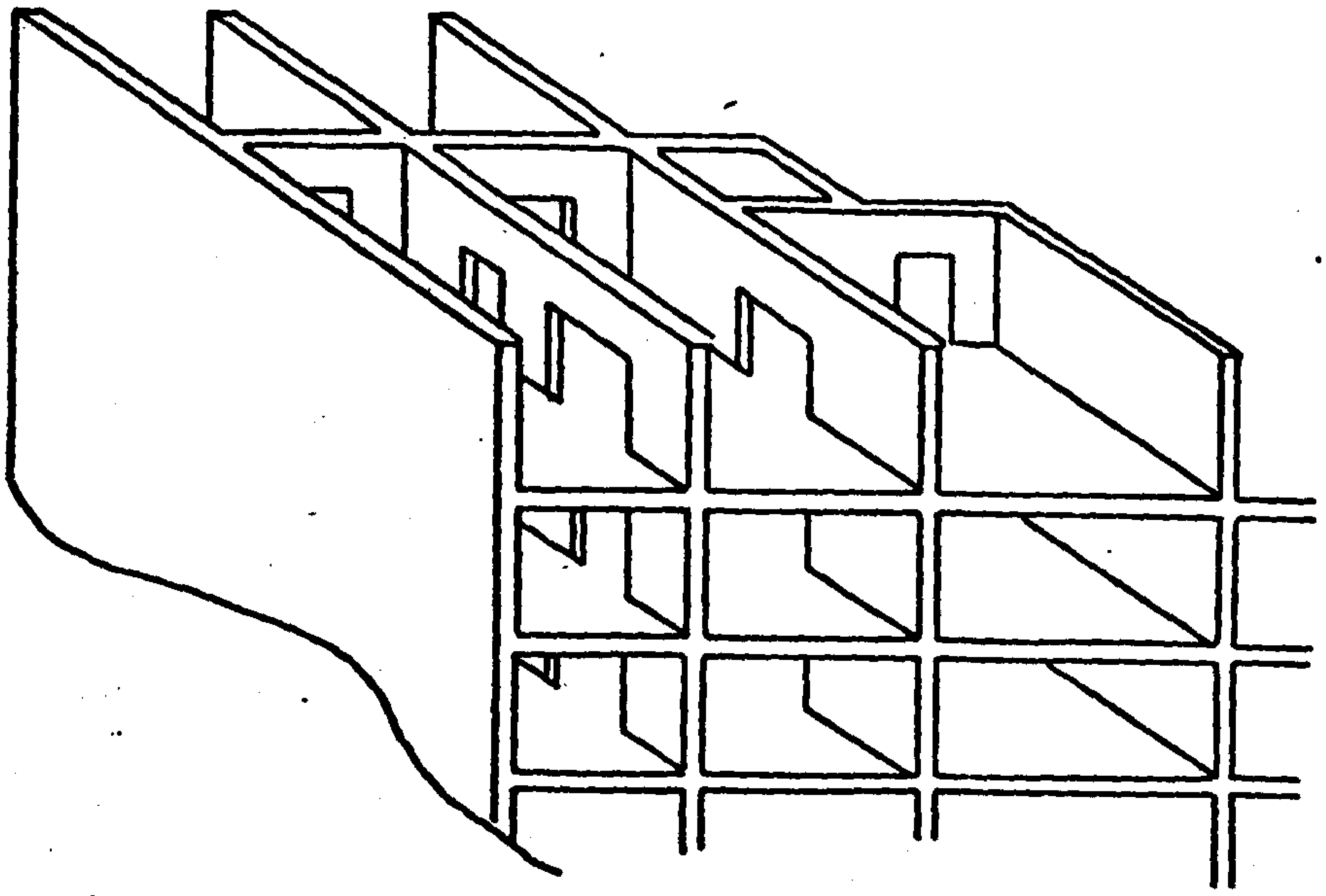


FIGURE 1.2 SLAB AND WALL SYSTEM

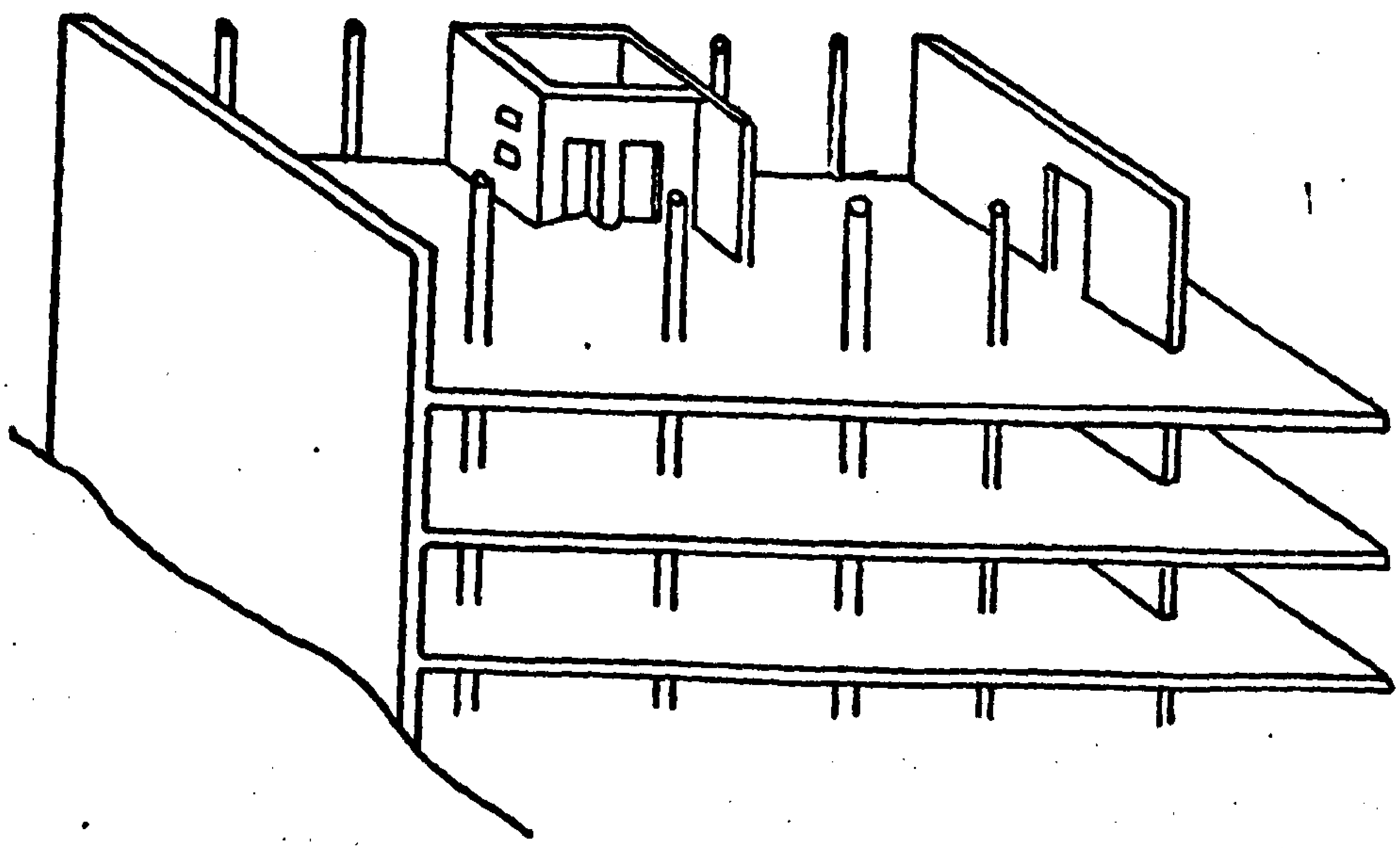


FIGURE 1.3 COMPLEX SYSTEM

to ensure adequate lateral stiffness to resist wind and earthquake forces as well as normal vertical loads and to make better use of space and materials. To fulfil these and other requirements such as fire and acoustic insulation and space division, 'shear-wall' systems were frequently used. These systems, often incorporating additional floors bearing on columns, as well as walls and shear cores, form many complex systems. In such systems the floor slabs minimise the storey heights and act as deep beams which transfer the lateral loads to the vertical load bearing units. Figure 1.3 shows as an example a complex system employing the shear-walls combined with the floor slabs and columns. Some tall modern build:ings comprise a central core containing elevators, stairwells, vertical ducts and other services equipment surrounded by floors supported by the core and facade columns. In some cases all the weight of the building is taken by the core, while in other cases facade columns and girders form a structural grid each giving its contribution to the stability of the building under lateral loadings. The use of central core system is suitable for areas in which the foundations are in close proximity to the foundations of other build:ings and additionally maximises open space at the base of the structure. Figure 1.4 shows an example of central core buildings. There are a number of potential tubular systems available, such as a tubular system utilising only the exterior walls, a tubular system utilising the interior wall but coupled to the exterior wall, the bundle tube system and a tube-in-tube system consisting of a heavy central shear core coupled by floor decks to closely spaced perimeter columns known as mullions. Figure 1.5 indicates some potential plan shapes for hollow tubular buildings.

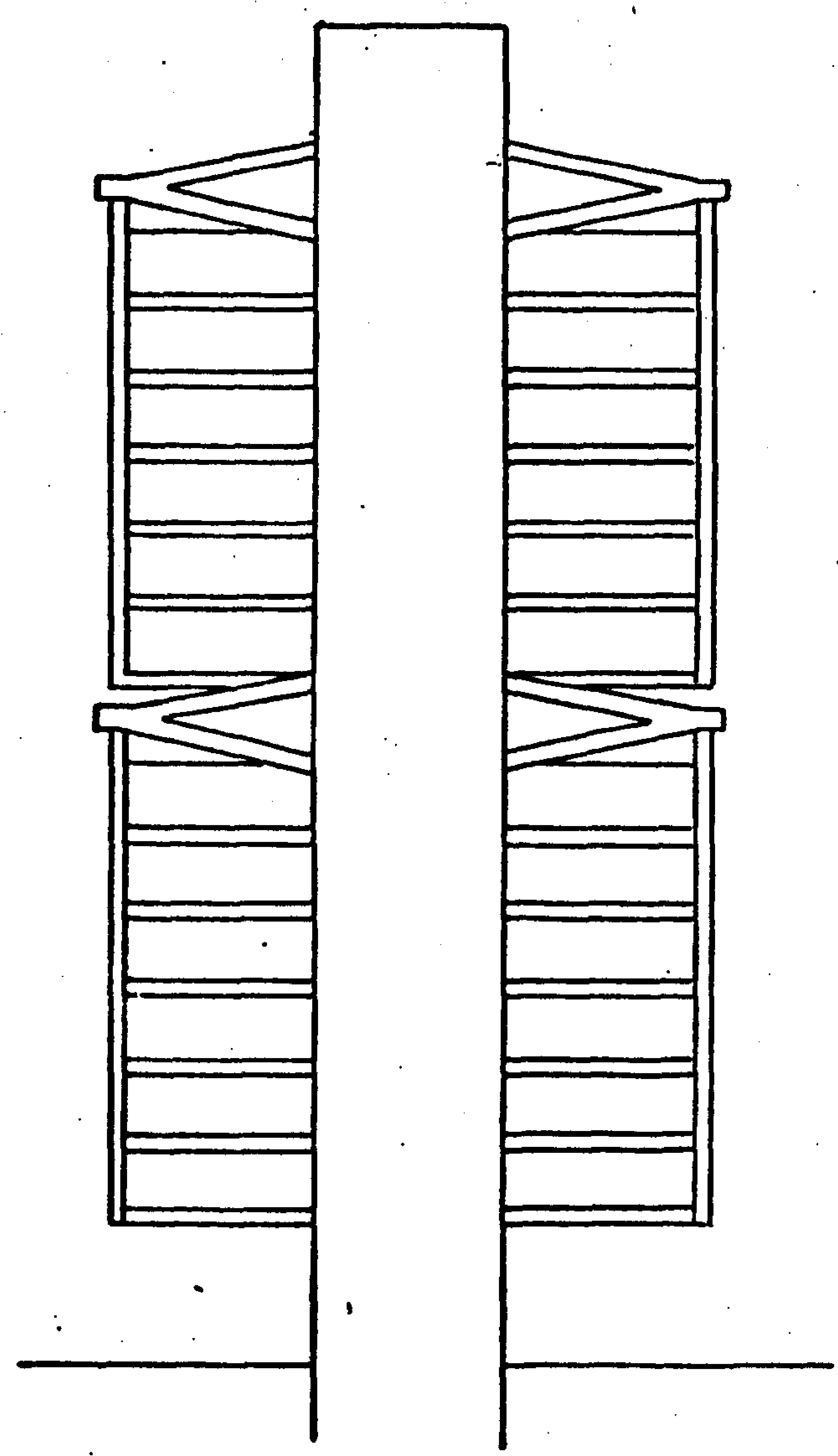


FIGURE 1.4 CENTRAL CORE SYSTEM

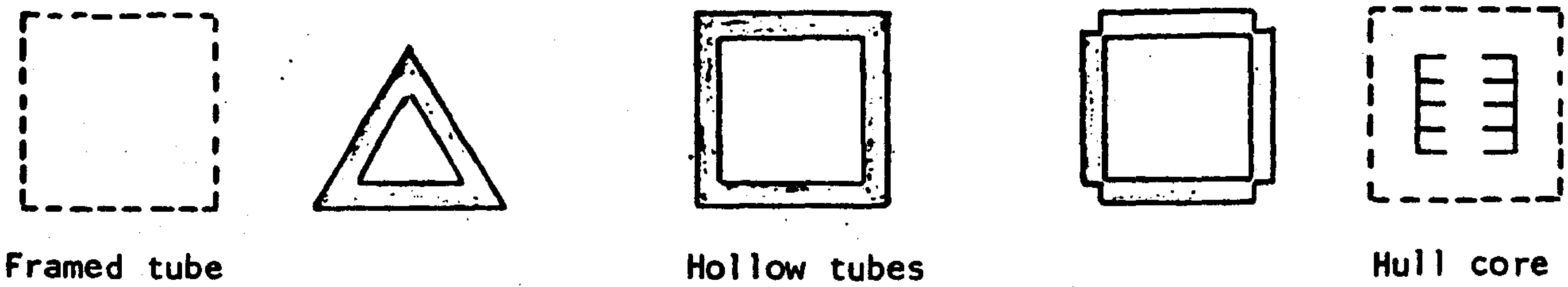


FIGURE 1.5 PLAN SHAPES

In steel the most common structural forms which can be framed are:

1. Rigid framing
2. Simple framing
3. Tube framing
4. and the staggered truss-system.

In the case of rigid framing structures, all connections are rigidly joined. The lateral stability depends on the stiffness of the beams and columns and especially important are the moment resisting connections. In the simple framing system a vertical bracing system is generally used to provide lateral stability. The most frequently employed bracing systems in this case are x-bracing, k-bracing and the warren truss. Figure 1.6 shows examples of these systems. When tube framing is used, lateral stability is achieved by the resulting rigid exterior wall. The interior beams are usually simply connected, the exterior columns are very closely spaced and the spandrel girders are rigidly connected to them. Figure 1.7 shows two commonly used types, the x-braced frame tube and the perforated shell tube. The tube framing system is an economical system for the construction of tall buildings and it lends itself very well to mass production of framing elements and often there is no penalty of increased lateral strength requirement for increase in height. Staggered-truss-system, see Figure 1.8, is the newest system of steel framing for tall buildings. It consists of a series of storey-high trusses placed so that each floor of the structure alternately rests on the top chord of one truss and hangs from the bottom of the next. In addition to carrying the vertical loads, the trusses carry wind loads through the web-members and floor-slabs to the base, thus reducing the wind-bracing requirements.

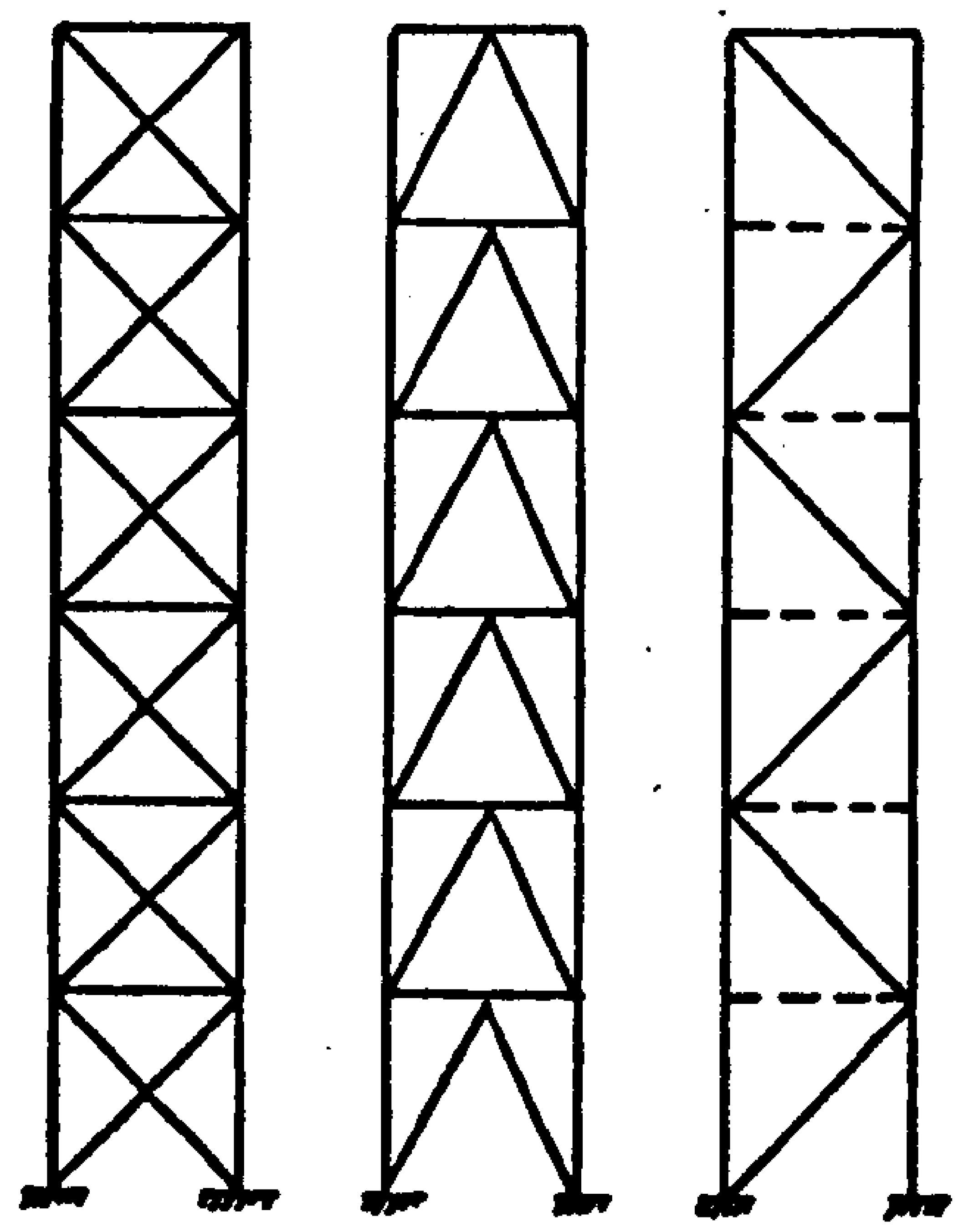


FIGURE 1.6

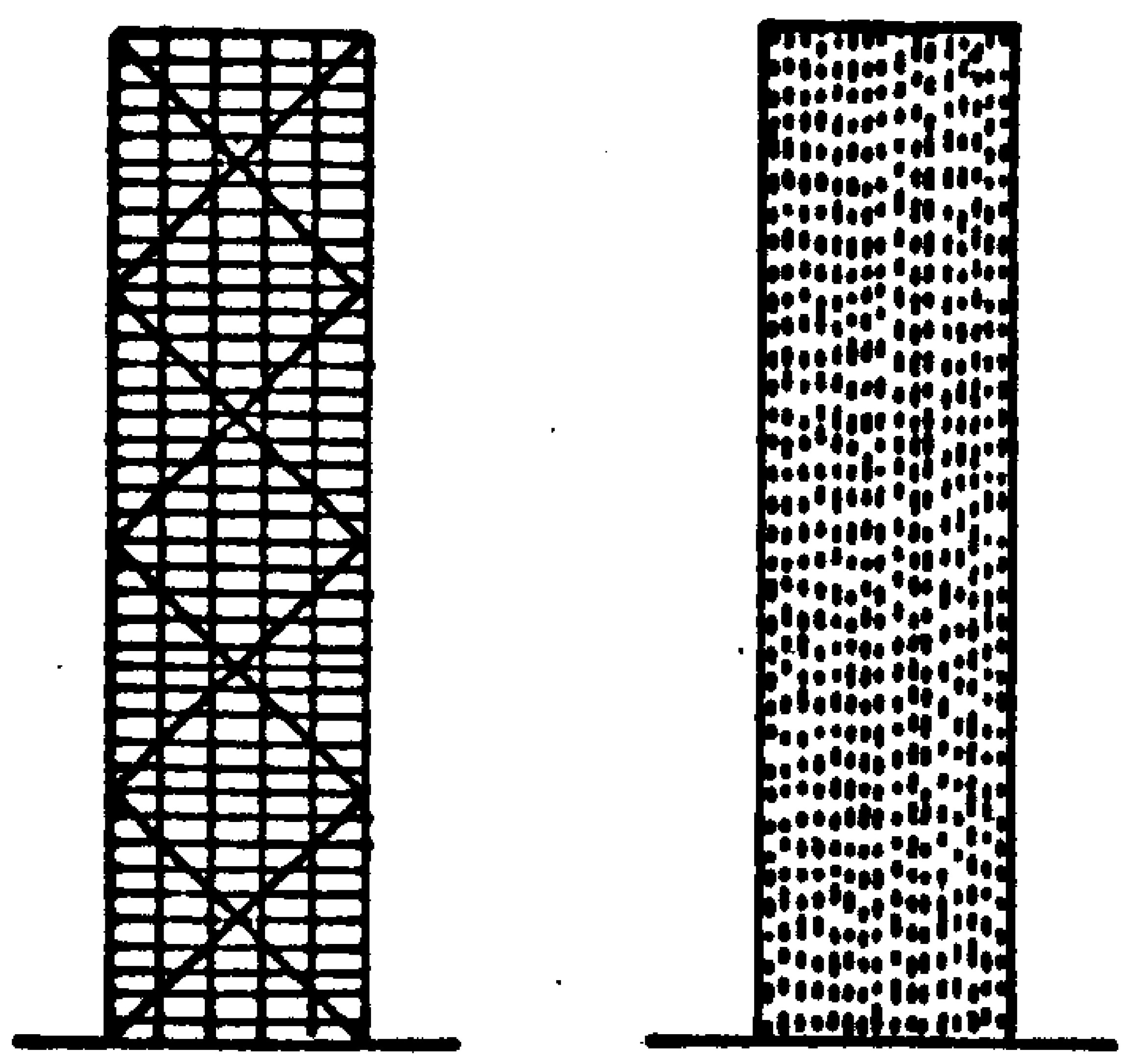


FIGURE 1.7

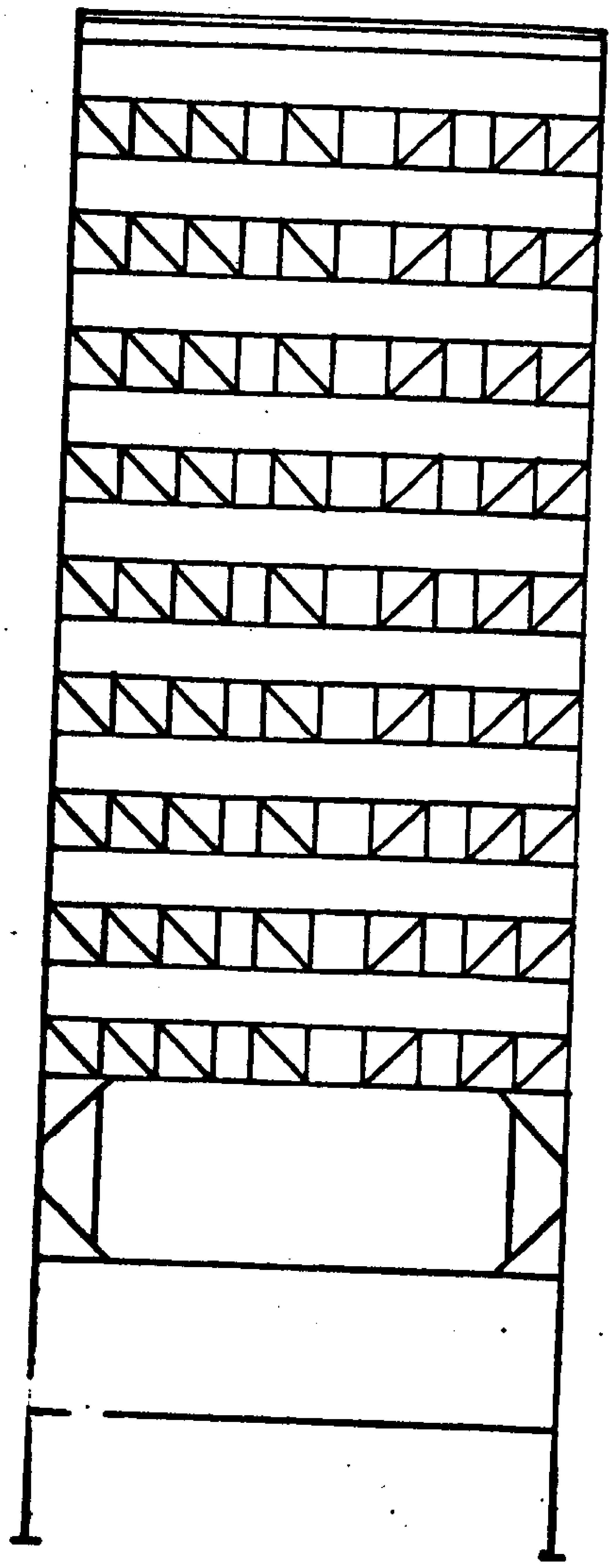


FIGURE 1.8 STAGGERED-TRUSS-SYSTEM

The optimum structural system of tall-building can be achieved by using a combination of materials and composite structural systems to make the best utilization of the material properties.

1.2b *Construction Systems and Techniques*

Construction of tall buildings is complex but with the development of better construction methods, contracting practice and construction techniques and technology, taller, more economical and better constructed buildings can be achieved. Just as the change from pyramid column footings and timber piles for foundations late last century and early this century gave way to steel grillages, rafts, concrete and steel piles and floating foundations, the changes in the foundation construction techniques in the last decade have made great, if not complete, changes in foundation design. New techniques such as tie-backs, slurry walls, high capacity piles, mind excavation, precast, prestressed sheet piling, core barrel caissons and others reflect the great advance that has been made in construction techniques.

Erection procedures using new types of equipment such as creeper stiffleg derricks, tower cranes, and stiffleg derricks mounted on platforms which are lifted within the structure, have reduced the time and cost of structural steel erection.

The most common method of erecting steel framed structures is tier by tier. Each tier represents a column height of usually two or three building floors. If the building can be reached from the ground to the roof by ground-based lifting equipment, the structure can be erected in vertical segments: usually a combination of the two methods is used. The push-up construction technique is used for the lesser height tall buildings. In this process the top floor is erected first at about ground level, then jacks push the complete structural frame upwards and additional steel is placed underneath and attached to that previously jacked.

In constructing tall concrete buildings, new forming techniques have helped in reducing costs and construction time. The types of forms which are used repetitively are gauged forms, flying forms and slip forming. Gauged forms often combine column, beam, slab and wall elements in the same large piece of formwork which is carried forward or up the building as a large unit. Flying forms are used for flat-slab work, built and handled in large areas. Slip forming is ideally suited to tall buildings having concrete cores and shear wall structures, and each storey of identical design. This type is good, and economic, if the vertical elements of the frame maintain uniform dimensions for a long vertical distance.

Hybrid-frame construction offers a number of combinations of concrete and steel frames. Such frames can have a steel frame braced by concrete cores and shear walls, a concrete core with a suspended steel frame, concrete cores with steel floor framing supported on them, steel frames with precast concrete modules inserted in the frame, and many other combinations. Each of the design solutions requires a careful selection on construction sequence and equipment. Allowance must be made for construction tolerance allowed in both steel and concrete when designing connections.

1.2c *Service Systems*

The service systems for tall buildings are mainly:

a) Mechanical systems

b) Electrical systems

c) Safety systems

d) Other technical services (water supply systems, plumbing systems, sewerage and draining systems, cleaning and waste disposal system).

The technical development and the standard of all these service systems, and their influence on tall building construction has to be part of the architectural idea at the very preliminary design stage of the architectural and structural concepts. The mechanical systems include two major systems, the transportation one and the heating, ventilation and air conditioning (HVAC) system.

In fact, without adequate vertical transportation facilities, reasonable accessibility to the high-rise building is impossible. In recent years there have been two developments in vertical transportation. The first being the double-deck elevator system which has the great advantage of increasing one directional capacities, but it has the disadvantage of the less efficient two-way peak period performance as well as the need to make the floor to floor heights exactly the same in order to permit accurate elevator floor leveling. The double-deck system is not efficient for buildings above 50 - 60 floors. The second and more recent is the sky-lobby system. It involves express elevators up to various zones, with local elevators running just within those vertical zones so that the local elevator bands are stacked one over the other. More transportation aspects have to be considered in the design of tall buildings such as the requirements of employees, residence, visitors' and servicing vehicles either delivering goods or taking away the rubbish, the requirement for parking and providing easy and clear access for emergency vehicles.

Each tall building, depending on its potential use, has its own unique particularities and the optimum combination of the HVAC systems. In general, a tall building must have air conditioning and air ventilation systems in order to make it independent of the weather situation at the heights which tall buildings construction now reaches.

In tall buildings an efficient electrical system is essential to safeguard the building against loss of normal electric supply or failure of parts of the internal electrical systems, and maintain the operation of the tall building services. Ensuring the supply of H. T. feeders from at least two different sources, coupling of transformer stations installed at the different concentration points in the building, and the use of emergency electrical generator sets are some of the means which can be used to overcome the effects of partial loss of electrical power supply.

In a tall building the rescuing of occupants and fire fighting must, in most instances, be accomplished from within the building itself. The greater the number of occupants and the height of the building, the more is the risk of fire and of smoke and gaseous transmission through service shafts. Shear walls, combined with horizontal and vertical zoning in a building can present an effective fire barrier and minimise fire spread. No additional protection is required on shear walls because of their inherent fire resistance.

Two methods or combination of them are usually used to over-:come the problems of water supply in tall buildings:

a) Using pumps to maintain constant pressure in risers with reducing valve sets breaking down pressure for different vertical zones.

b) Pumping of water up to house tanks at different levels with gravity feed from tanks to lower water supply points.

The waste drainage system must be carefully designed to allow for adequate venting.

Tall buildings with their high population rates give rise to waste disposal problems as the central waste collecting space and evacuation facilities must be sufficient to handle these very heavy loads. Often the siting of a tall building in a specific location can present problems not only regarding its internal efficiency but often equally formidable are the problems of the extra service loads on the local transport networks, sewerage systems, water and electrical supplies.

1.3 Force action on tall building structures

In contrast to low-rise buildings where the predominant force actions are vertical dead load and live loadings, in tall structures consideration of horizontal loads often assumes greater importance than vertical loadings. The principal force actions considered in the analysis and design of tall buildings are summarised below.

1.3a *Wind action and tall buildings*

The most frequently occurring horizontal loading to which tall buildings are subject is that due to wind action. Horizontal forces from this source depend upon the wind velocity, local topography, the building shape, height and roughness. The response of the building depends not only upon the wind forces, which can be extremely variable, but also upon the stiffness of the structure its damping & its mass distribution. Often uniform wind conditions create translational structural motion and rotational vibration due to nonsymmetry of the structure mass or stiffness, the wind incidence and vortex action. Often wind loads are examined to determine structural stresses, deflections and fatigue as well as the effect on non-structural cladding and glazing in every day and extreme storm conditions, but also important is the human reactions to building motion which is discussed later.

1.3b *Vertical dead and live loading*

Vertical dead loads in high rise buildings can be readily calculated, but live loading depends on a large number of factors including building use and time of day, but generally close estimates of minimum and maximum live loadings can be achieved in order to design for maximum vertical loads and prevent tension when high winds are acting at minimum live load conditions.

1.3c *Temperature movements*

Differential temperature movements on opposite sides of a building or between interior and exterior structural elements can cause both horizontal and vertical structural movements. Where such problems are envisaged the design can be made such that all structural elements are contained within the outer skin of the building, thus limiting differential temperature movements. Differential creep of vertical structural elements can cause similar effects to temperature movements.

1.3d *Earthquakes and tall buildings*

The unpredictable nature, range and magnitude of the forces from earthquakes are such that care has to be taken in designing tall buildings to achieve a structure which will behave satisfact-:orily and predictably in both the elastic and plastic ranges. Three approaches are available to estimate the design load due to earthquakes. The first is the equivalent static loading, the second is the dynamic analysis based on the response spectrum technique and finally the complete dynamic response analysis to obtain the time history response of the proposed structure. Two main criterion are required in the building:

1. The ability to sustain high deformations without appreciable loss of strength.
2. The ability to dissipate as quickly as possible in the face of a high impact of energy.

Examples from the last few years show that coupled shear-walls can be used in buildings to provide the structural ductility, restrict lateral drift and minimise earthquake damage and repair costs. (During the Yugoslavia earthquake in 1963 the Party Head-:quarters suffered only minor damage, similarly with the Indian Hill Medical Centre 1971). Using the ductile moment-resisting frames major structural distress can be avoided but extensive internal damage to the non-structural components and finishes can happen.

Structures should be designed such that, if plastic deform-:ations should occur, they should be restricted to elements which can most easily and economically be repaired.

The analysis and design of a tall building in a seismically active area depends on the local environmental soil conditions and the earthquake history of this area. The design usually rests on the premise that the gravity load carrying walls will be the ones to be damaged and that the structure will have an overall ductility factor of at least four.

1.4 Tall buildings and human reactions

The human reactions to tall buildings environment can be divided into three important criteria; the occupants' comfort, the sociological consequences of tall buildings and the environ-:mental aspects.

1.4a *Comfort criteria*

The occupants' comfort includes problems of vibrations due to tall building wind induced motion, earthquakes, mechanical services, activity on nearby construction sites, neighbouring factories, rail and road traffic and other reasons.

For the wind induced motion of buildings the criterion for upper magnitudes of horizontal motion of buildings is an alarming experience. The level of complaints due to such alarm is dependent upon:

1. The return period, the shorter the interval between occurrences the higher the complaints level.
2. The time over which motion of a particular intensity is sustained for each occurrence.

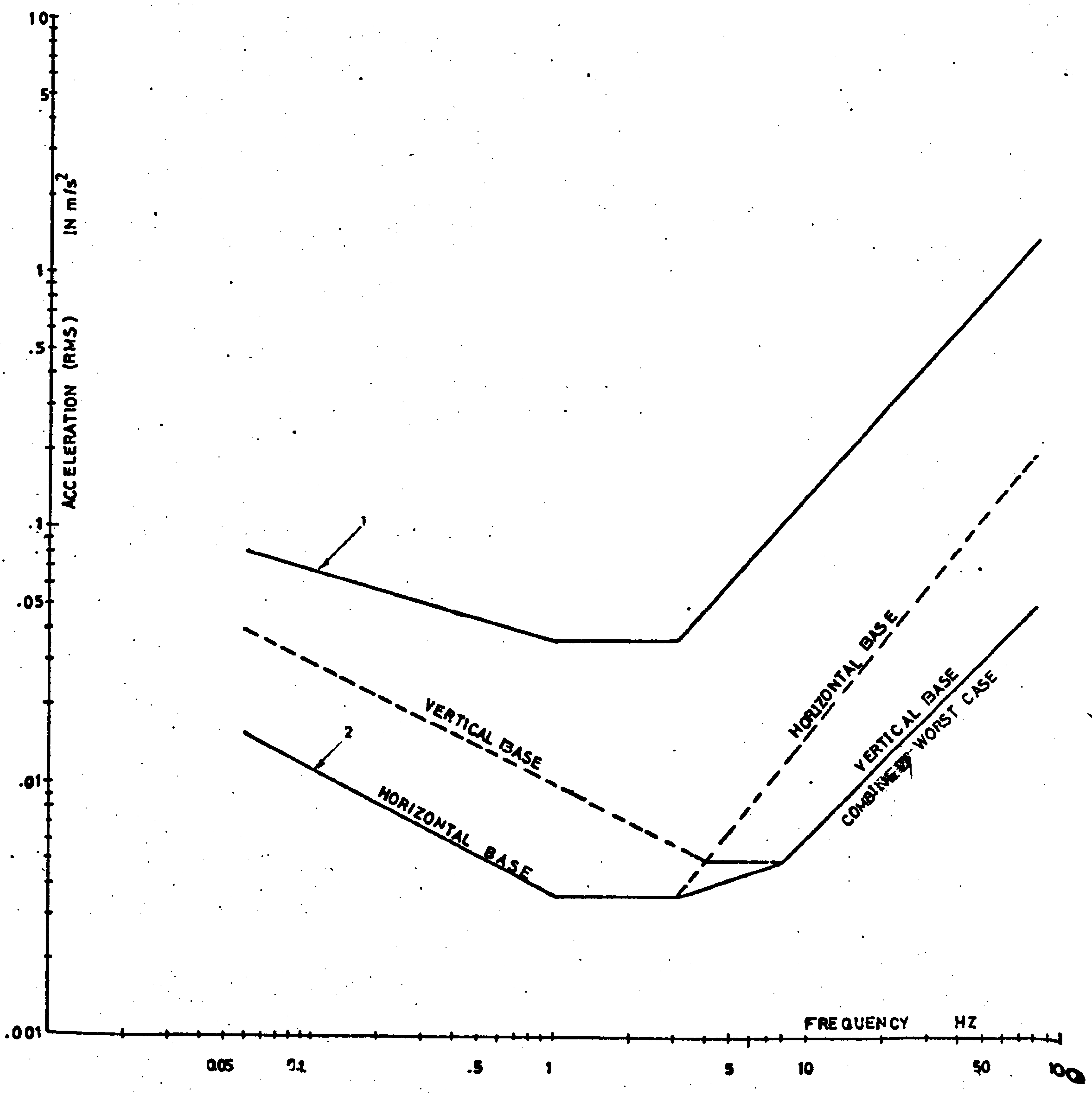
For buildings used for general purposes, suggested acceptable maximum magnitude of horizontal motion during the worst ten minutes of a wind storm with a return period of at least five years, in the range of 0.06 to 80 Hz, is shown in curve 1 Figure 1.9. For buildings in which precision work is carried out and for special buildings the criterion for regularly occurring building motion in the daily situation is related to the perception threshold of motion of people of average susceptibility or perceptibility. Irwin(1978) suggests as well that the lower threshold of perception of horizontal motion by human beings is as shown in curve 2 of Figure 1.9.

1.4b *Temperature and air supply*

Other comfort criteria include thermal comfort and air freshness. Temperature can be controlled to a certain extent by adjusting the sun-:light admission to the building but heating systems of various forms are used to supplement this in cold weather. In summer the air conditioning system allows for both turnround of air and cooling of the internal building environment.

1.4c *Socialogical Aspects*

The use of high rise buildings as regards to social aspects can be divided into two categories of work - office accommodation and apartment dwellings. In the first instance little objection is generally found in this form of accommodation; minor grievances, including sometimes remoteness from observable external activities and isolation from readily accessible shopping or mid-day recreation



ACCELERATION (RMS) IN m/s²
10
5
1
.5
.1
.05
.01
.001
1
2
VERTICAL BASE
HORIZONTAL BASE
HORIZONTAL BASE
VERTICAL BASE
COMBINED WORST CASE
FREQUENCY HZ
0.05
0.1
.5
1
5
10
50
100

FIGURE 1.9

areas, can be felt. On the other hand the acceptance of apartment dwellings seems, to a large extent, to depend upon the social background of those involved, the amenities provided, and the presence of children. Where the accommodation is of a luxury form with a high standard of amenities, there is often a high demand for apartments despite prohibitive rental charges. In high density populations such as Hong Kong and some other Asian cities, people seem to adapt more readily to more spartan high rise living than those in many Western societies, with the possible exception of Sweden where communal cooking and recreation areas in tall blocks tend to relieve the feeling of isolation such as is voiced by many in the United Kingdom. Often high rise accommodation is found undesirable by parents who do not feel confident about allowing their children out on their own in the ground level play areas when they live above about the third level. Another aspect of high rise living in some societies is the damage and disruption to services by vandals who often prey, undetected, on high rise apartment blocks.

1.5 Demolition

Although at the design stage demolition is usually regarded as a remote event contingency, plans should be made for this eventuality especially in some forms of precast and post-tensioned structures since severe damage by fire or earthquake could result in premature demolition of a building being carried out.

1.6 Present research work

The above discussion, although not fully comprehensive, is sufficient to indicate some of the important systems and concepts which accompany the design, analysis and erection of tall buildings, as well as their economic and social impact on the community. In the present research work, the main object is the analysis of coupled shear-walls, especially those connected only by floor slabs. In the following chapters details of this investigation are given, together with reviews of work carried out by other researchers.

CHAPTER TWO

EXPERIMENTAL INVESTIGATION

2.1 Introduction and past work

Test data from reinforced concrete models of shear wall structures is fairly sparse. They are restricted to 1: 25 models of complete shear wall buildings tested by Irwin and Young (1976) and by McKenzie (1976) to determine their overall behaviour when subject to lateral loading causing elastic and elasto-plastic behaviour of the structures, tests on walls coupled by beams carried out by Paulay (1975), tests on the torsional performance of coupled core structures carried out by Irwin and Young (1976) and Irwin and Bolton (1976), combined torsion and vertical load tests carried out by Irwin (1975) and tests on parts of the walls and single beam coupling walls at large scale by De Lisle (1971), Heidebrecht et al (1973) and by Irwin and Ord (1976). Other researchers have generally restricted their investigations to the elastic range and have employed models constructed from acrylic such as Coull and Irwin (1972), Holmes et al (1969) or from araldite Coull and Puri (1967) or brass Harrison, T. (1969). Qadeer and Stafford-Smith (1969) used asbestos cement models, while Coull and El Hag (1975) formed models combining steel and acrylic.

2.2 Present investigations

In this investigation it was intended to study the overall behaviour of complete shear wall buildings when subject to: only lateral loadings and working stresses; the dynamic response with and without vertical loading and the elasto-plastic cyclic load performance of these structures when subjected to near constant vertical forces and cyclic horizontal forces. In addition to monitoring the overall 'elastic' and elasto-plastic behaviour of the structures, the action of the floor slab coupling the vertical walls was also to be investigated in some detail. To fulfil the multi-purpose study it was decided to construct a complete 13-storey reinforced concrete model shear wall building structure at a scale of 1: 25 for investigation of the overall structural performance and subsequently construct a 2-storey high model at one-third full size, consisting of two walls coupled by floor slabs and reinforced in a similar manner to slab and wall structures designed for construction in an earthquake zone. This large scale model would then allow detailed investigation of the action of the floors coupling the walls both in the range of normal working stresses and when forced to cycle elasto-plastically.

2.3 Construction of models

The 13-storey, 1: 25 scale model shown in Figure 2.1 was constructed using expanded polystyrene void formers fixed in an outer shutter, the steel reinforcement (0.3% each way on a square grid) held in position between the expanded polystyrene units; the assembly vibrated at 50 Hz and 5 g's on a shake table while a concrete mix, with no additives, and comprising, by weight, 2:1:1 : 0.415 of dolerite aggregate, silica sand, ordinary Portland cement and water was inserted in a single continuous pour as described in detail by Irwin and Young (1976). The particle size distribution of the aggregate and sand are shown in Tables 2.1 and 2.2. This model was cured for thirty days in a temperature and humidity controlled room prior to fixing on the rig for instrumentation and testing.

The reinforcement incorporated into the 1:25 scale model comprises 1.019mm diameter mild steel wire welded in a 12mm square mesh. This mesh was placed centrally in the walls and slabs of the models, the slabs only being double reinforced on a 12mm broad strip at each wall opening. Details are shown in Figure 2.1b.

The two storey, single-bay one-third full scale model, details of which are given in Figures 2.2 - 2.4, was constructed in a continuous pour by fabricating the complete reinforcement cage on a base board and fixing and propping the shutters as the pour progressed. A series of rod vibrators were used to complete the concrete mix which was by weight 3.5: 1.25: 1.0: 0.45 of 10mm maximum size aggregate, whin sand, ordinary Portland cement, water with no additives. The particle size distribution of the aggregate is shown in Table 2.3. Twelve hours after casting the model the shutters were removed and wet hessian continuously draped over the model for a period of thirty days prior to testing.

Details of the reinforcement incorporated in the 1:3 scale model are given in Figure 2.2c. Tests were carried out on sample 12mm deep beams reinforced as for the 1:25 scale model floor slabs, 150mm cubes and a range of cylinders and an average value of the modulus of elasticity for the micro-concrete of 33.84 N/mm^2 and a Poisson's ratio of 0.14 were found.

For the 1:3 scale model results from six 150mm cubes gave the follow-:ing values:

Crushing strength	=	20 N/mm^2
Modulus of elasticity	≃	35.00 N/mm^2
Poisson's Ratio	≃	0.20

A number of 5mm long, 120Ω, electrical resistance strain gauges were cemented to smoothed surfaces of the 1:25 scale models using catalyst activated cement. The positions of these gauges are shown in Figures 2.9a, 2.9b and 2.17. 20mm long, 120Ω, electrical resistance foil strain gauges were attached to the 1:3 scale model at smoothed parts of the slab's surfaces using catylyst activated cement at the positions shown in Figures

TABLE 2.1 DOLERITE AGGREGATE PARTICLE SIZE DISTRIBUTION

Sieve size, mm	Percentage passing
4.75	100
2.40	29
1.20	5
0.60	0

TABLE 2.2 SILICA SAND PARTICLE SIZE DISTRIBUTION

Sieve size, mm	Percentage passing
2.40	100
1.20	93
0.60	56
0.30	22
0.15	3.5

TABLE 2.3 AGGREGATE PARTICLE SIZE DISTRIBUTION

Sieve size, mm	Percentage by weight passing
20	-
10	100
5.0	30
2.36	20
1.18	16
.600	12
.300	4
.150	0

2.36 and 2.37. Identical strain gauges as those attached to the models were fixed to concrete beams cast at the same time as the models and these gauges were used as compensating gauges in the tests described below.

Once these gauges had been attached to the 1:25 scale model it was fixed to the rig as shown in Figure 2.1c.

2.4 Test procedure

2.4a 1: 25 scale model structure - 13-storeys

(i) Dynamic Tests:

A series of dynamic tests were carried out on this structure to determine its translational natural frequencies and mode shapes for the case when the structure was subjected only to a horizontal sinusoidal exciting force supplied by a 25 watt electro-magnetic actuator driven by a power amplifier and signal generator, and, subsequently, when the model was subjected to a vertical load by means of twin tie rods of steel channels as shown in Figures 2.5 - 2.6. The response of the model during these tests was monitored by means of four accelerometers, whose positioning on the model could be varied and whose signals were amplified through a calibrated system of head and charge amplifiers and recorded on tape by a multi-track FM tape recorder. A visual check during testing was also made by means of a two channel storage oscilloscope. The results of these tests are shown in Figures 2.7 and 2.8.

(ii) Static Tests

The strain gauges were now wired to Peekel constant resistance switch boxes and strain reader together with the compensating gauges attached to the small unstressed beams and a series of 0.01mm per division dial gauges fixed to a frame adjacent to the model positioned to monitor displacements of the model, including any slippage of the base. A translational lateral load was now applied at level 10 by means of a manually pumped long stroke hydraulic ram acting on a calibrated load cell which was placed between the end of the ram and the model. This load position was chosen since it roughly corresponds to the centre of gravity of a horizontal earthquake loading on a tall building. Records of the small base displacements of the model allowed their contribution to be deducted from the overall displacement of the model. These incremental load tests were carried out to a strict time schedule of two minutes between load applications to minimise the effects of creep of the concrete. At each increment of load the dial gauge readings were noted and the strain readings, corrected for temperature and humidity by the incorporation into the circuit of the dummy gauges attached to the 12mm thick unstressed reinforced concrete slabs, were also taken. These tests were repeated for a number of magnitudes of vertical loadings from 0 kN to 20 kN and the test results are given in Figure 2.10 From the above test results a vertical load of 20 kN was chosen and applied to the model which was then subjected to an elasto-plastic cyclic load test. The vertical load was applied as before by twin tie rods acting on a sandwich to two channels compressing two calibrated load cells which were used to monitor the vertical load acting on the model.

In the elasto-plastic test the model was cycled six times. It was loaded in small increments and when plastic deformations started the load was increased until the deflection of the top point of the model reached twice the deflection at yield. The load was then quickly reversed and was applied in increments as before until the deflection reached around three times the deflection at yield, and so on until failure. Cracking appeared in the walls near the base in the first cycle. For example, the results obtained from this particular model for the strain distribution in the floor slab (for slabs in the third and fourth floor) during the first and last cycles, are shown in Figures 2.11 - 2.13. Figure 2.14 shows a comparison between the maximum strains in levels 3 and 4 for the six cycles. In Figure 2.15 a comparison is given between three strain gauges placed as shown in Figure 2.9b.

2.4b 1:25 scale model structure - 9-storeys

Examination of the 13-storey model upon completion of the above tests revealed that cracking was confined to the first two storeys and it was decided to cast the model into a new R.C. base to form a 9-storey model for a second series of tests. No dynamic tests were carried out but an extensive number of strain gauges were attached to the slab at level 2 of the model - see Figure 2.17. A series of dial gauges were placed to measure the model deflections due to translational lateral load applied at level 7. The load was applied in small increments as for the previous model. The same precautions to minimize the effects of creep and to correct the strain readings for humidity and temperature changes were taken. A vertical load of 20 kN was chosen and applied to the model as before. The model was then subjected to an elasto-plastic cyclic load test in the same way as the previous model. In this case the model was cycled nine times. Figures 2.18 and 2.19 show the load deflection relationship for the dial gauges at top level and at level 2. Cracking again appeared in the walls near the case in the third cycle. Figures 2.20 - 2.33 indicate, as examples, some of the results obtained during the first and fourth cycles.

2.4c 1:3 scale model

In order to test this model a very heavy rigid space frame of 203 x 203mm x 86kg/m (Figure 2.4) universal column sections was welded together and positioned as shown in Figure 2.34, with its base bolted through the cellular strong floor. The model was fixed to two 40mm diameter pivot spindles housed in steel tubes, which in turn were held in position in the model by means of additional reinforcement. The model was placed on its side on a number of machined steel rollers and loads applied horizontally, as shown in Figure 2.35, by means of a hand pumped large hydraulic ram acting on a load cell attached to a heavy triangulated steel loading bracket, also bolted through the strong floor. Strains and move-:ments of the model were monitored by means of the system of 0.01mm per division dial gauges and the grid of strain gauges as shown in Figures 2.36 - 2.39, and compensating gauges wired to Peekel constant resistance switch boxes and manually balanced strain reader. The load acting through the calibrated load cell was obtained using a digital voltmeter powered by a constant voltage supply. A series of low strain tests were carried out to determine the action of the floor slabs and also to detect any movements or faults in the test system. Once the low strain tests had been completed the model was then subjected to an elasto-plastic cyclic load test for eight cycles before it was finally subjected to repeated loading from one direction until it could not stand any more of this kind of loading. Cracking started in the slabs at the end of the low strain tests when the load was between 3.5 and 3.75 tons. The elasto-plastic tests were carried out in the same way as for the previous two models with the model loaded in increments at two minute intervals until it reached twice the yield, the load then quickly reversed and applied until the deflection of the model reached three times the deflection at yield, and so on. At each load increment the strains of the model were recorded and deflections of the model and supporting structure noted. The record of the movements of the support structure allowed their contribution to the overall structural displacements to be found and adjustments to be made.

Additional sets of results for the three models are given in Appendix 1.

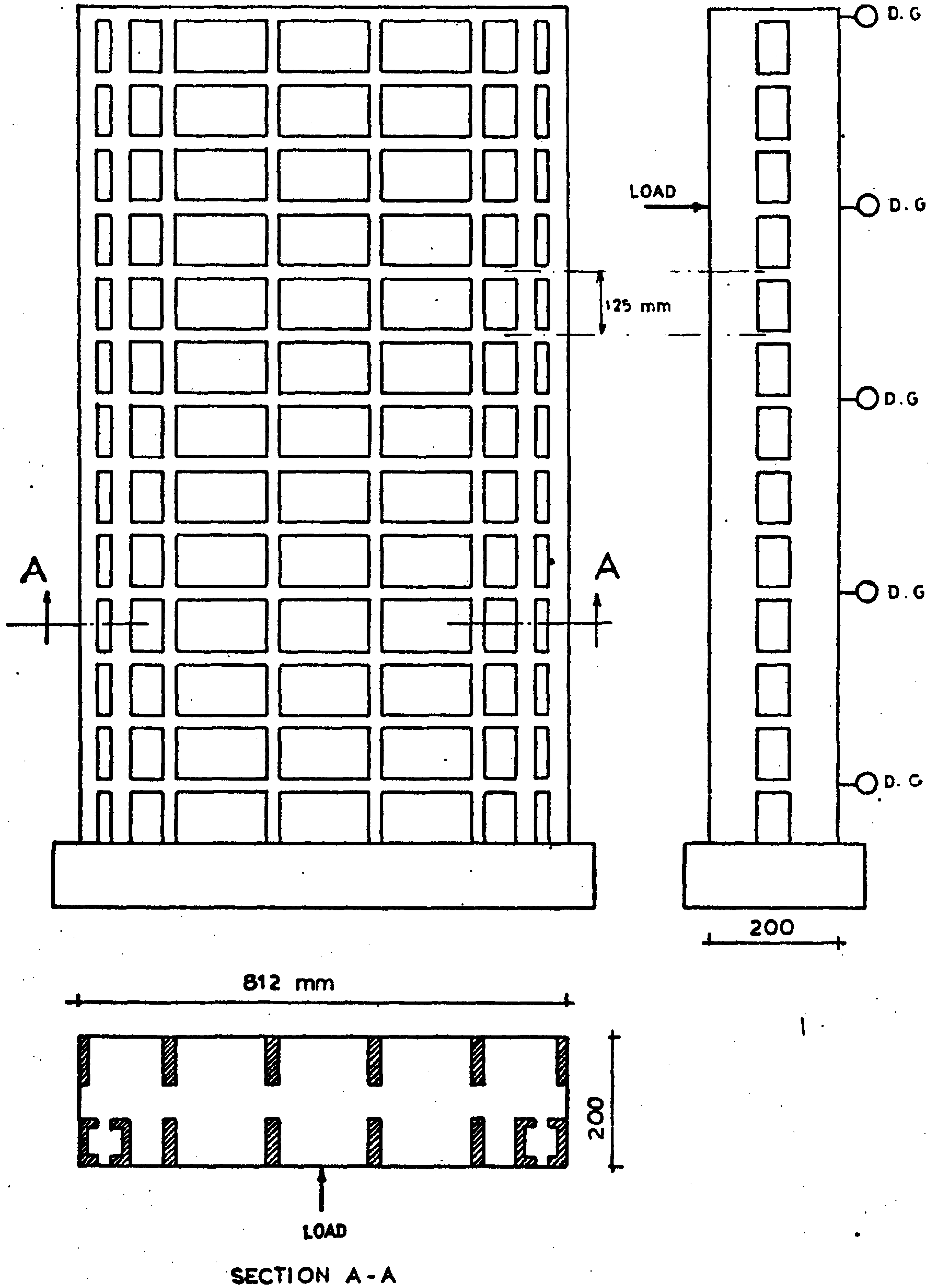
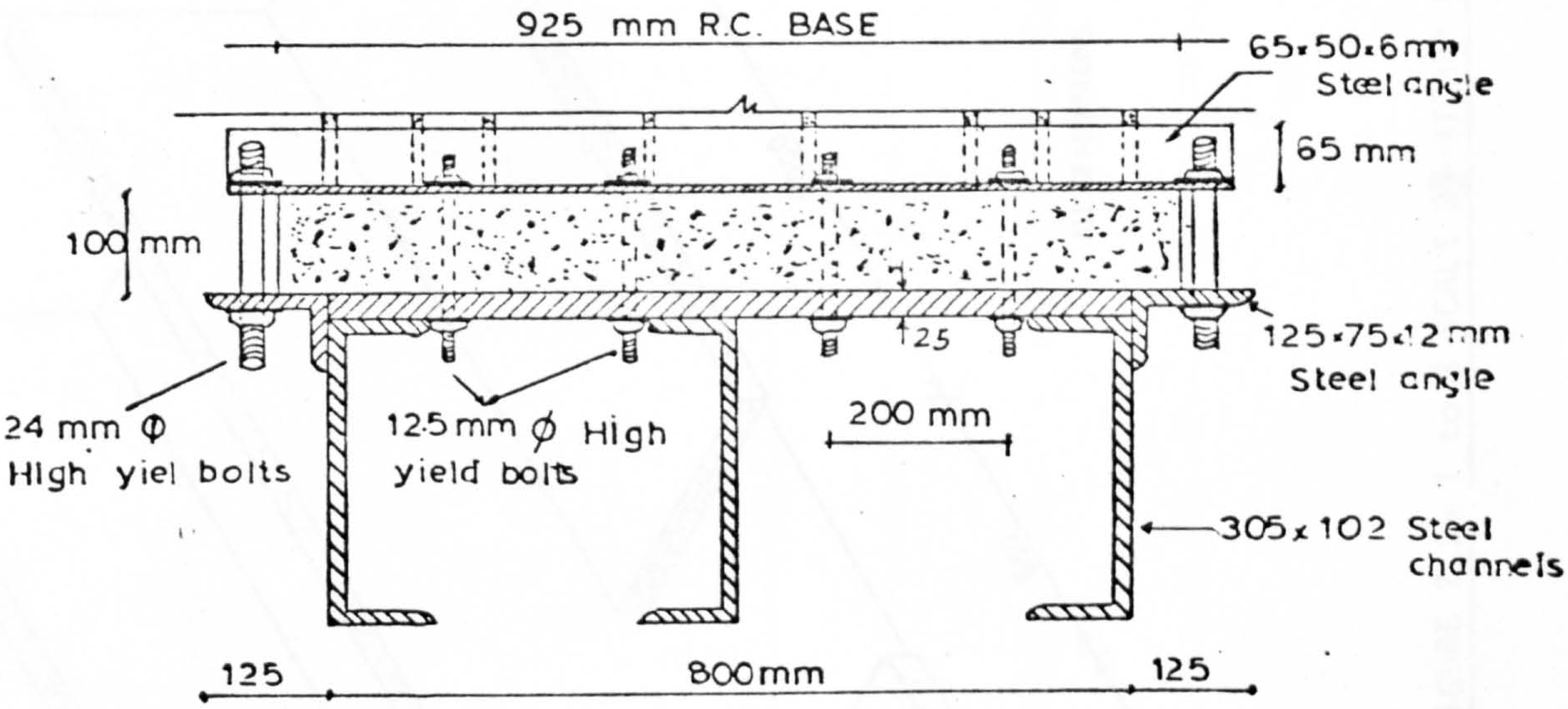
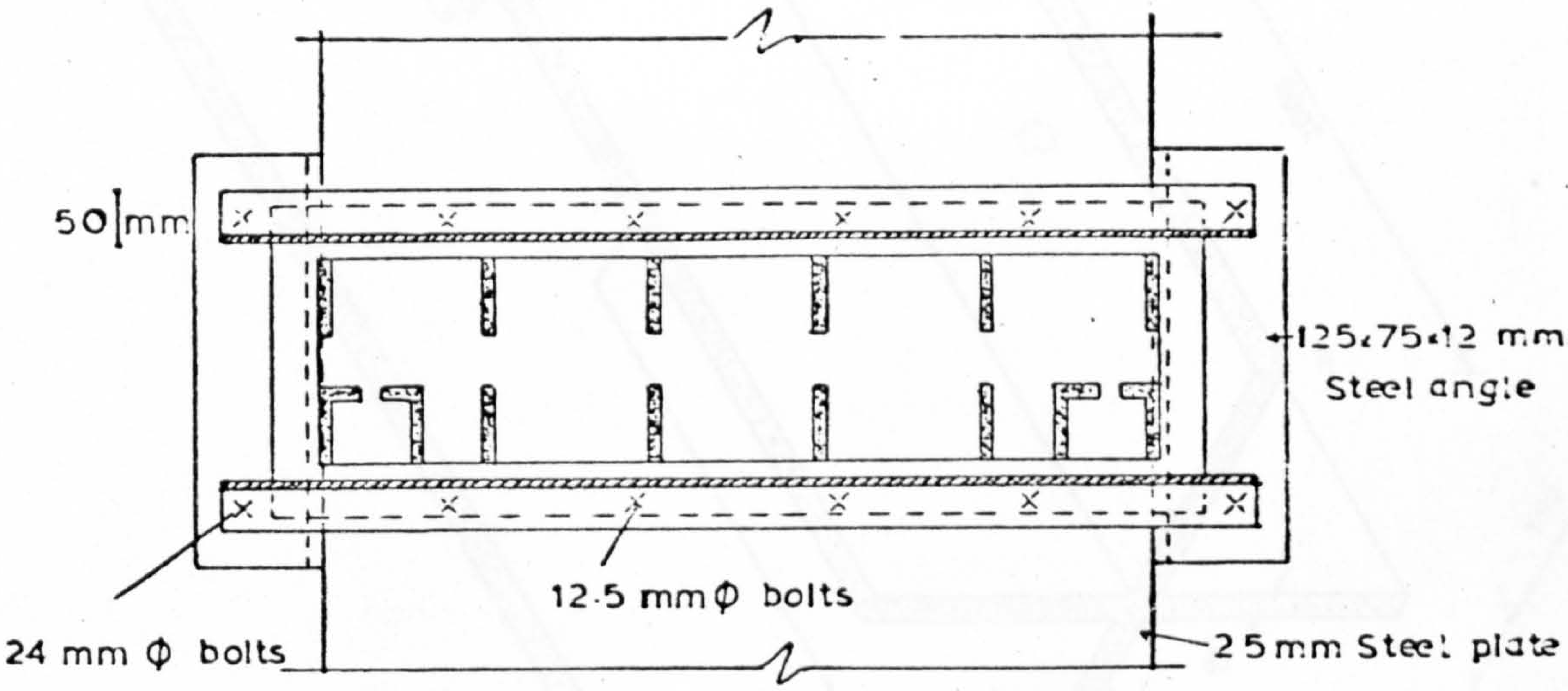


FIGURE 2.1a 13-STOREY 1 to 25 SCALE MODEL



a- ELEVATION



b. PLAN

FIGURE 2.1c BASE FIXING FOR SMALL MODELS

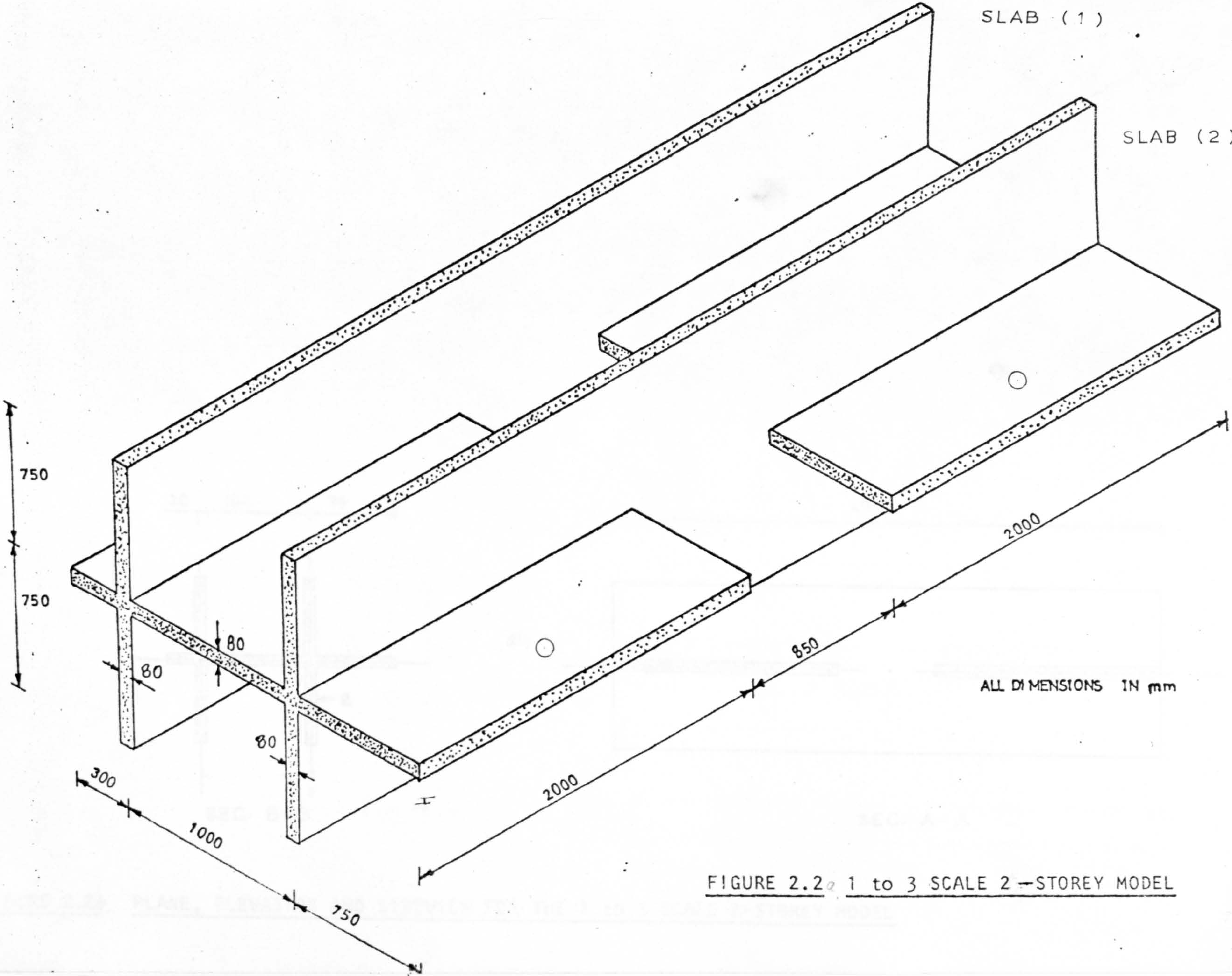


FIGURE 2.2a 1 to 3 SCALE 2-STOREY MODEL

ALL DIMENSIONS IN Cm

Scale 1: 50

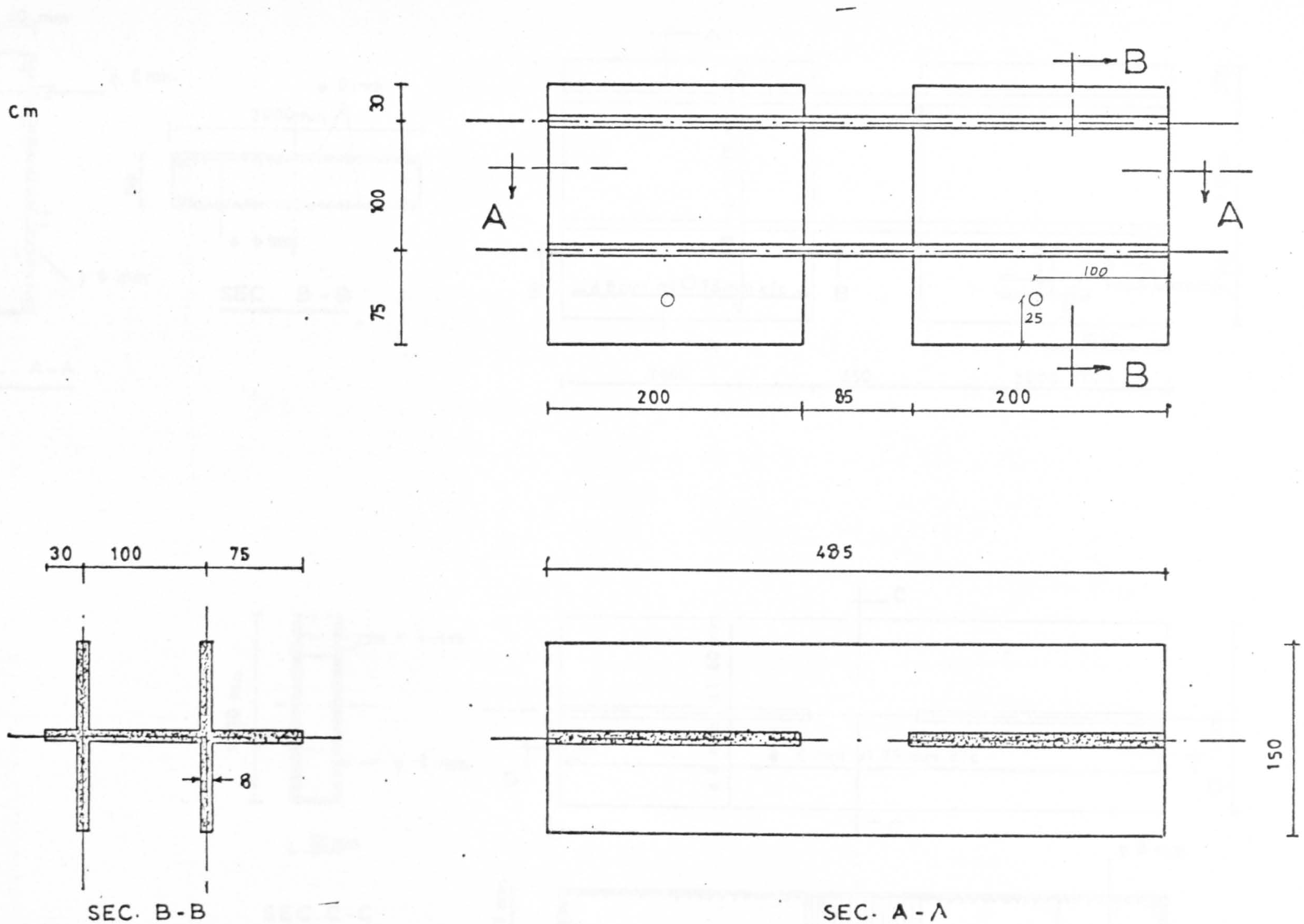


FIGURE 2.26 PLANE, ELEVATION AND SIDEVIEW FOR THE 1 to 3 SCALE 2-STOREY MODEL

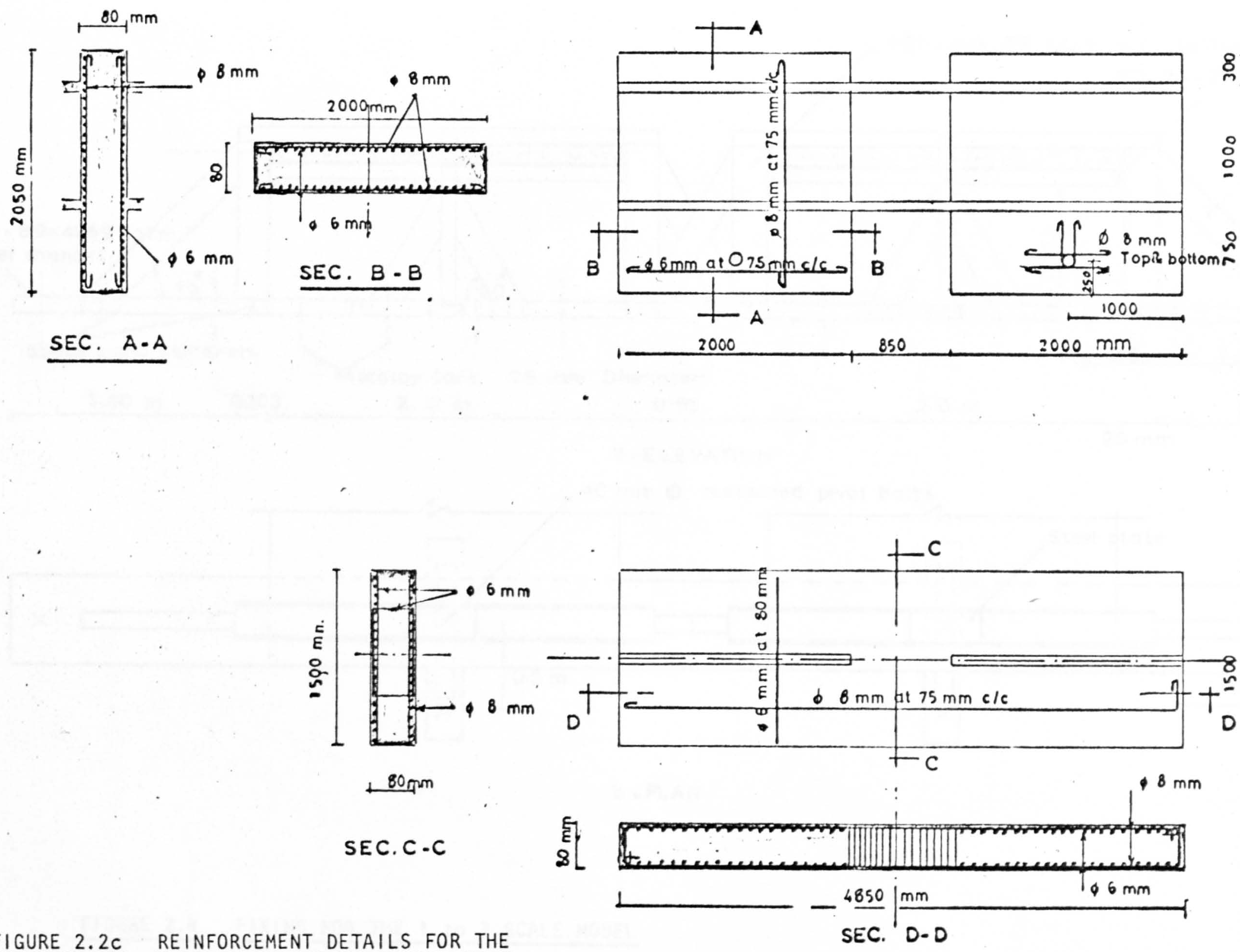


FIGURE 2.2c REINFORCEMENT DETAILS FOR THE 1 to 3 SCALE MODEL

FIGURE 2.4 FIXING FOR THE 1 to 3 SCALE MODEL

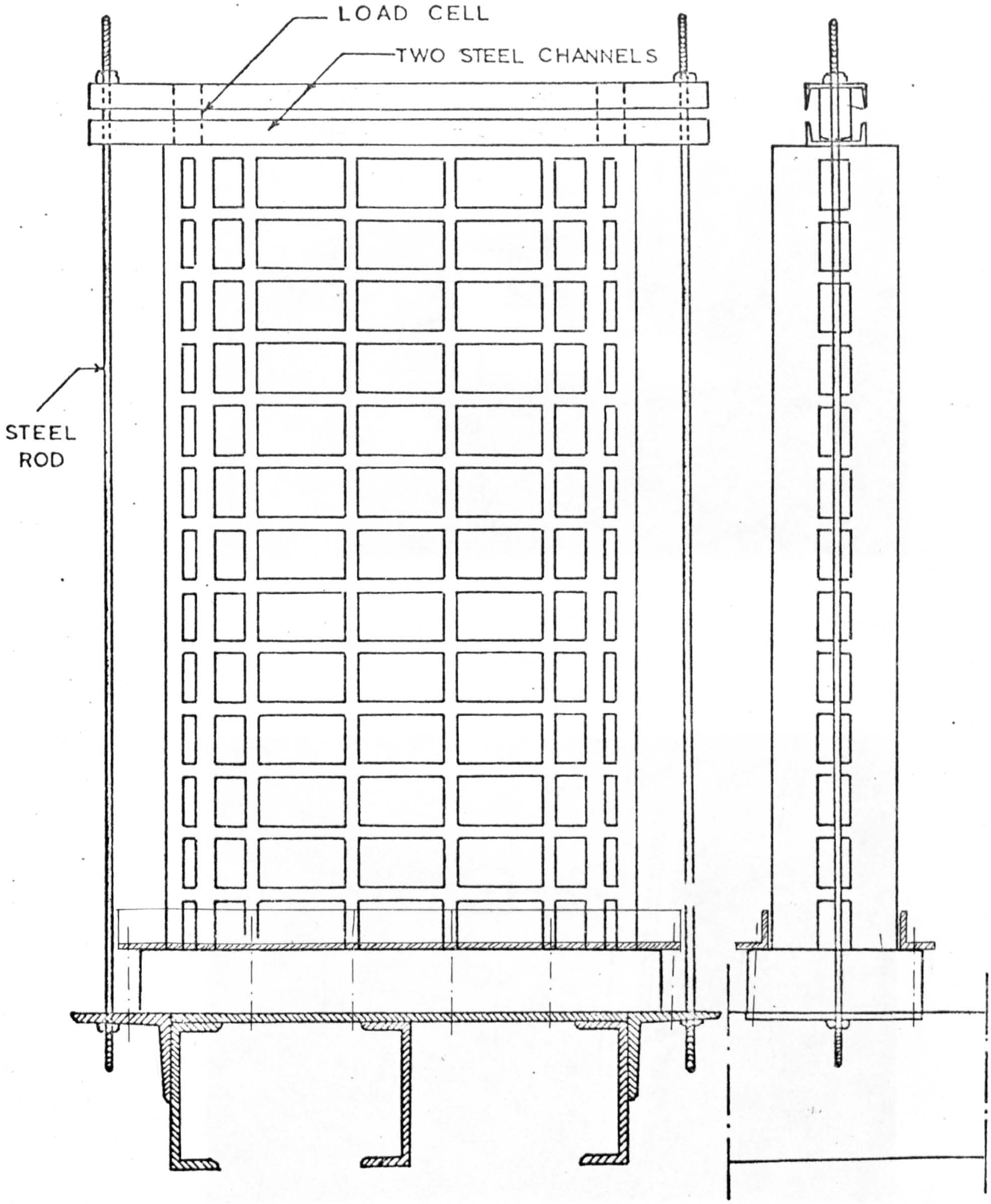


FIGURE 2.5 APPLICATION OF VERTICAL LOAD (1:25 SCALE MODELS)

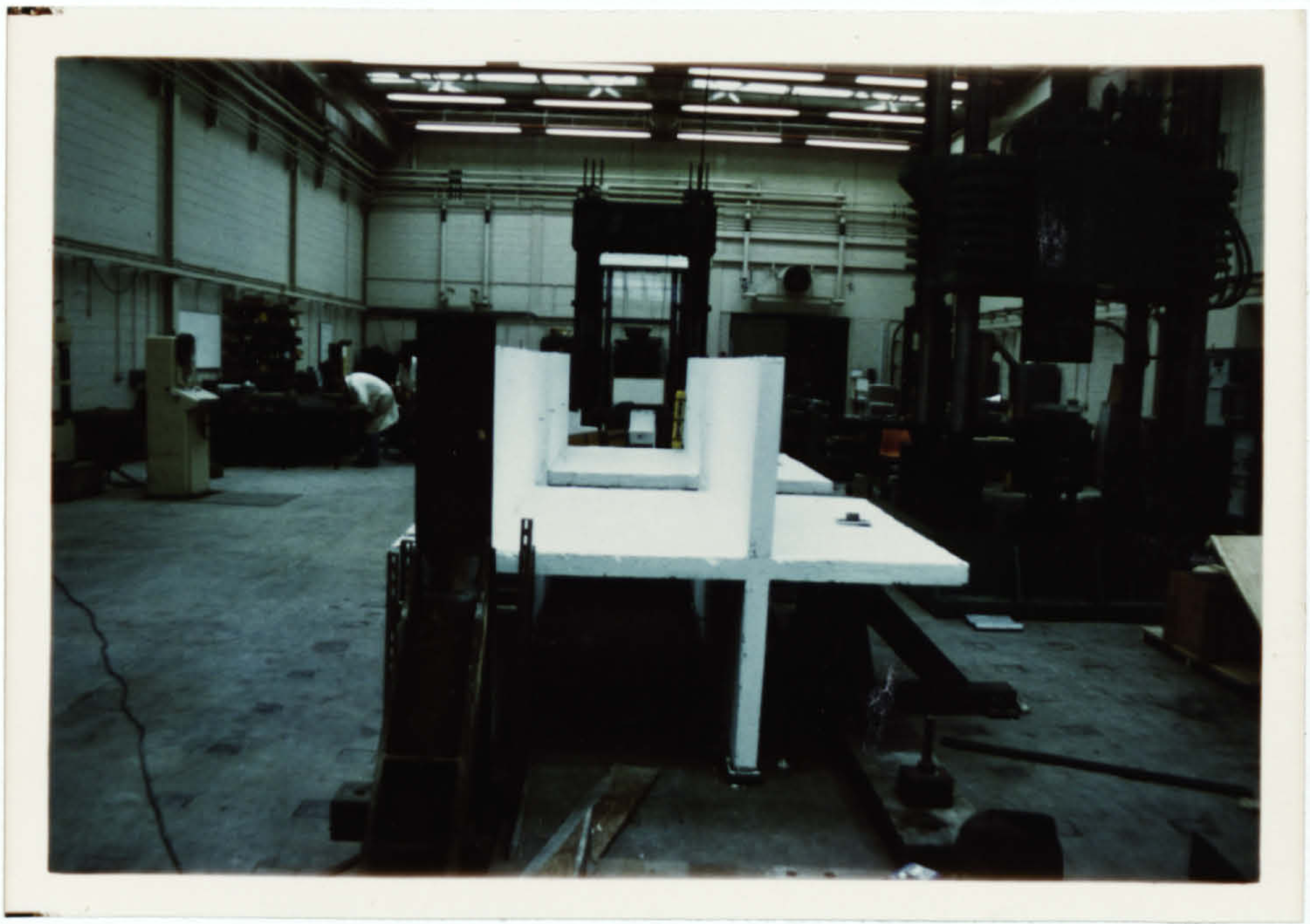


FIGURE 2.3 ONE-THIRD FULL SCALE MODEL (PRELIMINARY TESTS WITH FIRST STAGE SUPPORTS)

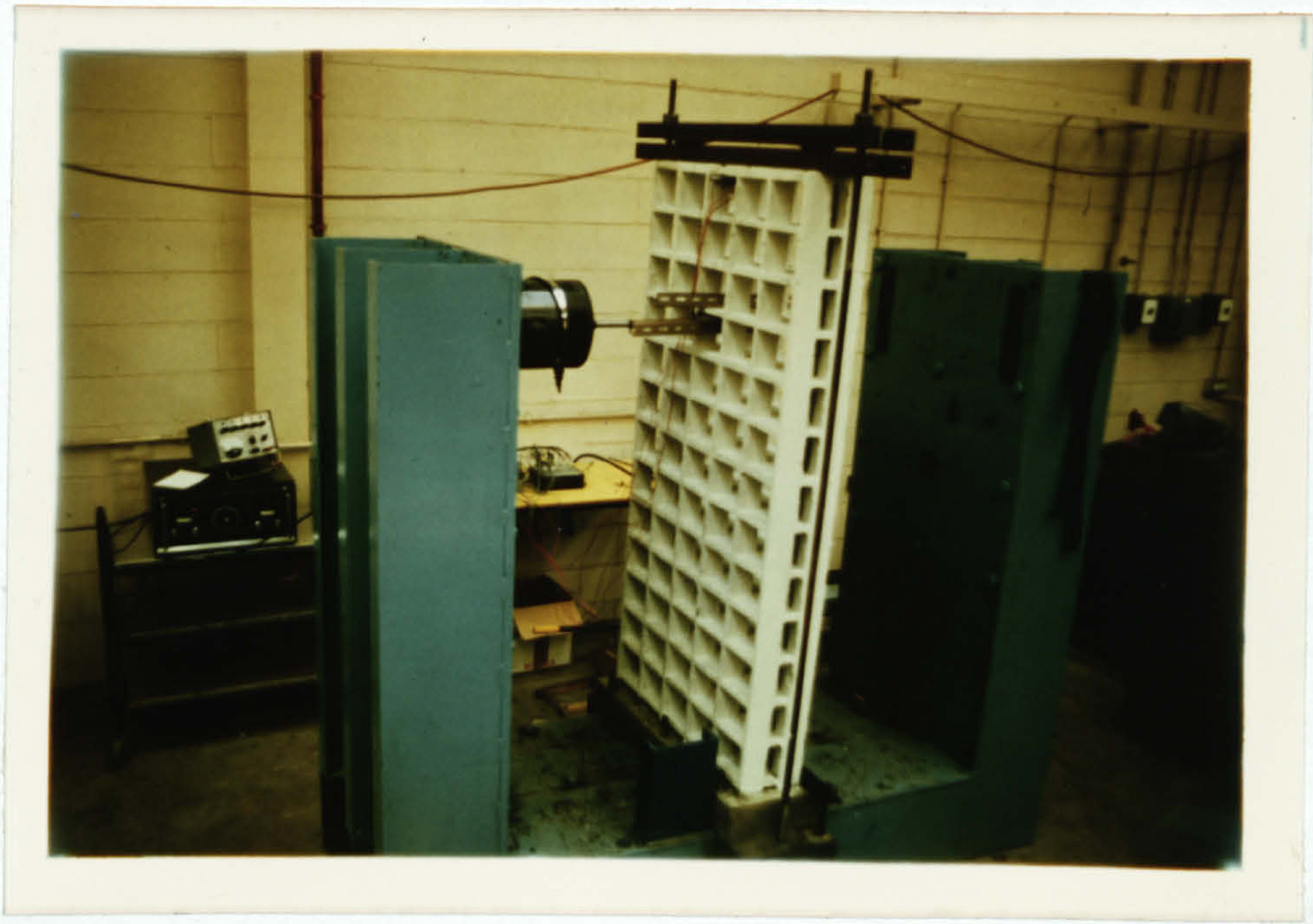


FIGURE 2.6 1:25 SCALE 13-STOREY MODEL SUBJECT TO DYNAMIC AND VERTICAL LOADING

FIGURE 2.7(a) RESPONSE OF MODEL TO SINUSOIDAL INPUT

FIGURE 2.7(b). RESPONSE OF MODEL, TO SINUSOIDAL INPUT

FIGURE 2.8(a) RESPONSE OF MODEL TO SINUSOIDAL INPUT

FIGURE 2.8(b) RESPONSE OF MODEL TO SINUSOIDAL INPUT

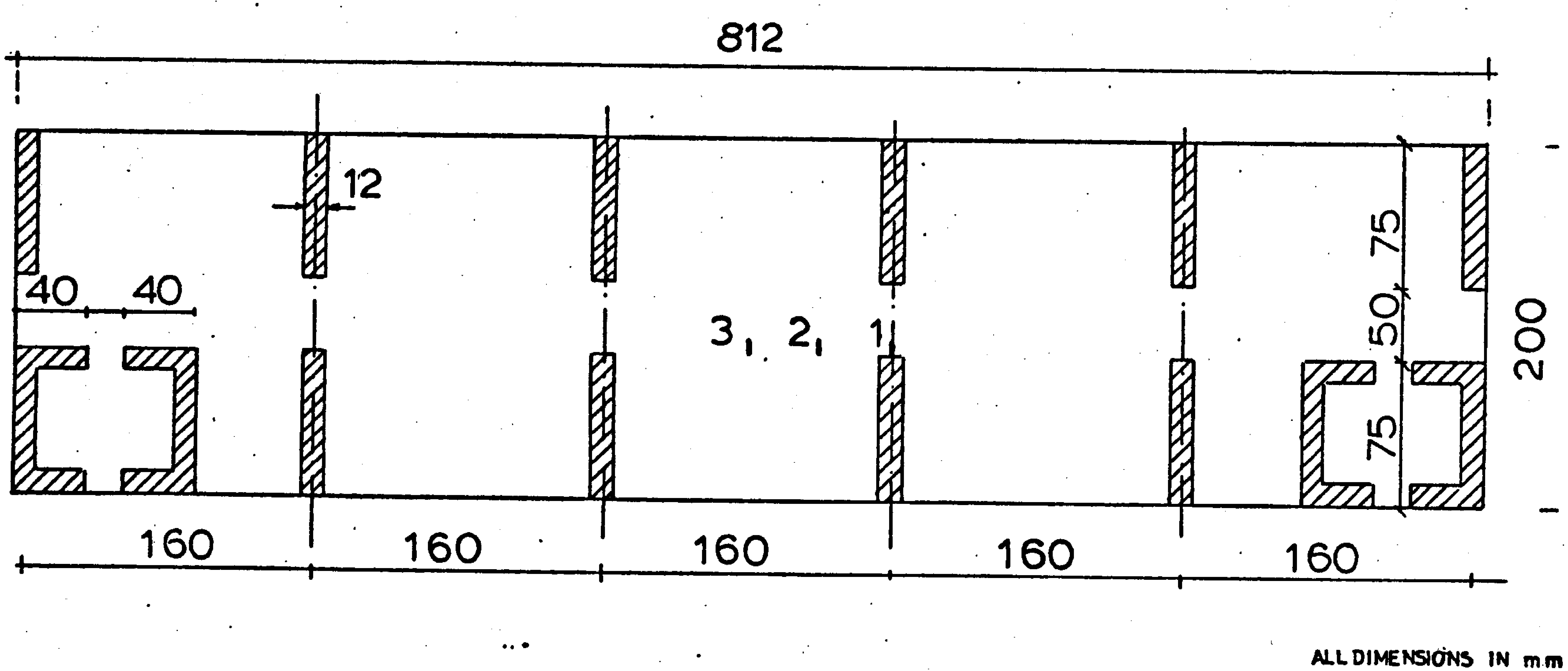


FIGURE 2.9(a) STRAIN GAUGES IN THE FOURTH LEVEL (13-STOREY MODEL)

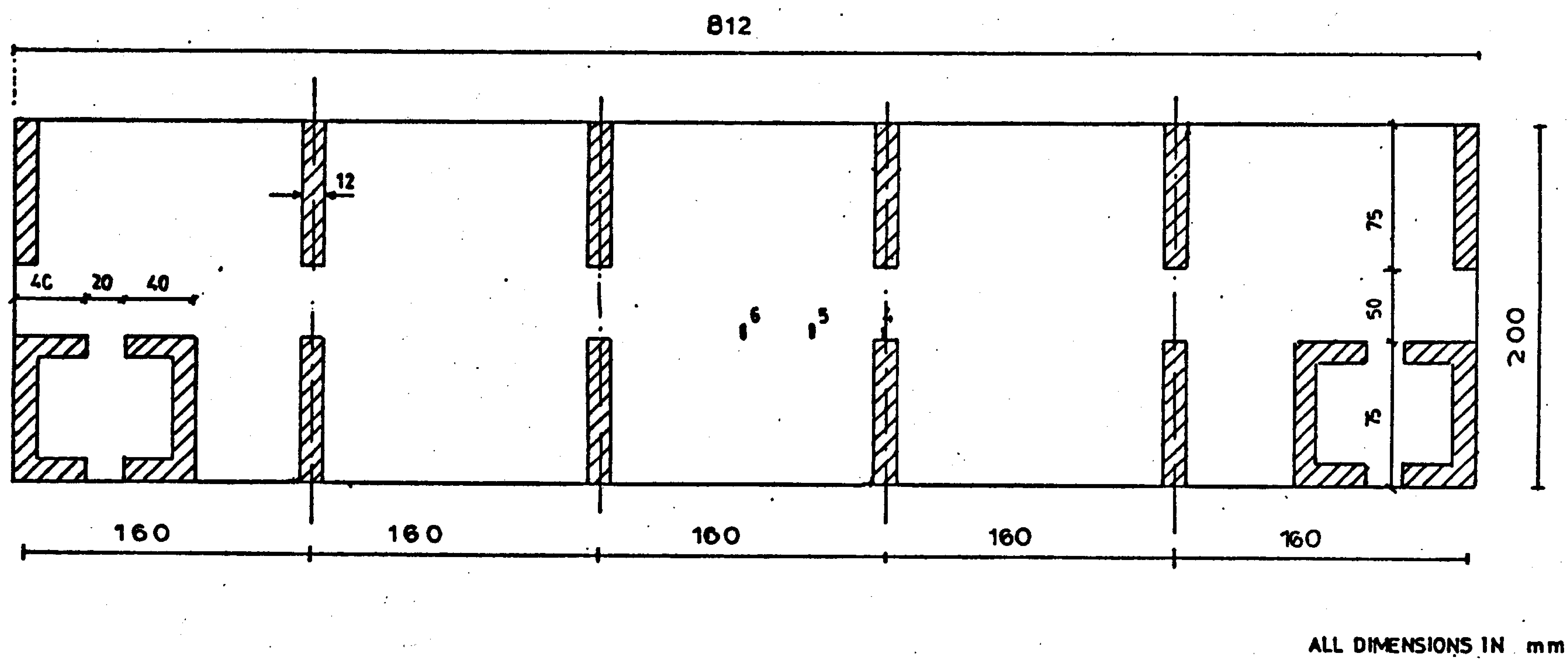


FIGURE 2.9(b) STRAIN GAUGES IN THE THIRD LEVEL (13-STOREY MODEL)

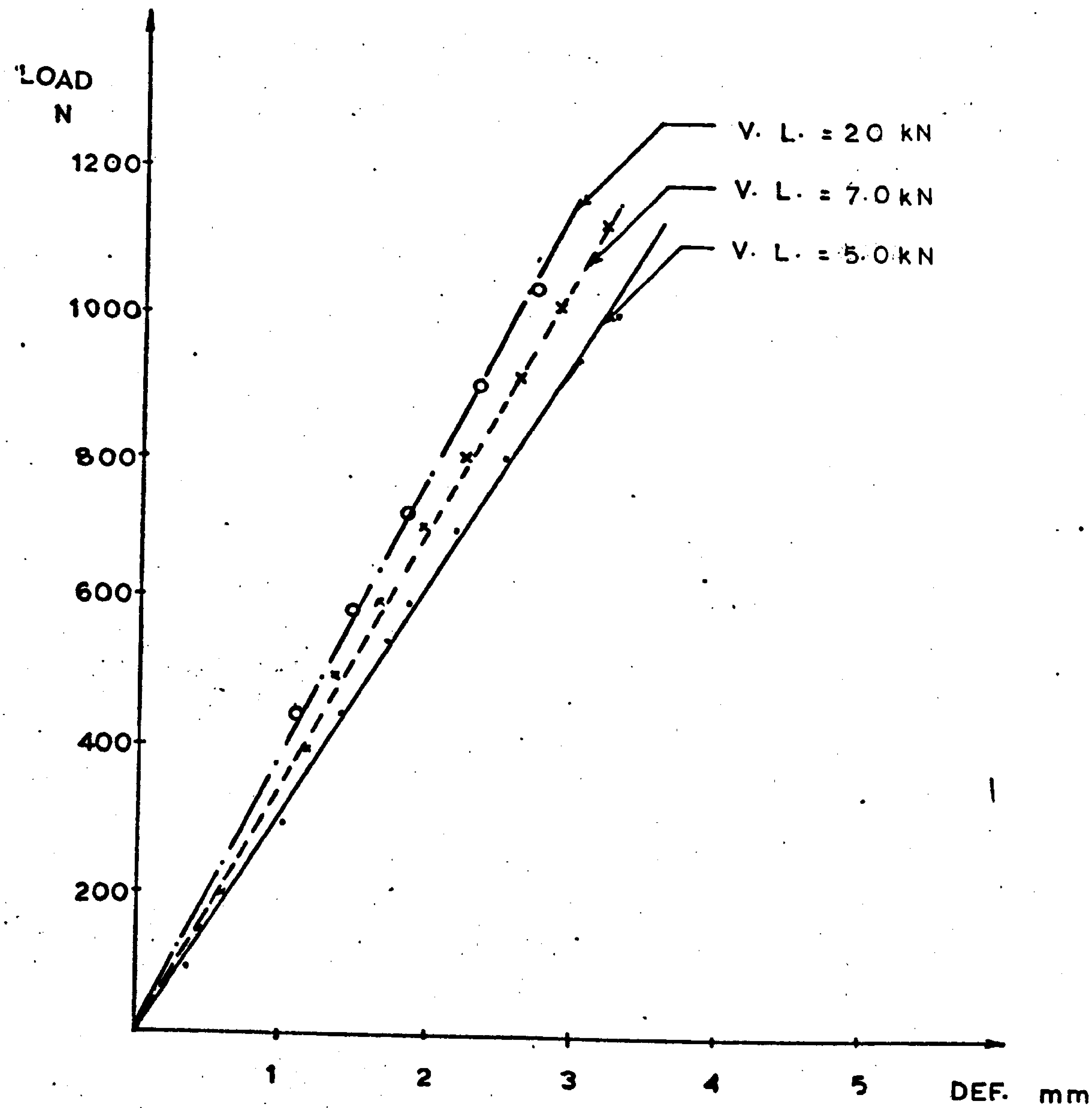


FIGURE 2.10 THE EFFECT OF DIFFERENT VERTICAL LOAD VALUES ON THE DEFLECTION OF THE TOP POINT (13-STOREY MODEL)

FIGURE 2.11 STRAIN DISTRIBUTION ALONG THE LINE A-A WHILE THE MODEL STILL ELASTIC (13-STOREY MODEL)

FIGURE 2.12 STRAIN DISTRIBUTION DURING LAST LOADING (13-STOREY MODEL)

FIGURE 2.13 STRAIN DISTRIBUTION DURING LAST LOADING (13-STOREY MODEL) CONT.

FIGURE 2.14 COMPARISON BETWEEN THE MAXIMUM STRAINS IN LEVELS 3 AND 4 (13-STOREY MODEL)

FIGURE 2.15 A COMPARISON BETWEEN THE THREE STRAIN GAUGES SHOWN (13-STOREY MODEL)

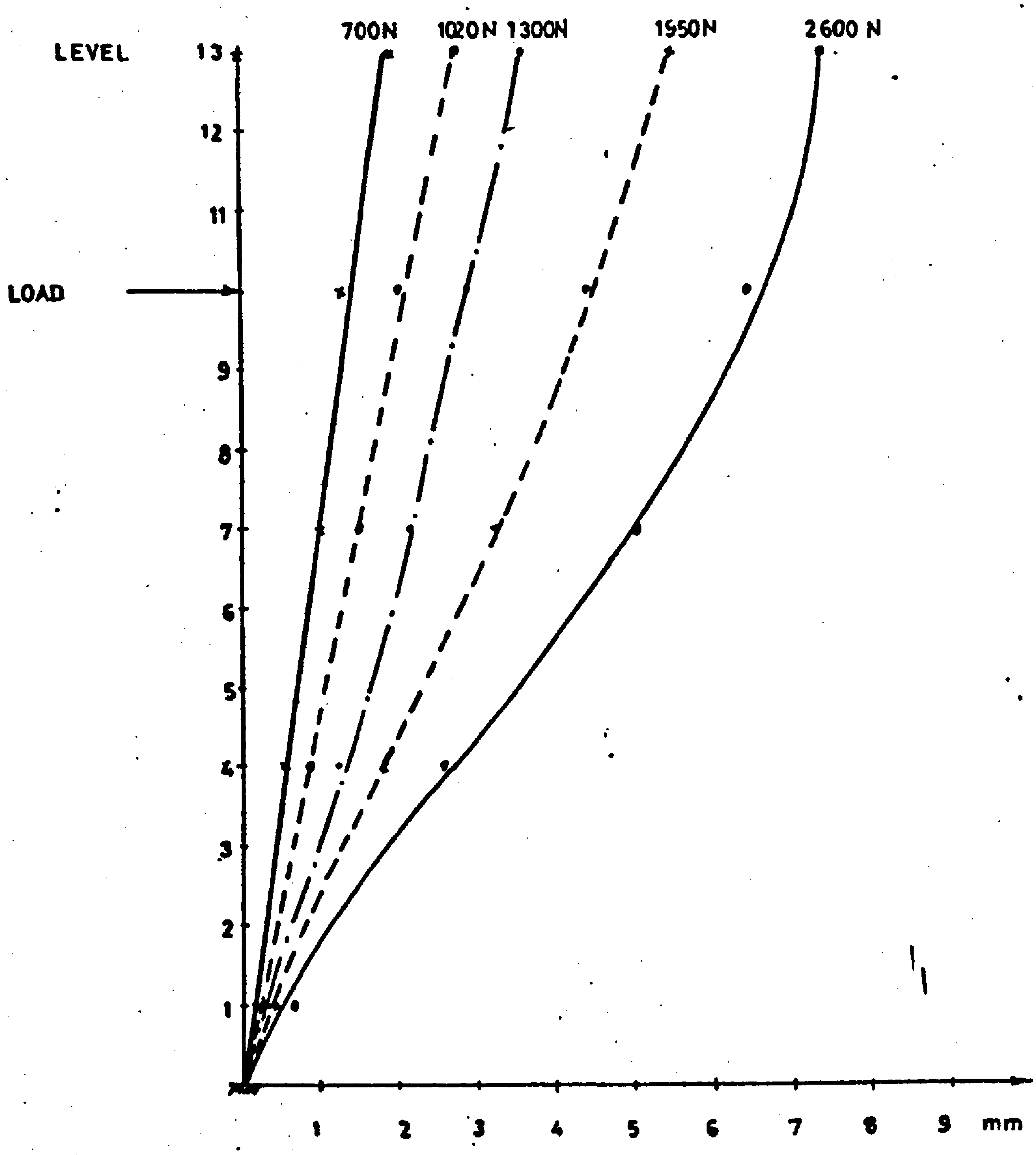


FIGURE 2.16 DEFLECTION OF THE 13-STOREY MODEL SUBJECT TO POINT LOAD

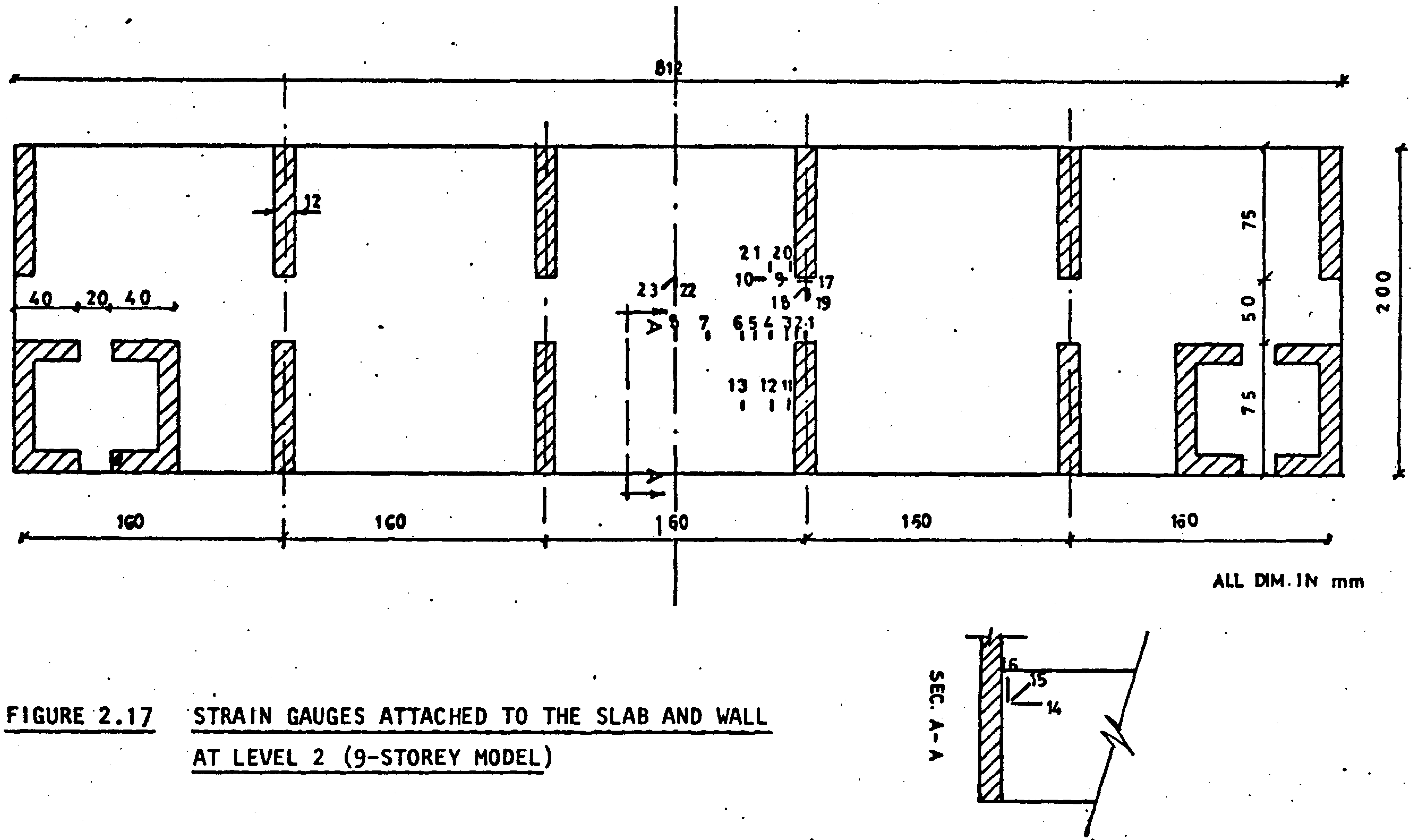


FIGURE 2.17 STRAIN GAUGES ATTACHED TO THE SLAB AND WALL AT LEVEL 2 (9-STOREY MODEL)

FIGURE 2.18 LOAD DEFLECTION CYCLE FOR DIAL GAUGE AT TOP LEVEL (9-STOREY MODEL)

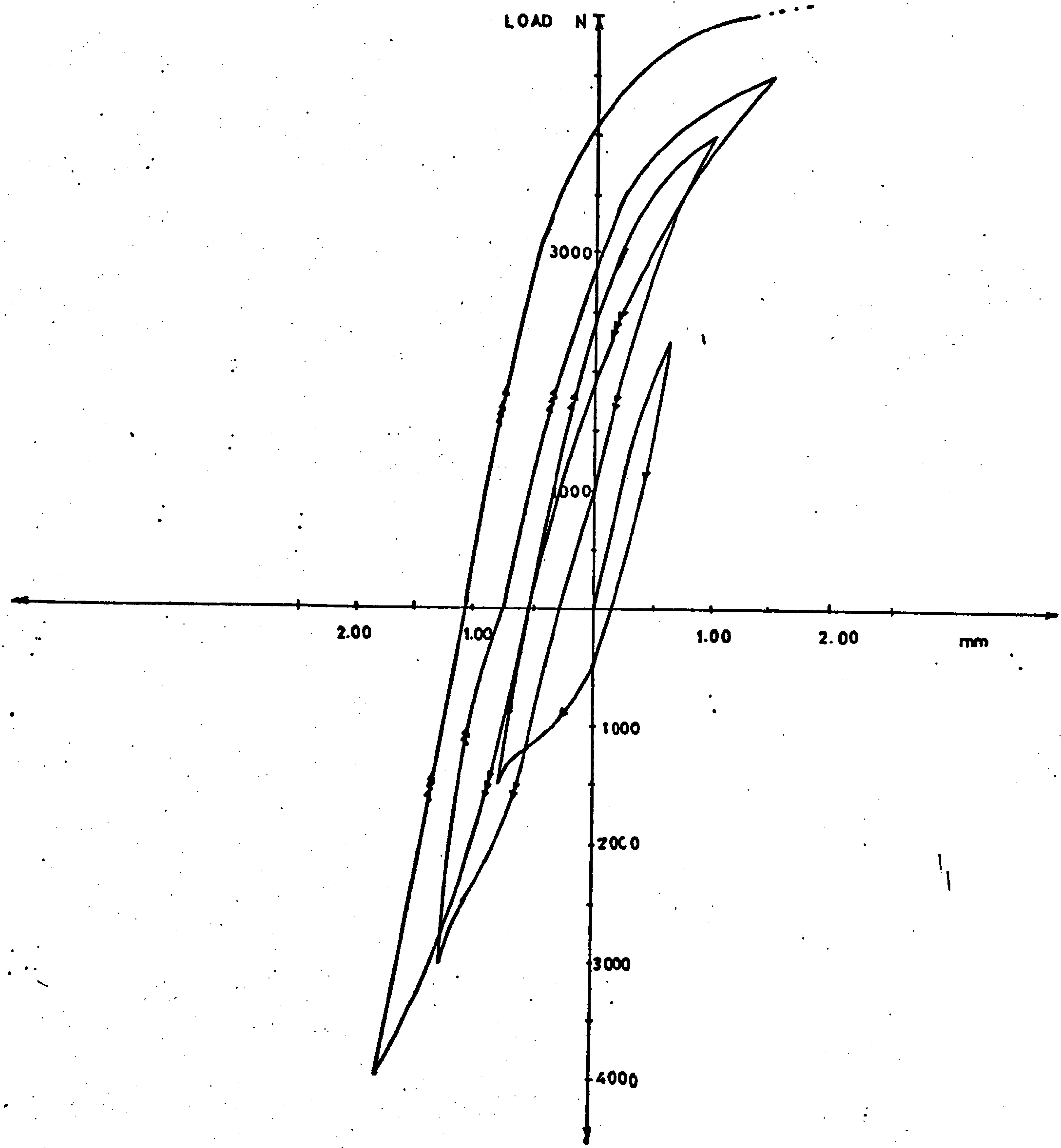


FIGURE 2.19 LOAD DEFLECTION CYCLES FOR DIAL GAUGE AT LEVEL 2 (9-STOREY MODEL)

FIGURE 2.20 LOAD STRAIN CYCLES FOR STRAIN GAUGE No. 1

(9-STOREY MODEL)

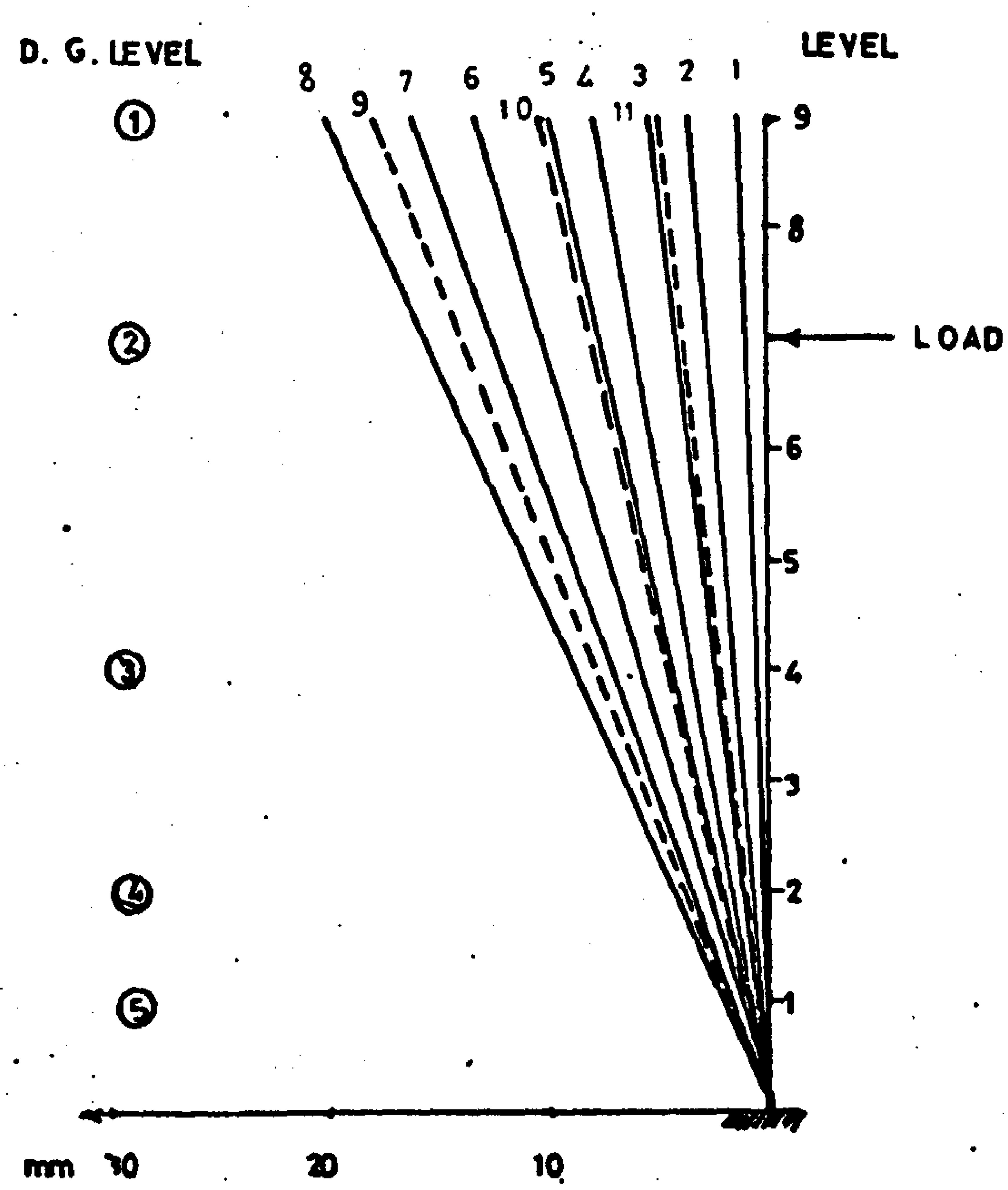


FIGURE 2.21 DEFLECTIONS OF MODEL SUBJECT TO POINT LOAD

(9-STOREY MODEL)

FIGURE 2.22 STRAIN DISTRIBUTION DURING FIRST LOADING (9-STOREY MODEL)

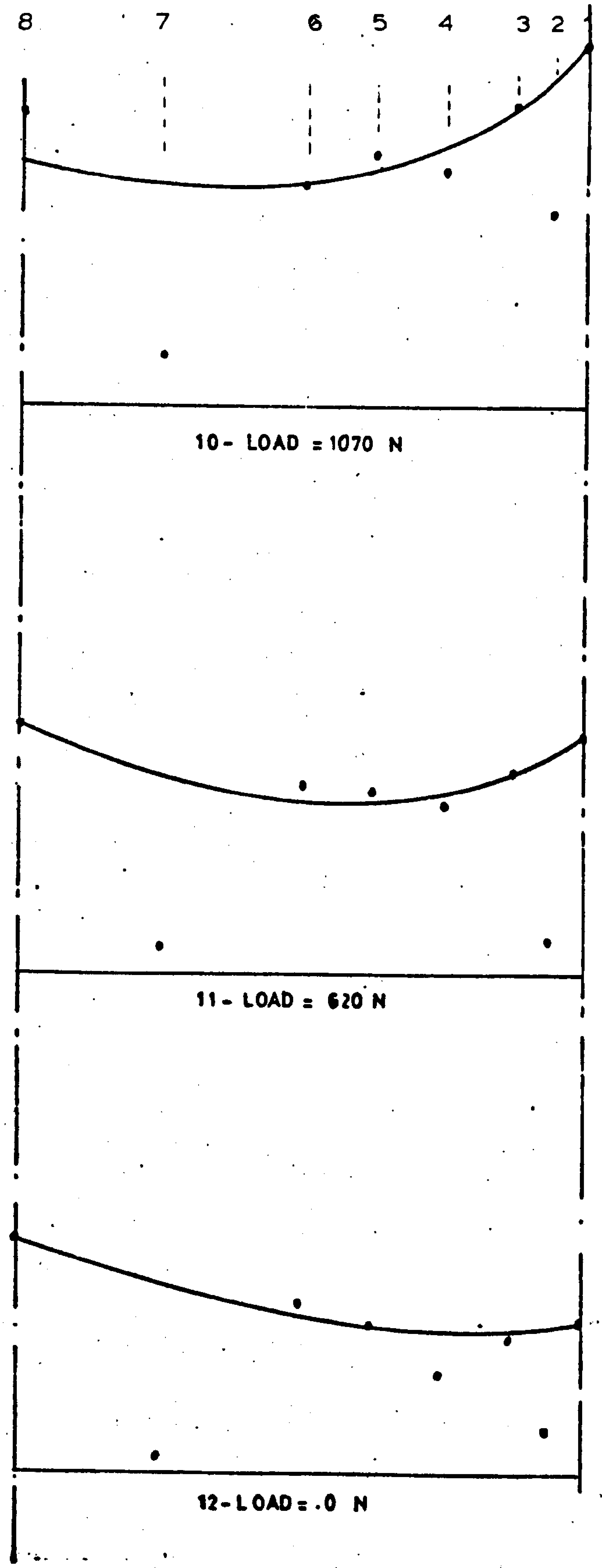


FIGURE 2.23 STRAIN DISTRIBUTION DURING FIRST LOADING (CONT.)
(9-STOREY MODEL)

FIGURE 2.24 STRAINS OF VARIOUS POINTS DURING FIRST LOADING (9-STOREY MODEL).

FIGURE 2.25 STRAINS OF VARIOUS POINTS DURING FIRST LOADING (9-STOREY MODEL)

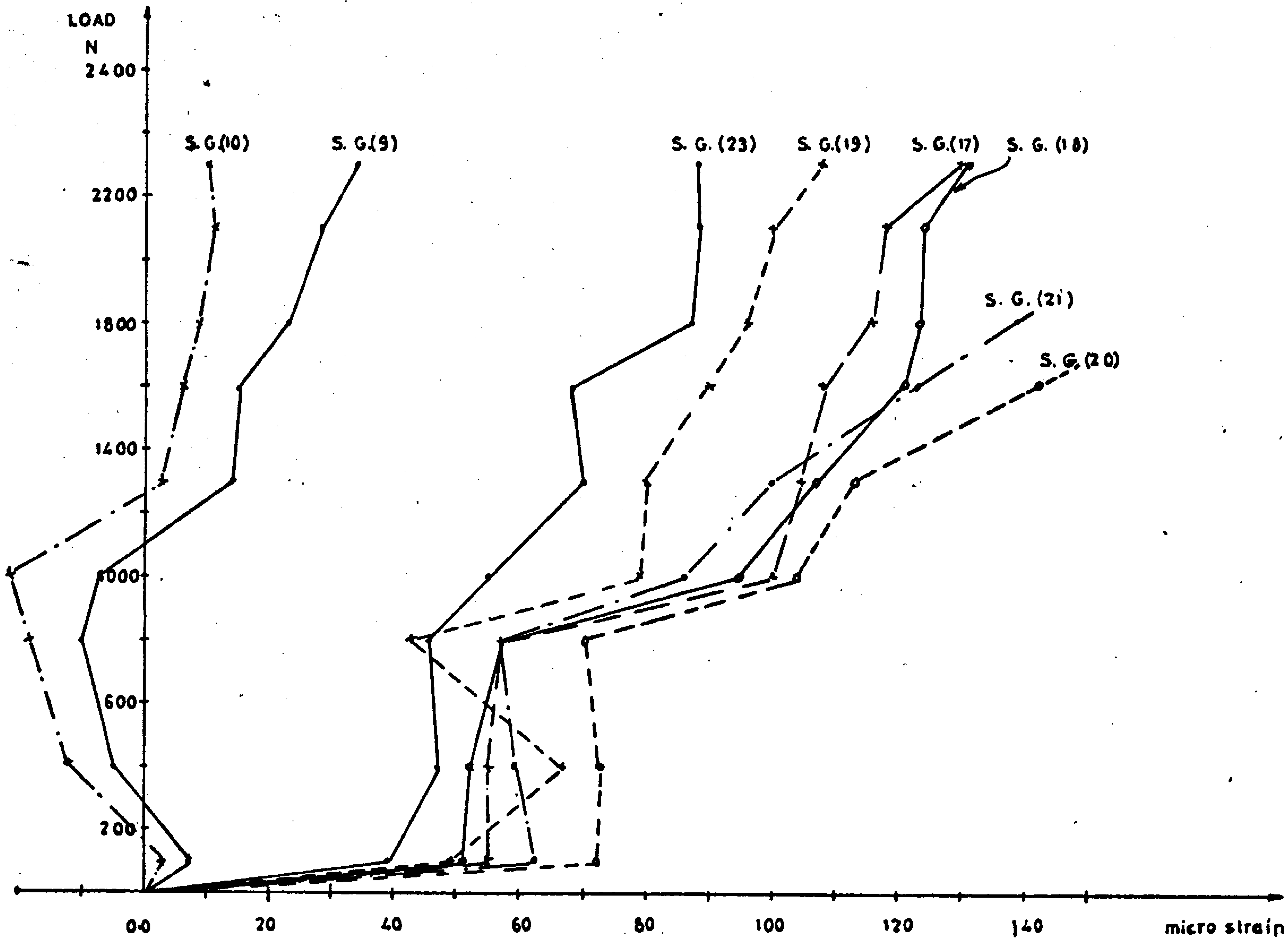


FIGURE 2.26 STRAINS OF VARIOUS POINTS DURING FIRST LOADING (9-STOREY MODEL)

FIGURE 2.27 DEFLECTION OF MODEL DURING FIRST LOADING (9-STOREY MODEL)

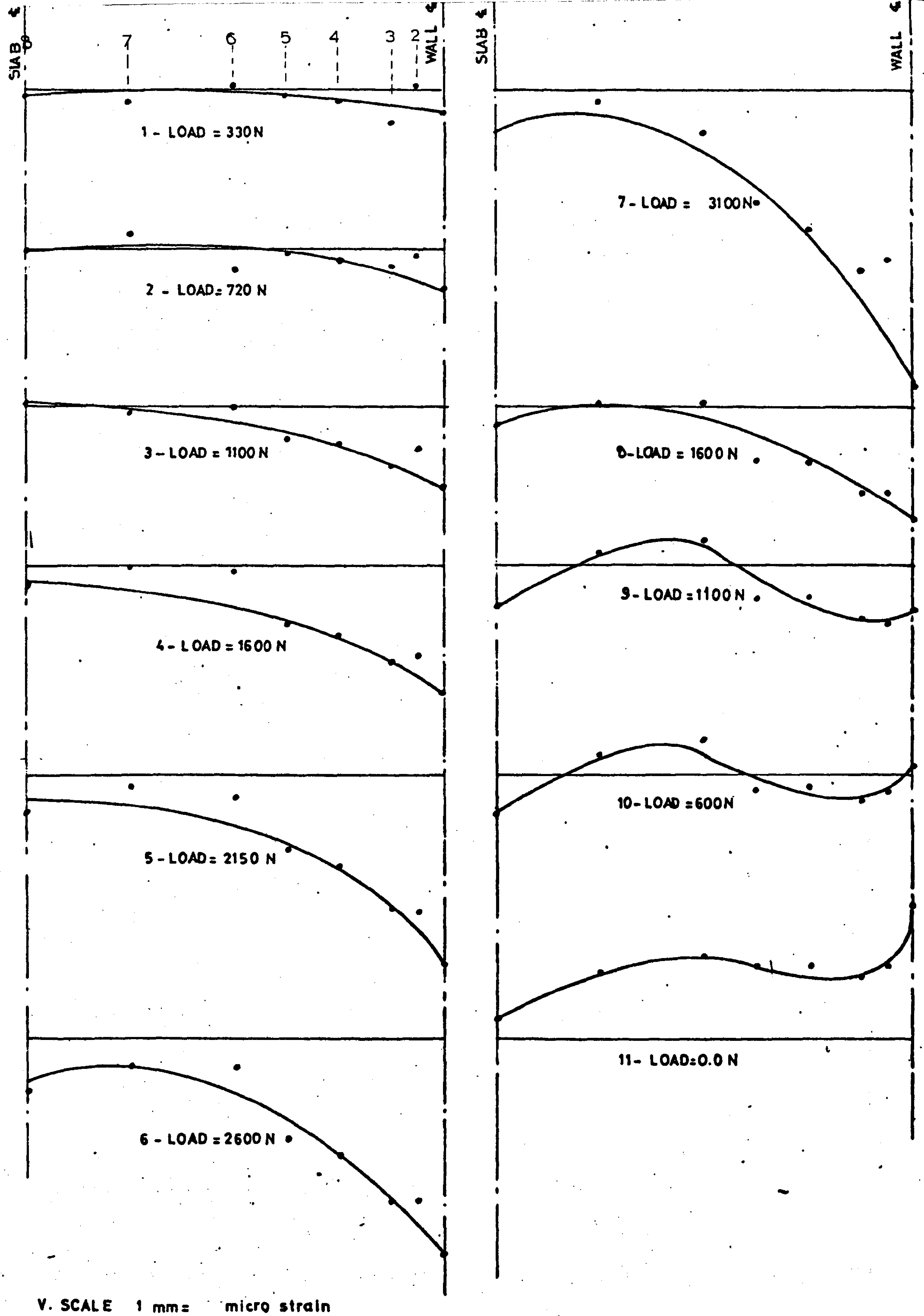


FIGURE 2.28 STRAIN DISTRIBUTION DURING FOURTH LOADING (9-STOREY MODEL)

FIGURE 2.29 STRAINS OF VARIOUS POINTS DURING FOURTH LOADING (9-STOREY MODEL)

FIGURE 2.30 STRAINS OF VARIOUS POINTS DURING FOURTH LOADING (9-STOREY MODEL)

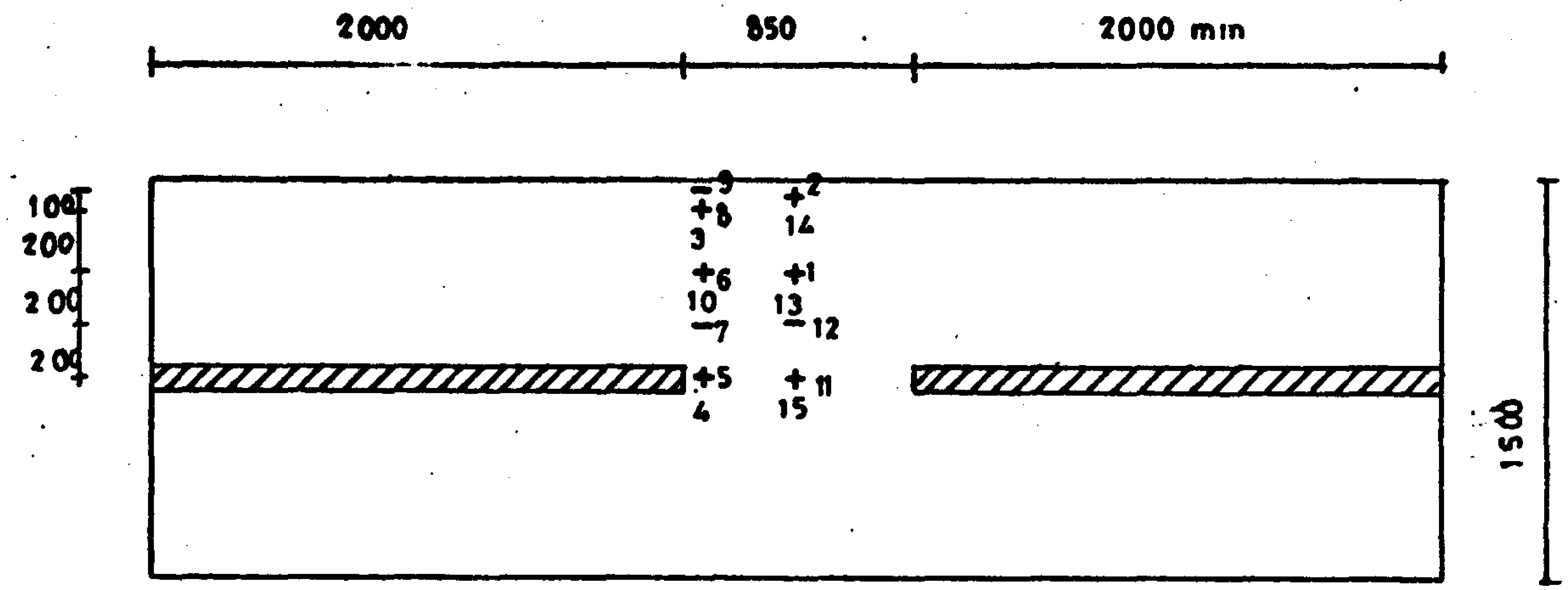
FIGURE 2.31 STRAINS OF VARIOUS POINTS DURING FOURTH LOADING (9-STOREY MODEL)

FIGURE 2.32 STRAINS OF VARIOUS POINTS DURING FOURTH LOADING (9-STOREY MODEL)

FIGURE 2.34 1:3 SCALE MODEL AND THE HEAVY RIGID FRAME

FIGURE 2.35 APPLICATION OF LOADS (SHOWING ONE LOAD BRACKET)

FIGURE 2.33 LOAD-DEFLECTION RELATIONSHIP FOR POINTS AT VARIOUS LEVELS DURING FOURTH LOADING (9-STOREY MODEL)



– HORIZONTAL

+ VERTICAL & HORI.

FIGURE 2.36 STRAIN GAUGES GRID AT SLAB 1 (2-STOREY MODEL)

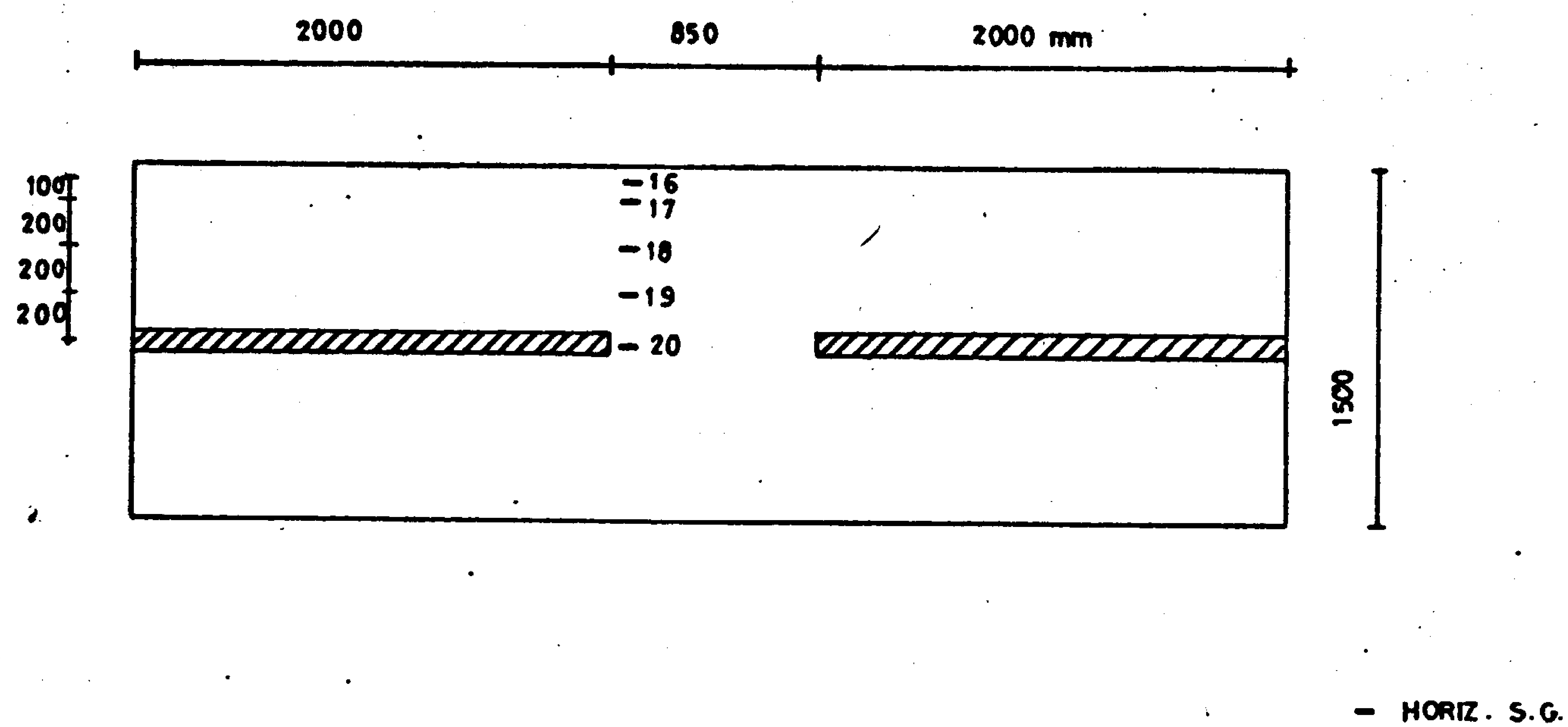


FIGURE 2.37 STRAIN GAUGES AT SLAB 2 (2-STOREY MODEL)

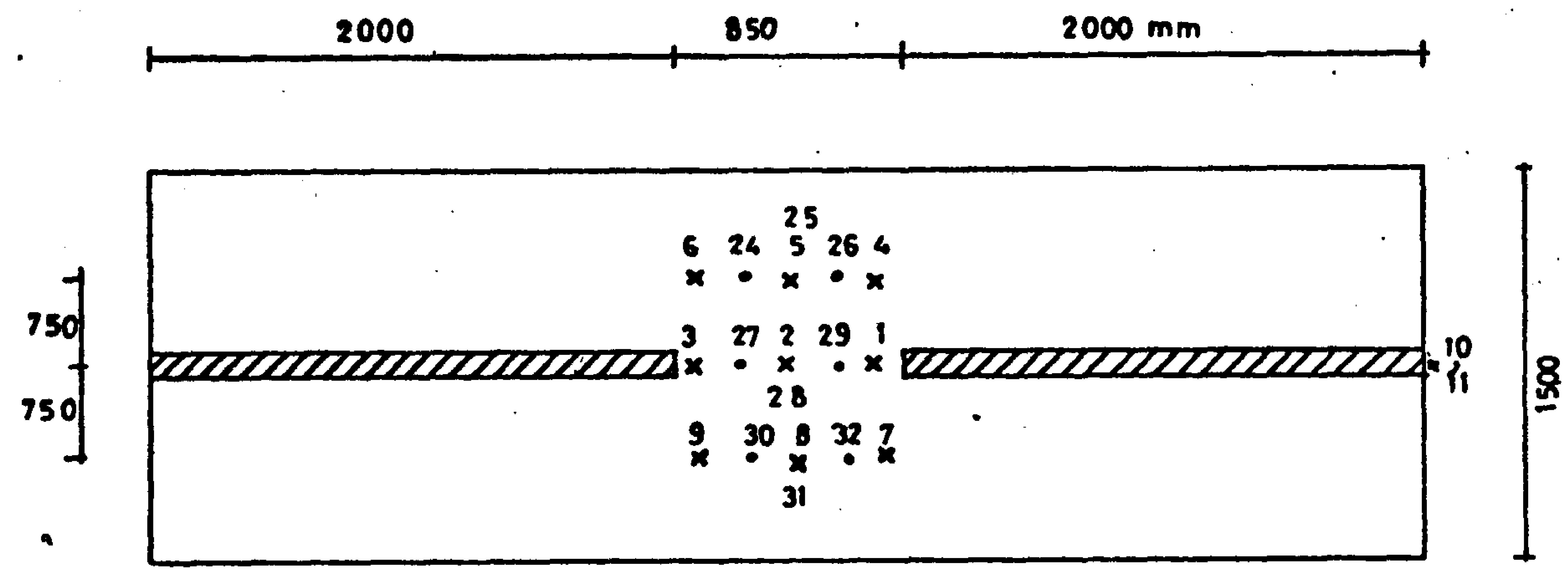


FIGURE 2.38 DIAL GAUGES ON SLAB 1 (2-STOREY MODEL)

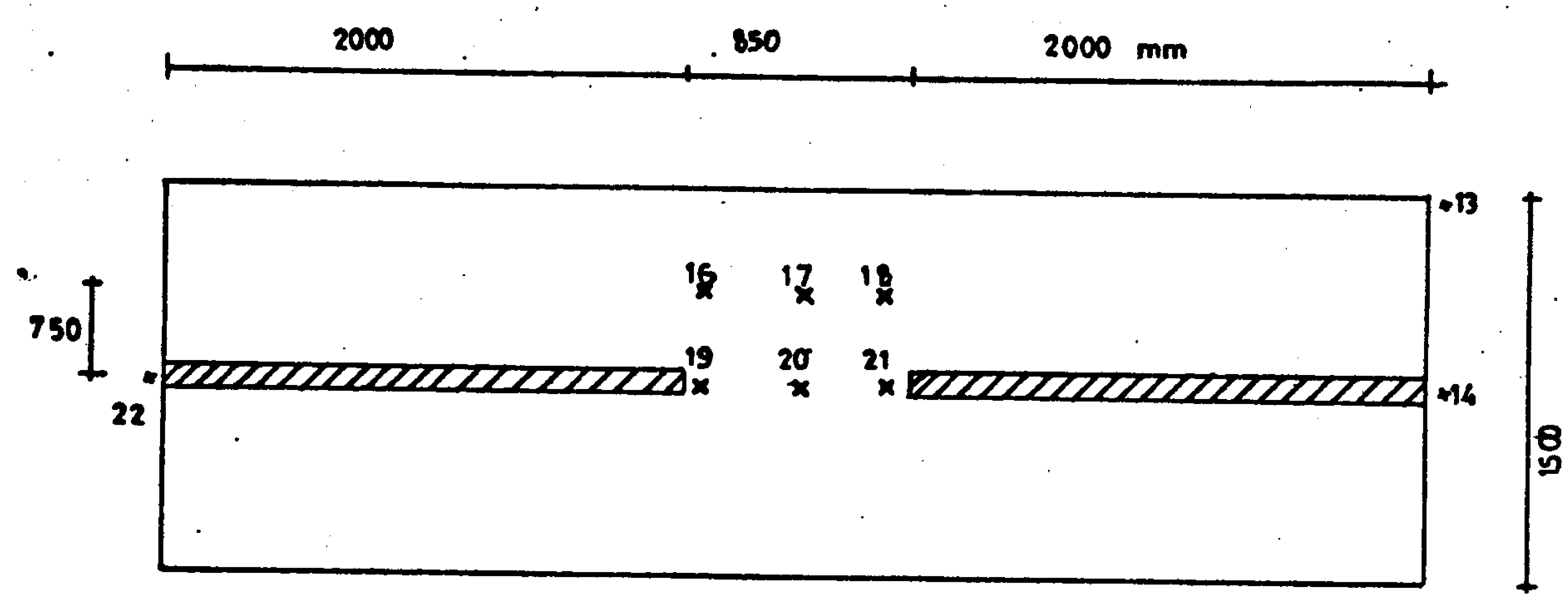


FIGURE 2.39 DIAL GAUGES ON SLAB 2 (2-STOREY MODEL)

FIGURE 2.40 LOAD DEFLECTION CYCLES FOR POINT 1 (2-STOREY MODEL)

FIGURE 2.41 LOAD DEFLECTION CYCLES FOR POINT 2 (2-STOREY MODEL)

FIGURE 2.42 LOAD DEFLECTION CYCLES FOR POINT 3 (2-STOREY MODEL)

FIGURE 2.43 LOAD STRAIN CYCLES FOR STRAIN GAUGE No 6 (2-STOREY MODEL)

FIGURE 2.44 LOAD STRAIN CYCLES FOR STRAIN GAUGE No 7 (2-STOREY MODEL)

REMARK: VERY SMALL STRAINS

FIGURE 2.45 LOAD STRAIN CYCLE FOR STRAIN GAUGE No 5 (2-STOREY MODEL)

FIGURE 2.46 LOAD STRAIN CYCLES FOR STRAIN GAUGES No 4, 10, 3 (2-STOREY MODEL)

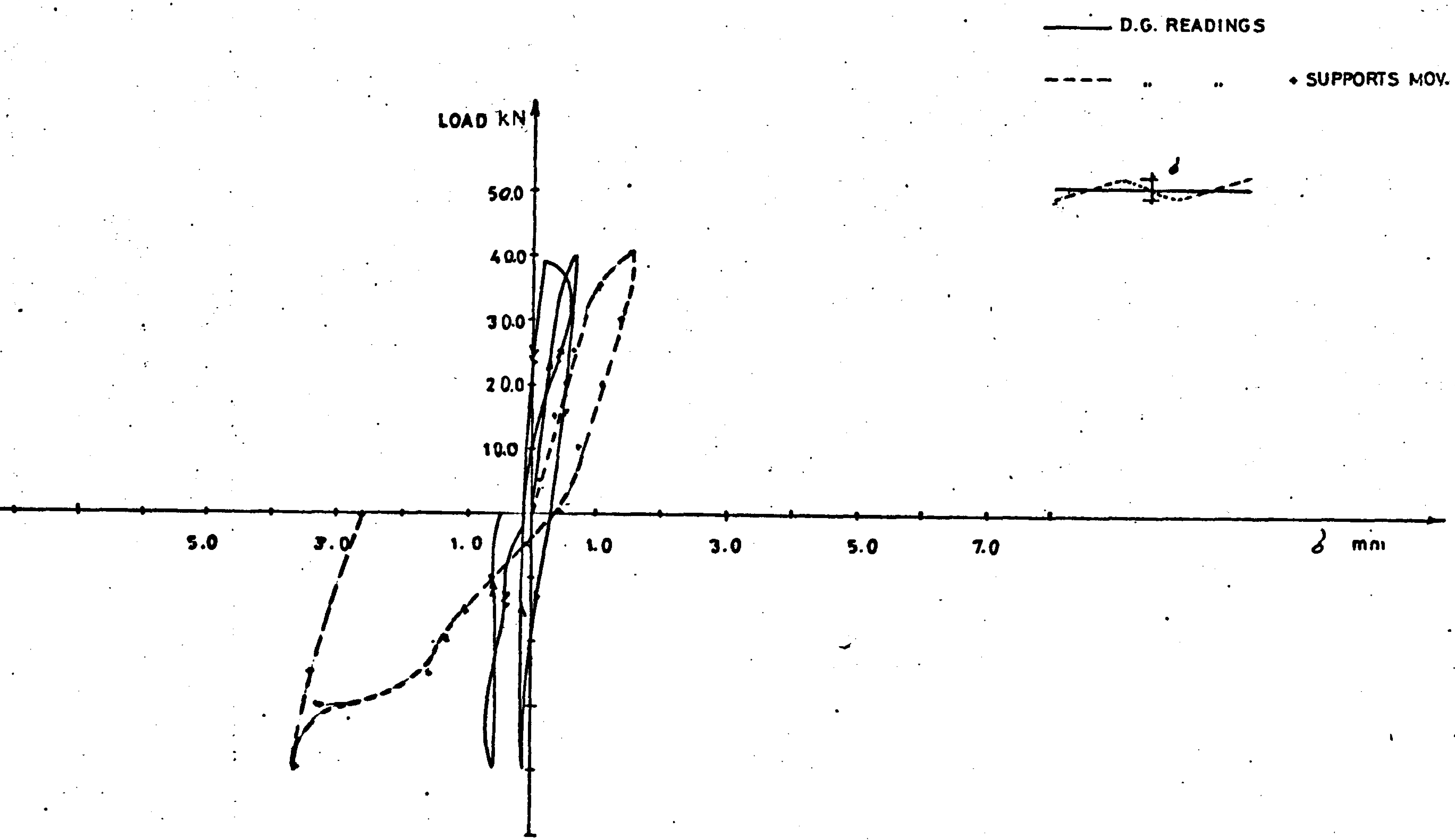


FIGURE 2.47 LOAD DEFLECTION CYCLES FOR δ (2-STOREY MODEL)

FIGURE 2.48 LOAD DEFLECTION CYCLE FOR DIAL GAUGE No 11 (2-STOREY MODEL)

FIGURE 2.49 LOAD DEFLECTION CYCLE FOR DIAL GAUGE No 14 WITH SUPPORT MOVEMENTS (2-STOREY MODEL)

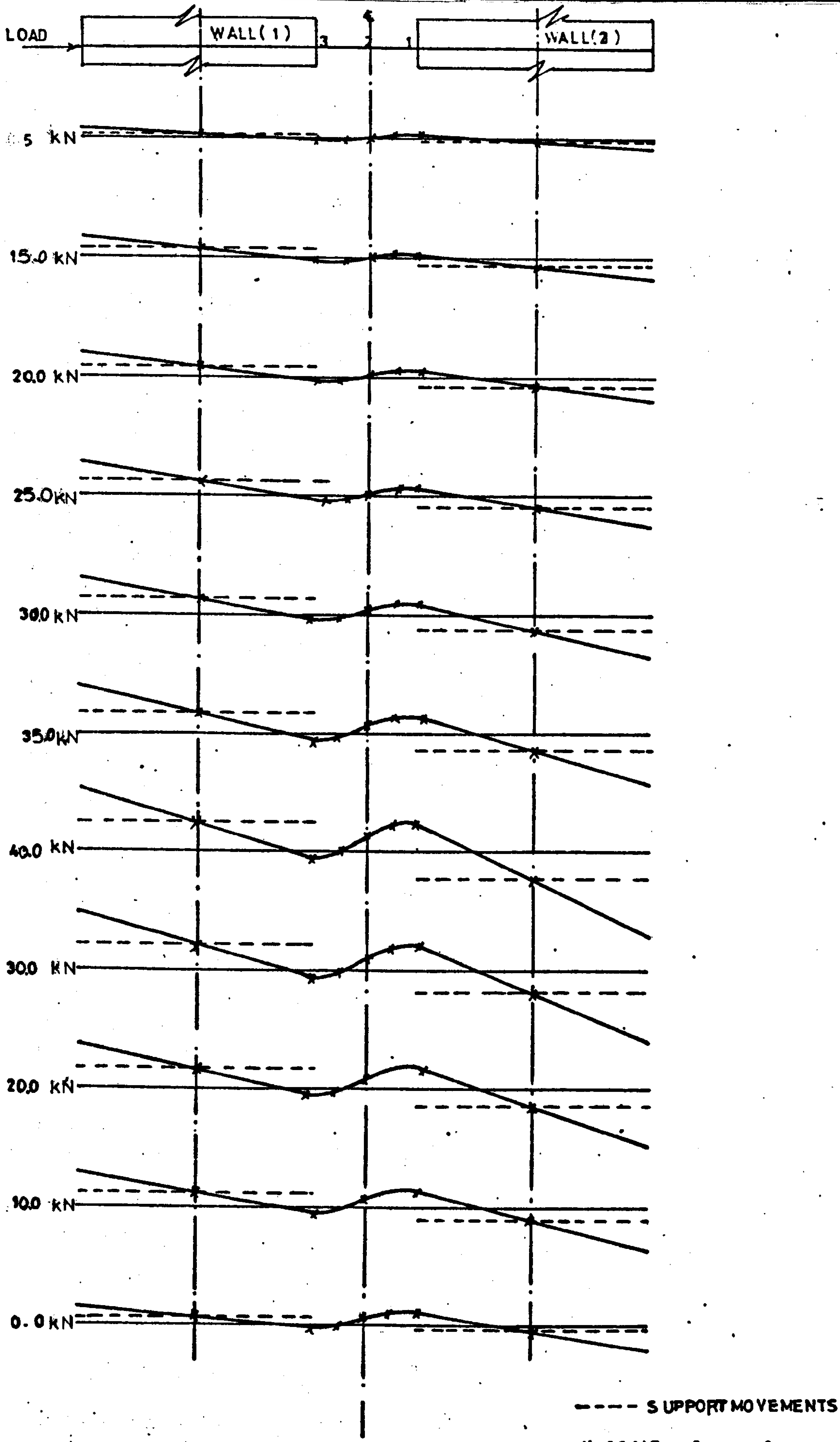


FIGURE 2.50 SLAB 1 DEFORMATION DURING FIRST LOADING (2-STOREY MODEL)

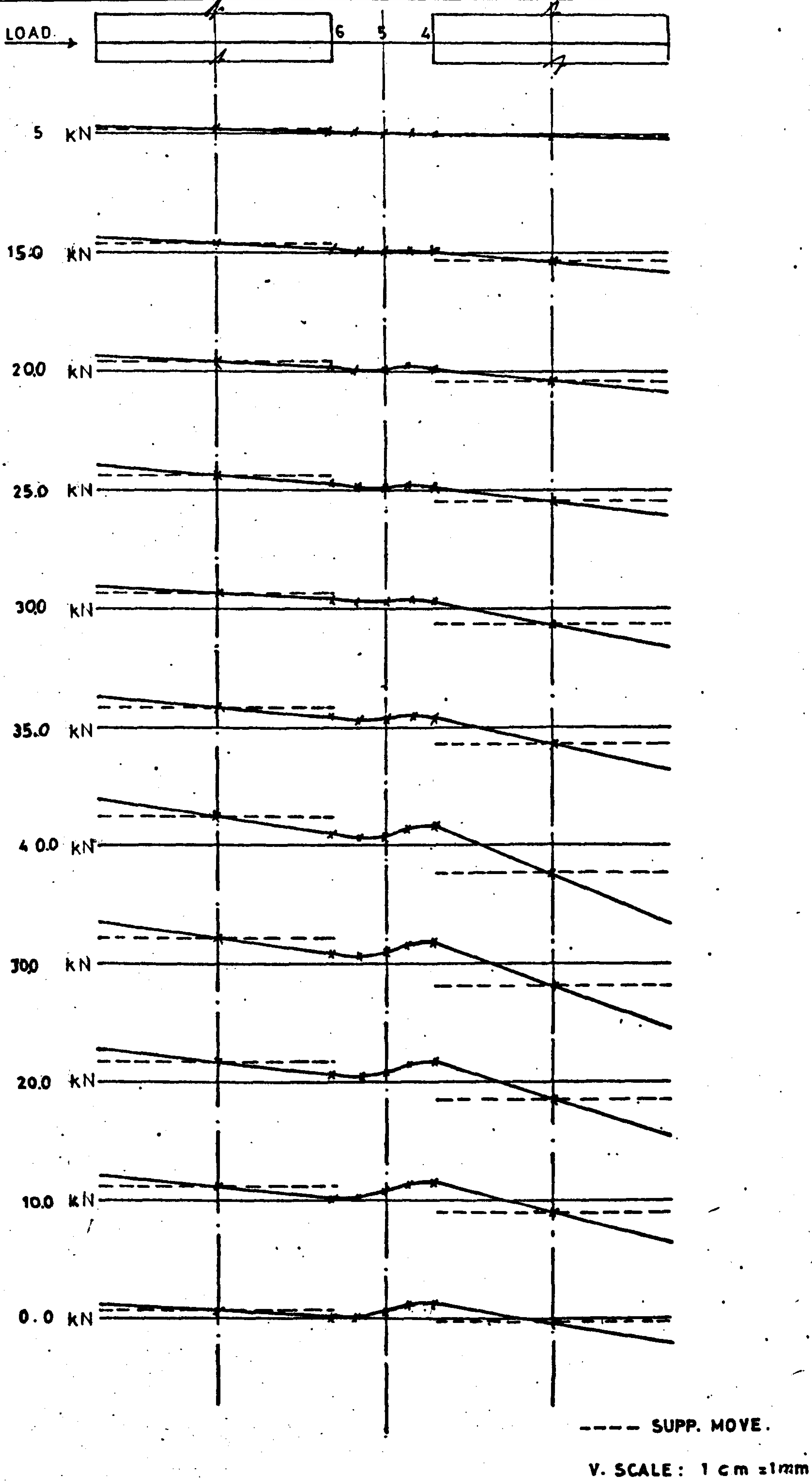


FIGURE 2.51 SLAB 1 DEFORMATION DURING FIRST LOADING (2-STOREY MODEL)

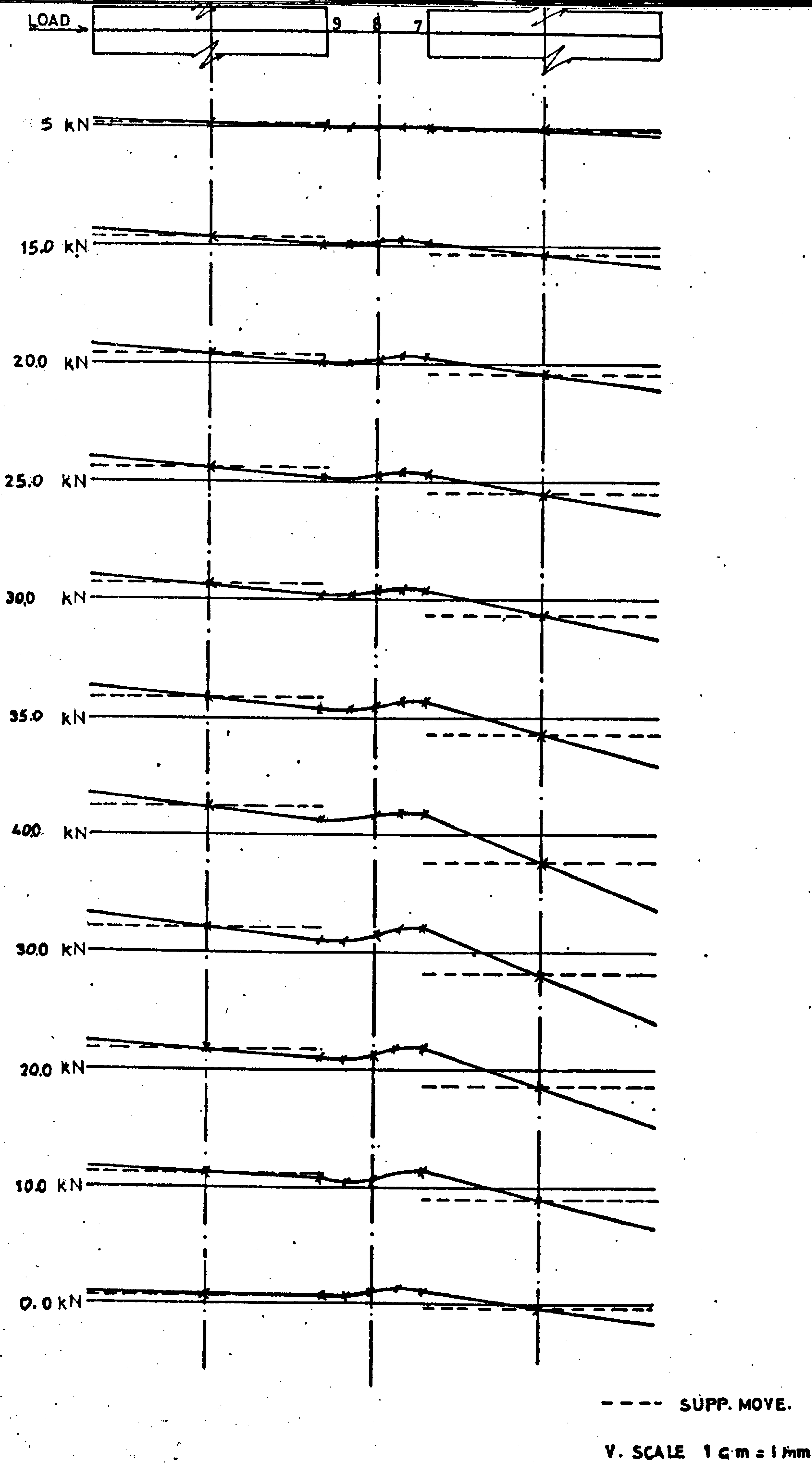


FIGURE 2.52 SLAB 1 DEFORMATION DURING FIRST LOADING (2-STOREY MODEL)

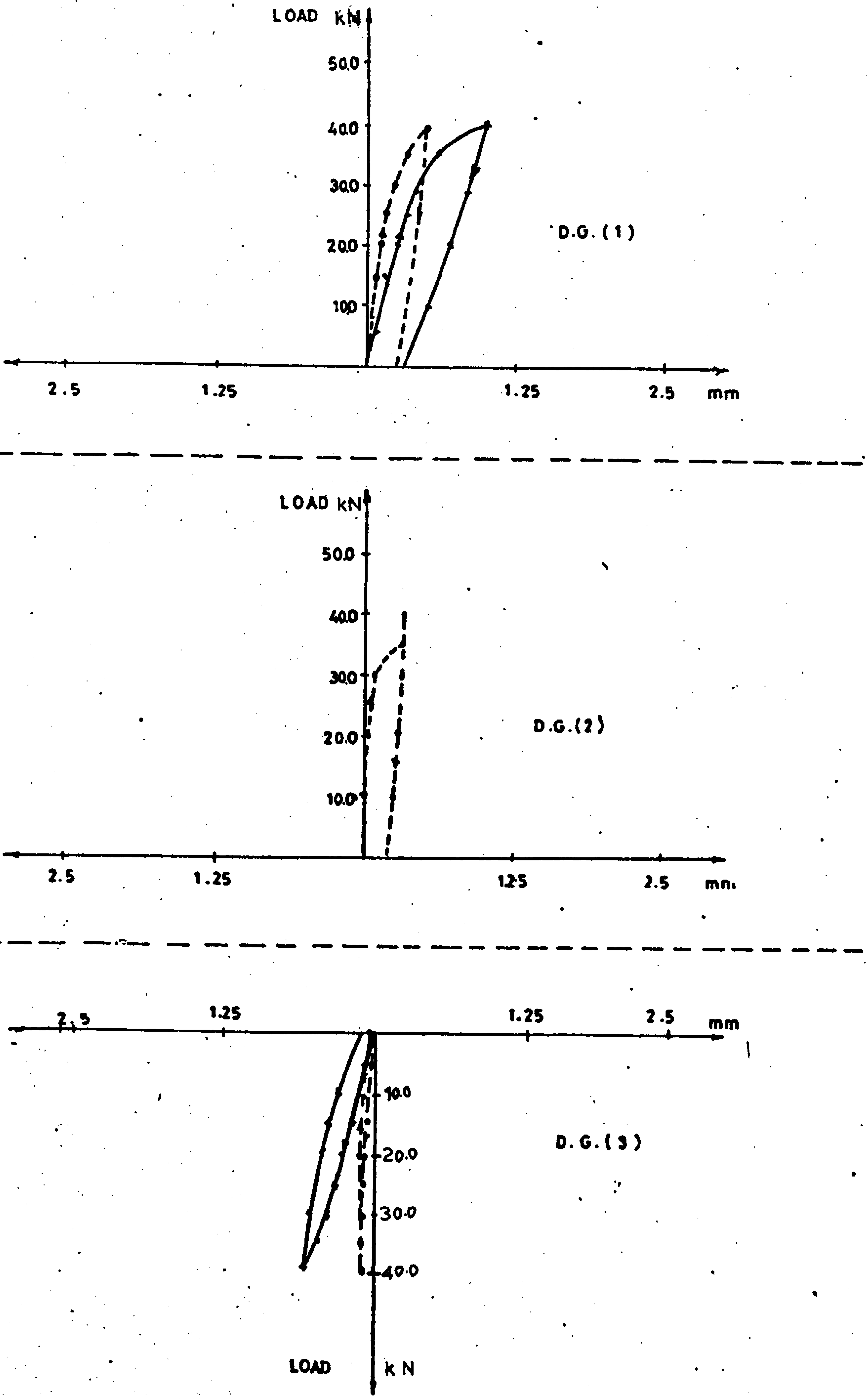


FIGURE 2.53 DEFLECTION OF VARIOUS POINTS OF SLAB 1 DURING FIRST LOADING (2-STOREY MODEL)

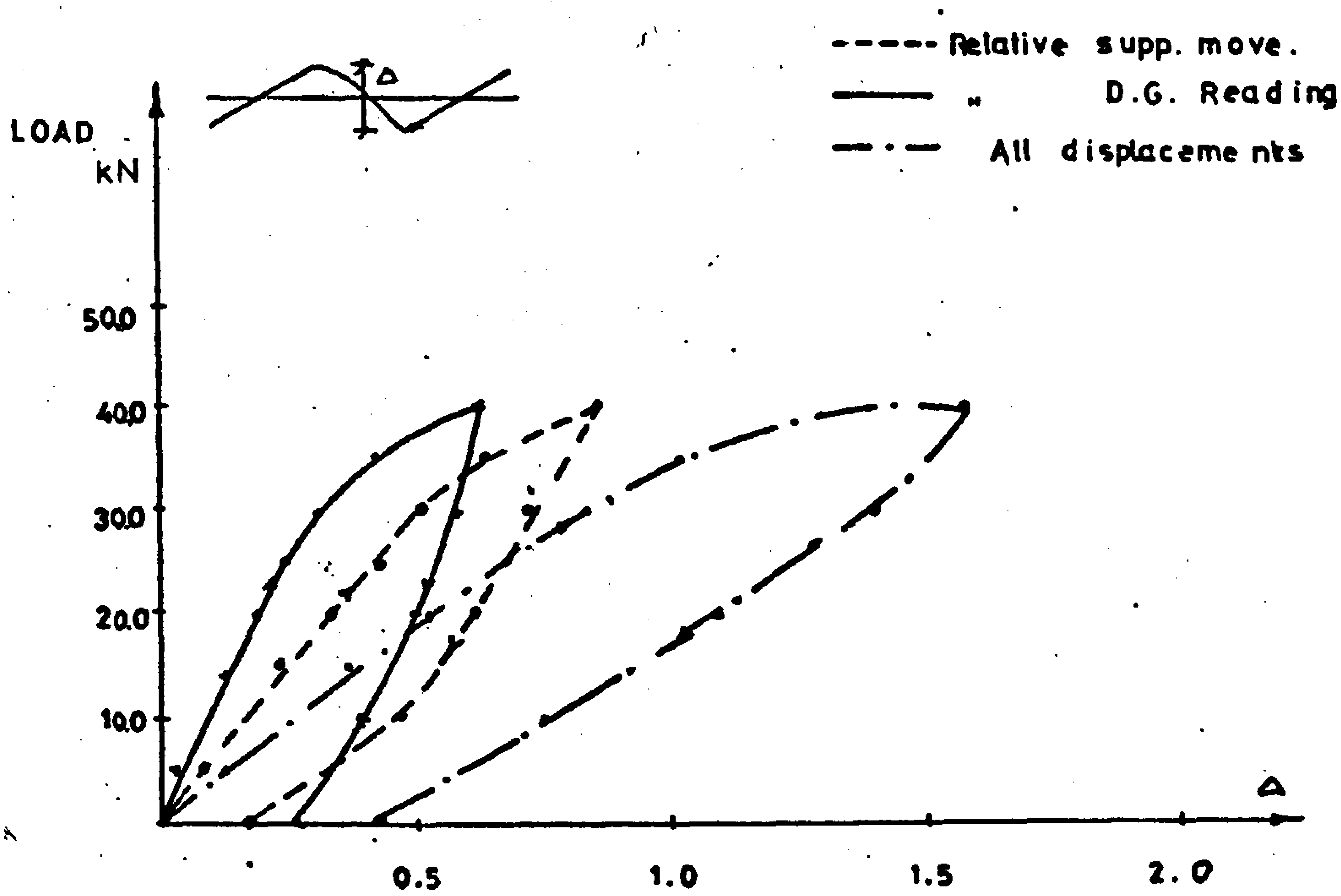


FIGURE 2.54 RELATIVE WALL DEFLECTIONS DURING FIRST LOADING (2-STOREY MODEL)

FIGURE 2.55 CHANGES OF STRAINS AT EACH LOAD INCREMENT DURING FIRST LOADING

FIGURE 2.56 CHANGES OF STRAINS AT EACH LOAD INCREMENT DURING FIRST LOADING

FIGURE 2.57 ACCUMULATED STRAINS FOR STRAIN GAUGES 5 TO 9 DURING FIRST LOADING (2-STOREY MODEL)

FIGURE 2.58 ACCUMULATED STRAINS FOR STRAIN GAUGES 16 TO 20 DURING FIRST LOADING (2-STOREY MODEL)

(2-STOREY MODEL)

FIGURE 2.59 ACCUMULATED STRAINS DURING FIRST LOADING

V. SCALE: 1 mm = 5 micro strain

FIGURE 2.60 LOAD STRAINS RELATIONSHIP FOR FIRST LOADINGS

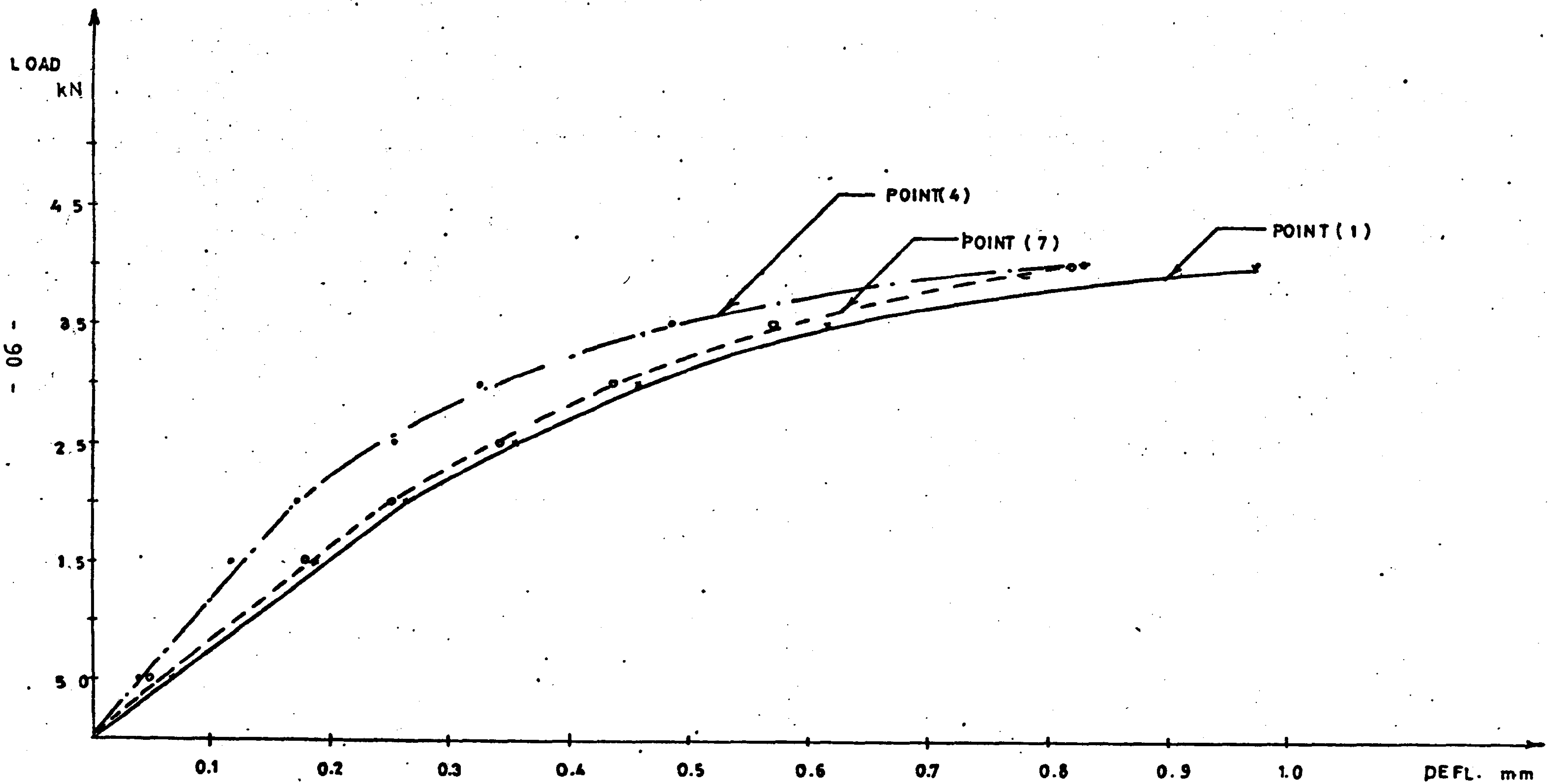


FIGURE 2.61 DEFLECTION OF VARIOUS SLAB POINTS DURING FIRST LOADING (2-STOREY MODEL)

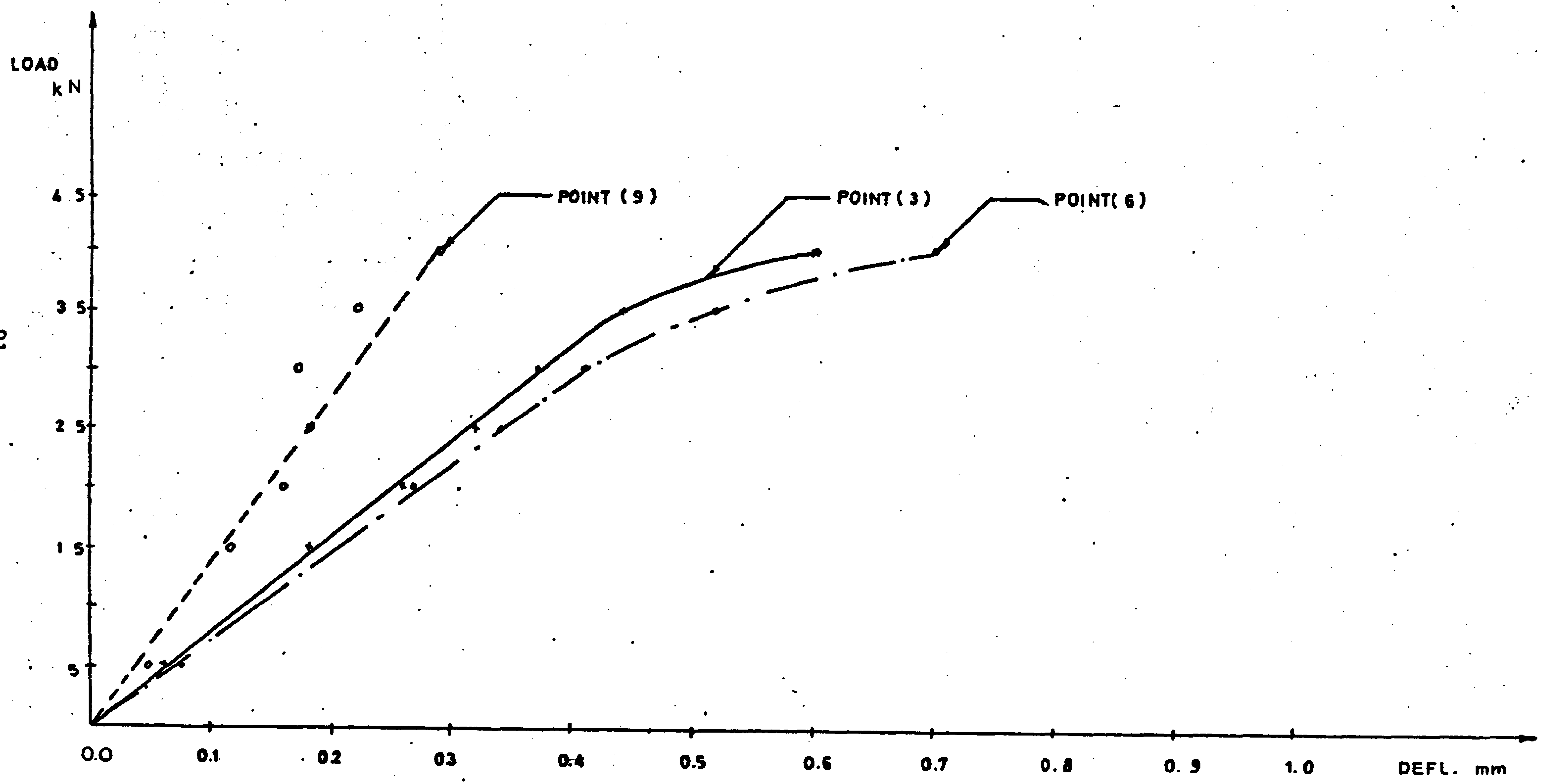


FIGURE 2.62 DEFLECTION OF VARIOUS SLAB POINTS DURING FIRST LOADING (2-STOREY MODEL)

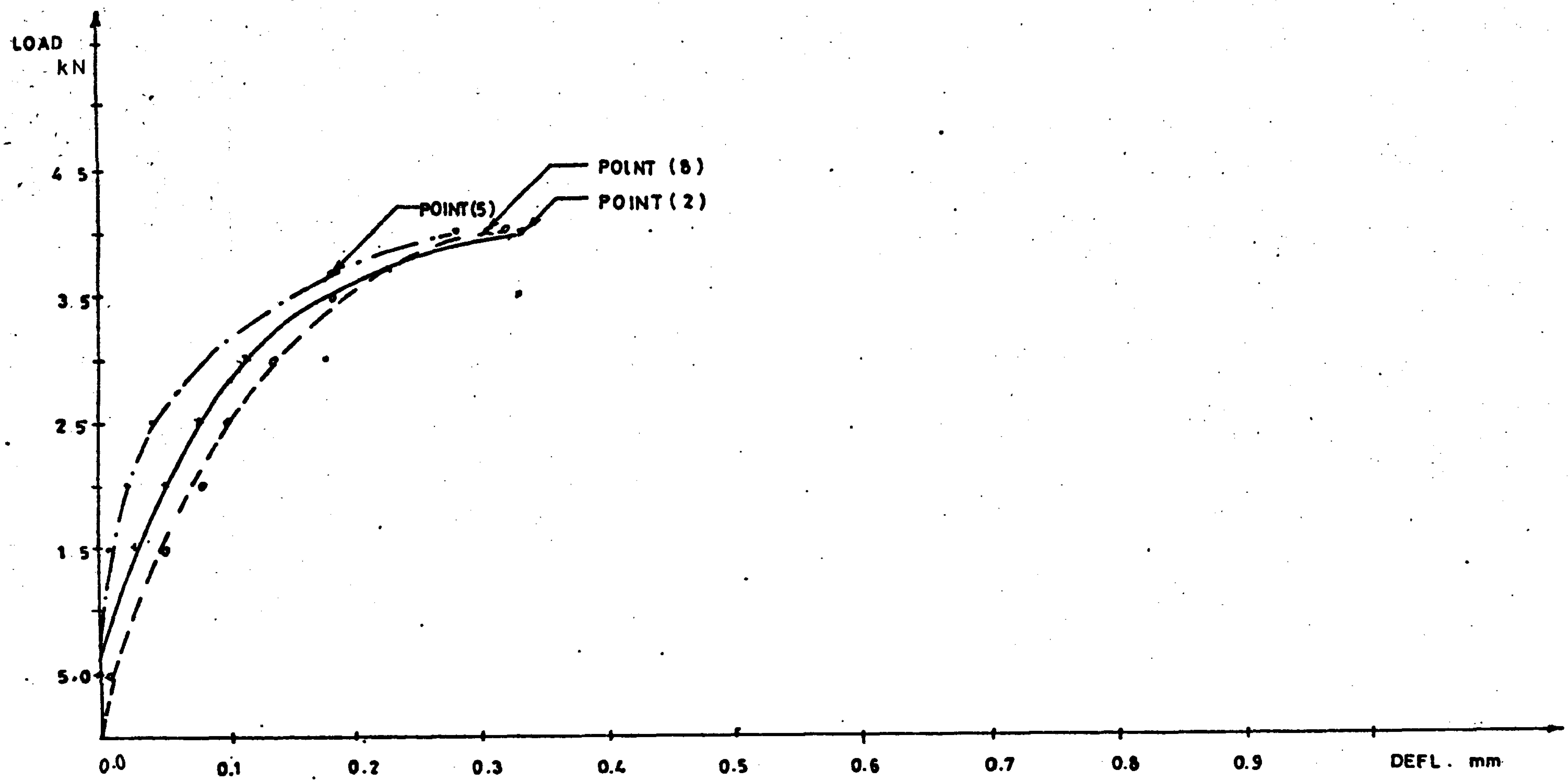


FIGURE 2.63 DEFLECTION OF VARIOUS SLAB POINTS DURING FIRST LOADING (2-STOREY MODEL)

FIGURE 2.64 SLAB 1 DEFORMATION DURING FOURTH LOADING (2-STOREY MODEL)

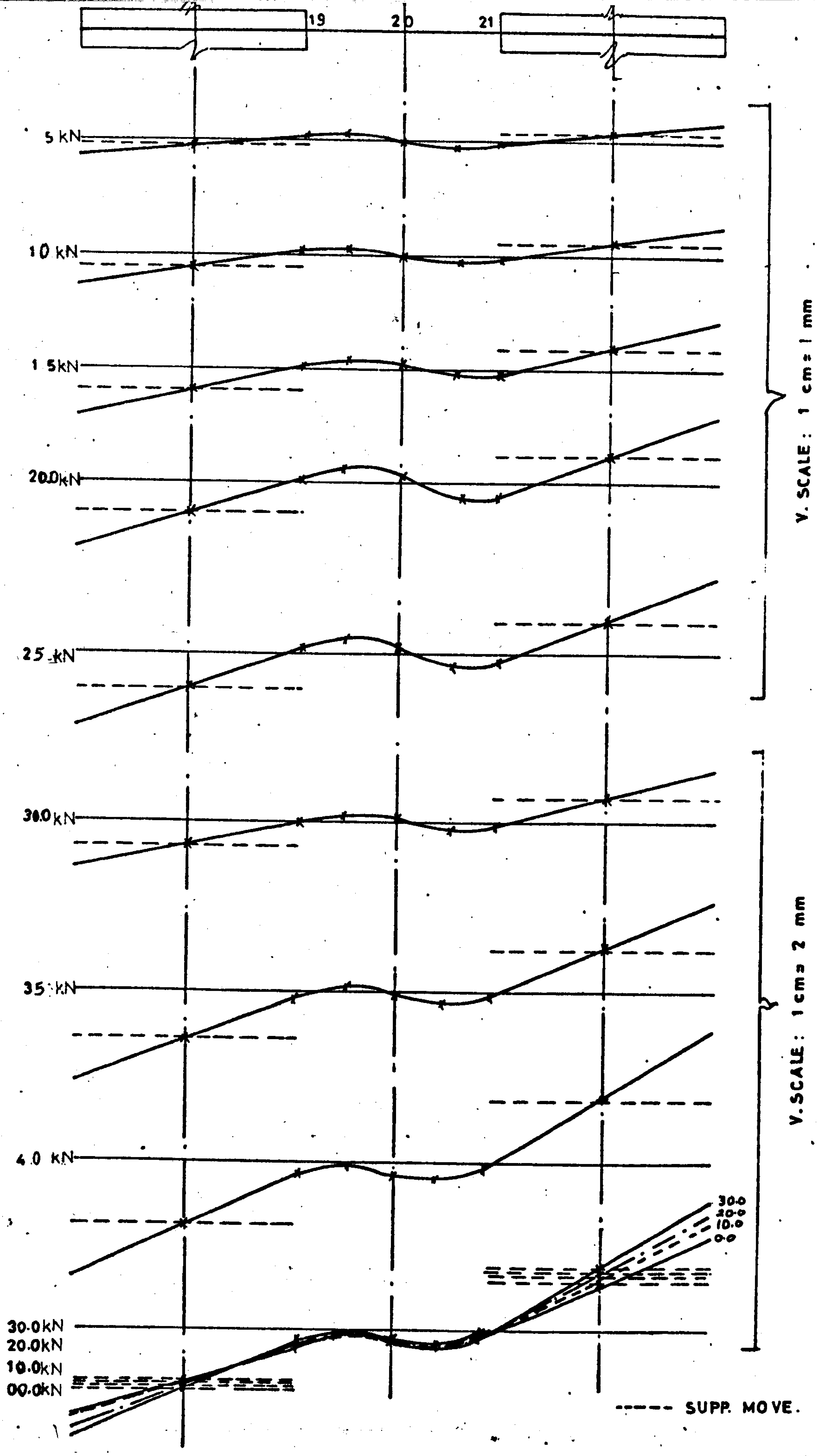
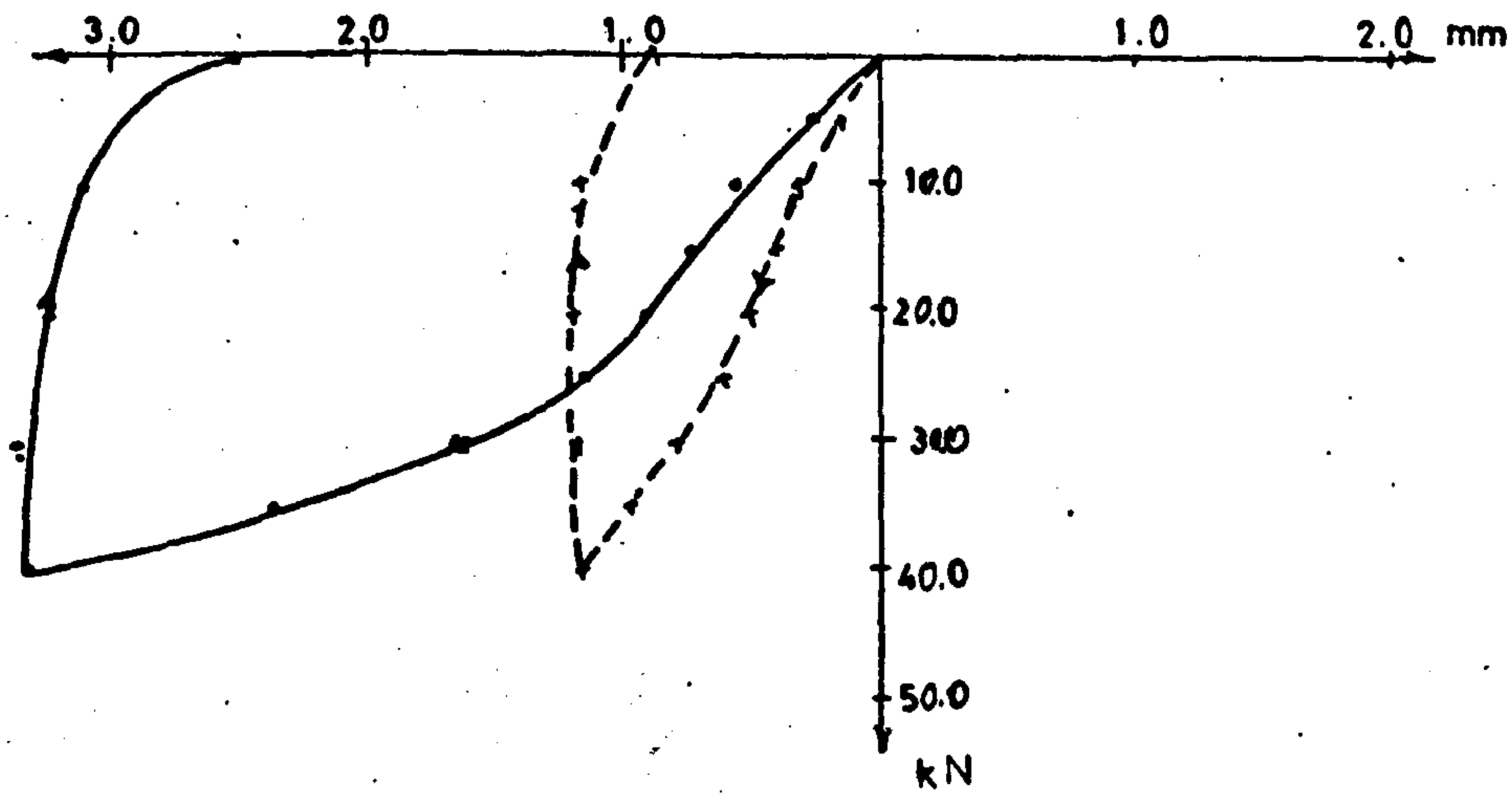
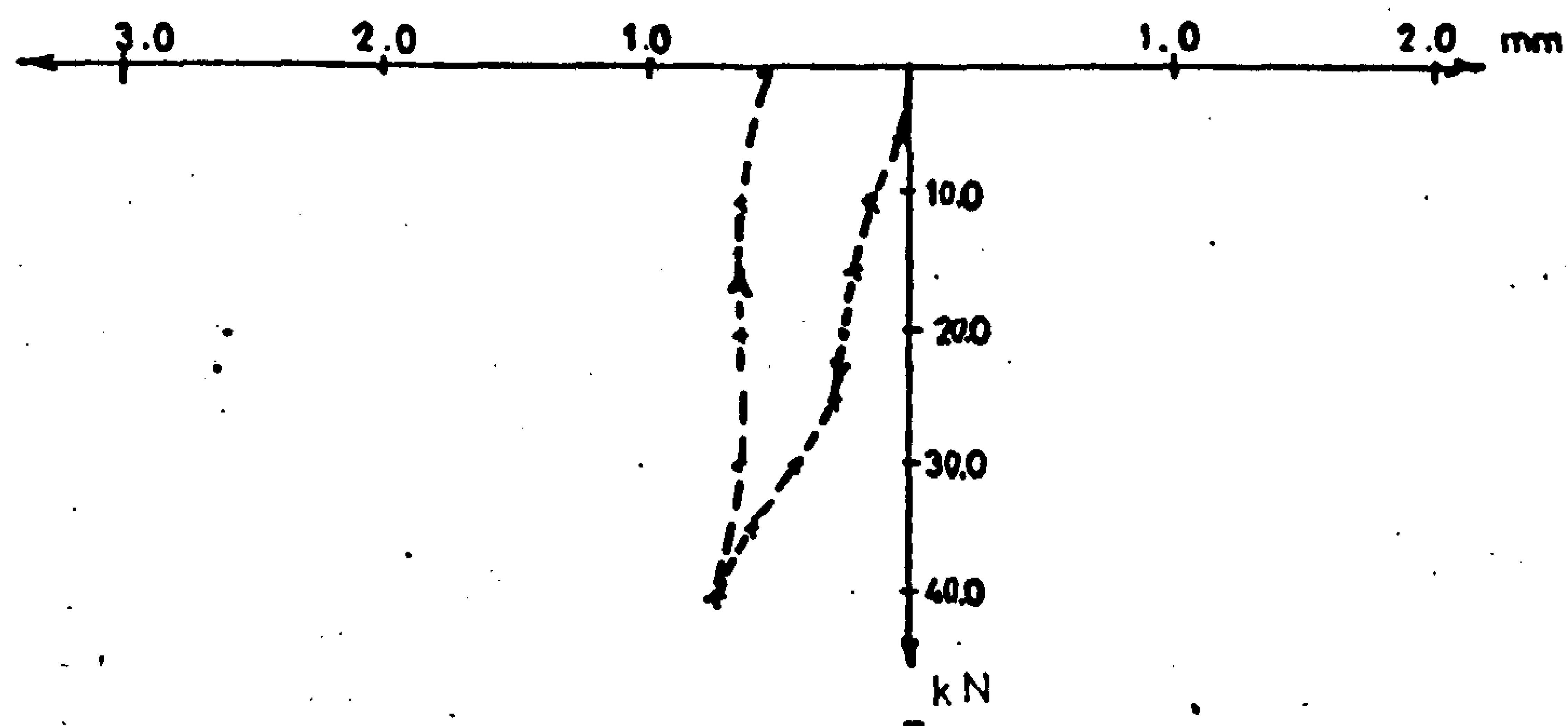


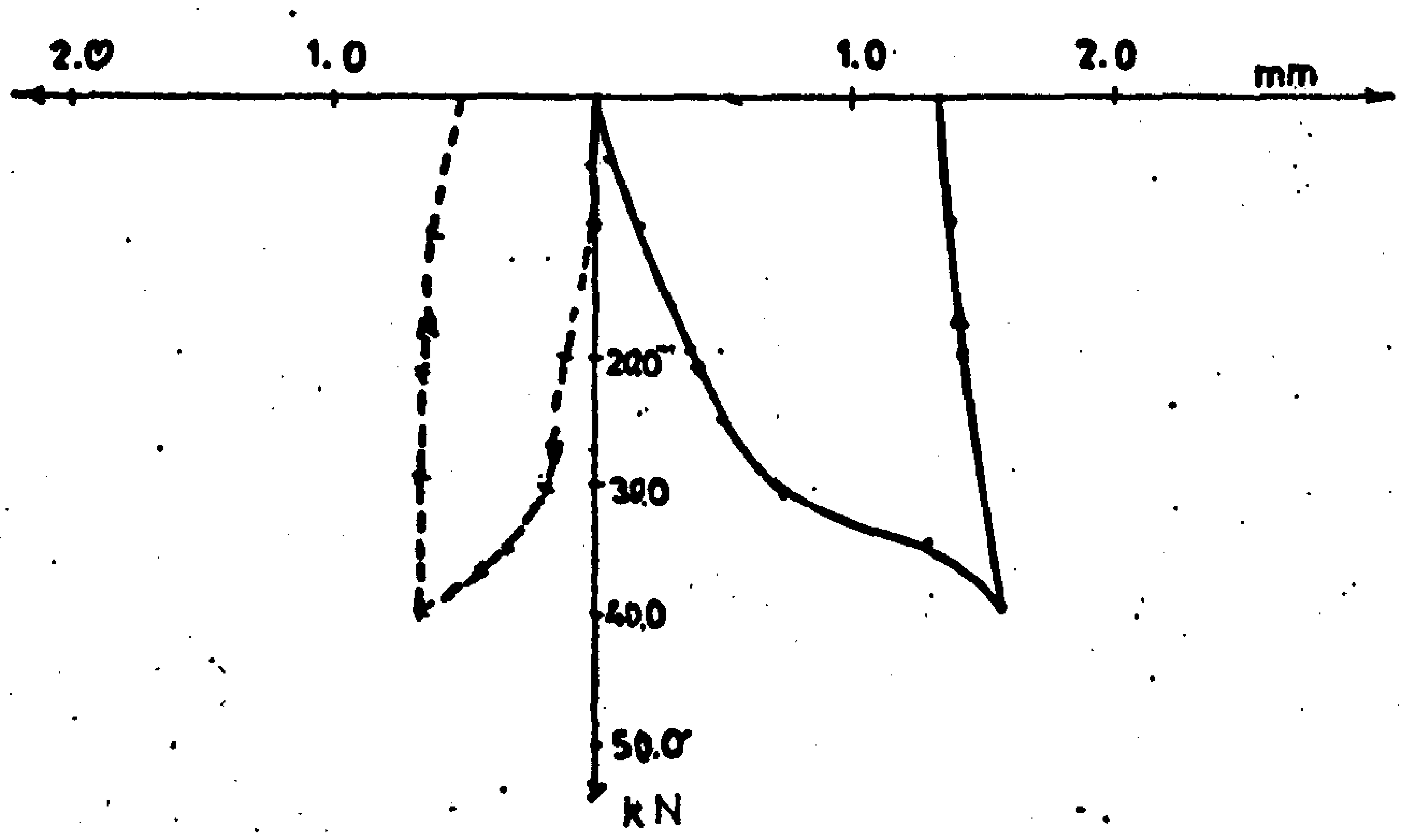
FIGURE 2.65 SLAB 2 DEFORMATION DURING FOURTH LOADING (2-STOREY MODEL)



D.G. (1)



D.G. (2)



D. G. (3)

FIGURE 2.66 DEFLECTION OF VARIOUS POINTS OF SLAB 1 DURING FOURTH LOADING (2-STOREY MODEL)

FIGURE 2.67 RELATIVE WALLS DEFLECTION DURING FOURTH LOADING (2-STOREY MODEL)

FIGURE 2.68 LOAD STRAINS RELATIONSHIP FOR FOURTH LOADING

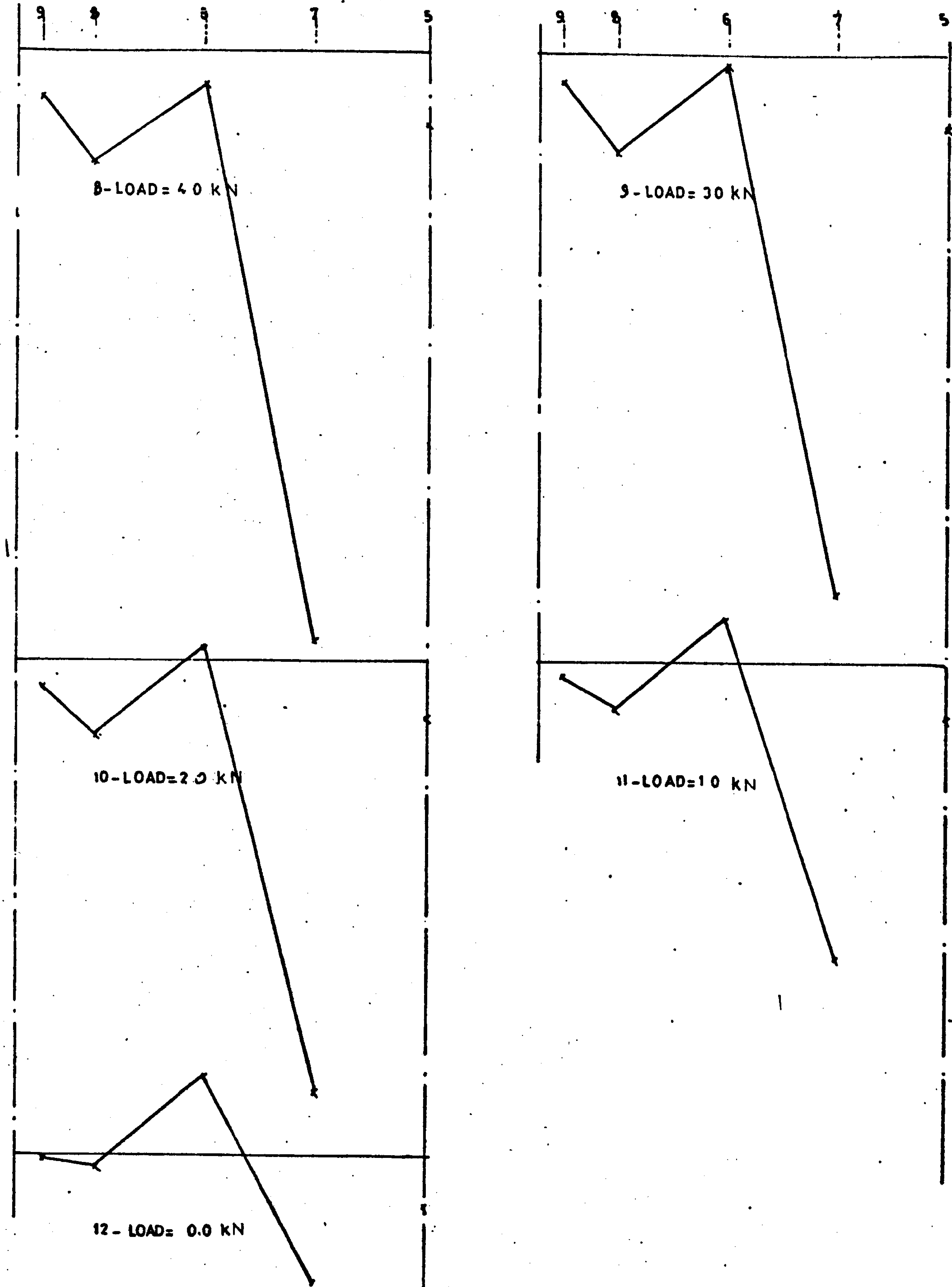
V. SCALE: 1 mm = 5 micro strain

FIGURE 2.69 CHANGES OF STRAINS AT EACH LOAD INCREMENT DURING FOURTH LOADING

FIGURE 2.70 CHANGES OF STRAINS DURING EACH LOAD INCREMENT DURING FOURTH LOADING

(2- STOREY MODEL)

FIGURE 2.71 ACCUMULATED STRAINS DURING FOURTH LOADING



(2-STOREY MODEL)

FIGURE 2.72 ACCUMULATED STRAINS DURING FOURTH LOADING

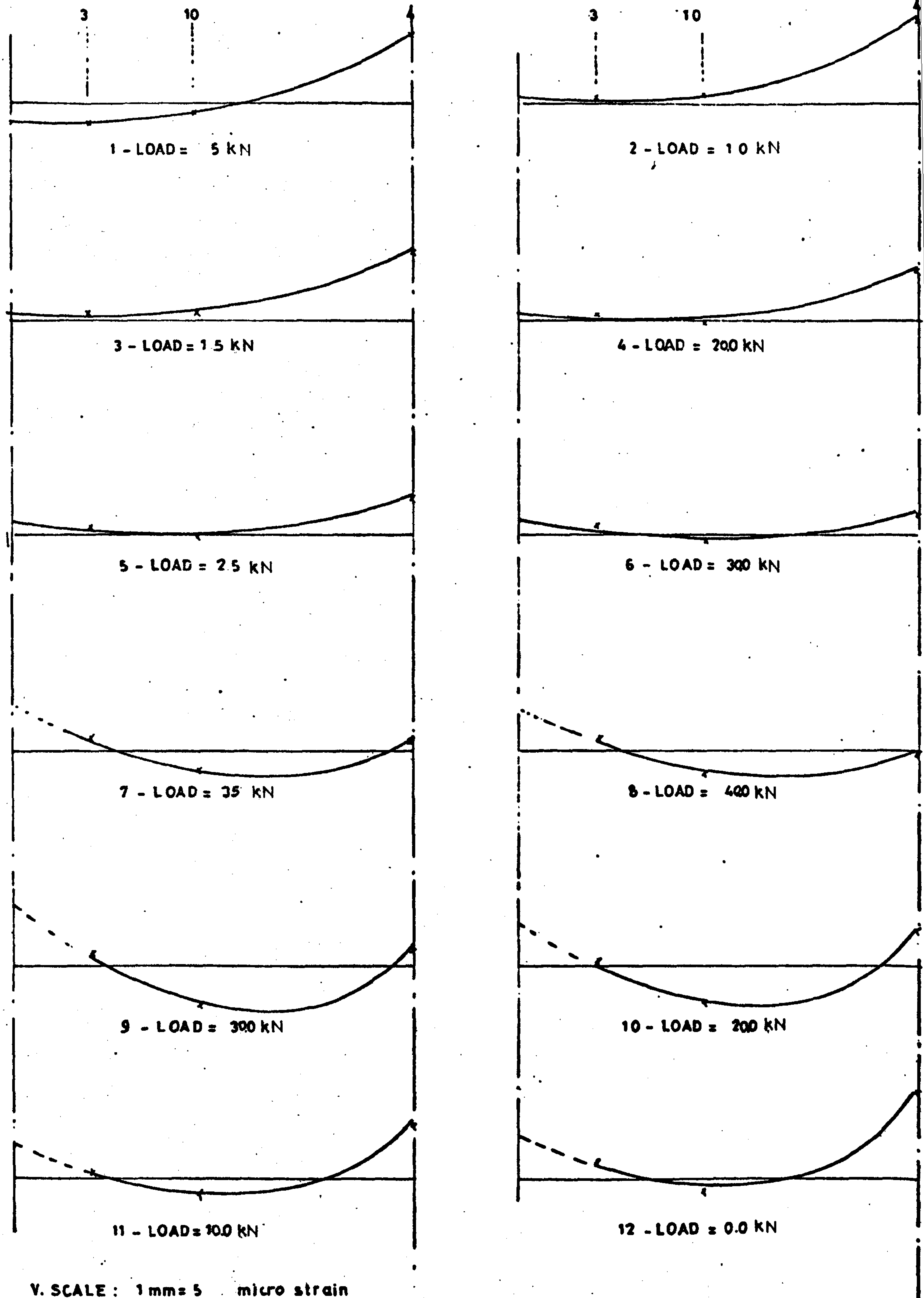


FIGURE 2.73 ACCUMULATED STRAINS DURING FOURTH LOADING (2-STOREY MODEL)

CHAPTER THREE

ANALYSIS OF COUPLED SHEAR WALLS CONNECTED BY SLABS

3.1 Introduction

In the structural design of tall buildings often the effect of lateral seismic and wind loads control the design and selection of the basic structural form. As shown in Chapter One it is convenient to use shear walls and floor slabs as an efficient structural system to resist the lateral environment load in many multi-storey buildings. This regular system of walls and slabs allows maximum use of industrialised building techniques enabling work on site to progress speedily and economically. In this Chapter elastic static and dynamic analyses of shear wall systems, especially shear walls coupled with floor slabs, are presented. Use has been made of the great improvements in the digital computer facilities to combine the Finite Element method with the continuous connection technique in the analysis of complete shear wall buildings both for overall analysis of the whole structures subject to various forms of loading and in the analysis of critical parts of the structures.

3.2 Past work

A review of the papers and work on shear wall structures for the period prior to 1970 was comprehensively presented by Irwin (1970), Puri (1967) and Coull and Stafford Smith in 1967. The work and papers published since then are briefly reviewed in this section.

In 1969 Schwaighofer and Microys presented in their paper a method for the analysis of diverse shear wall structures using standard computer programs. In their methods they used the established frame methods to analyse shear walls with any arbitrary number and size of bands of openings.

Coull and Irwin (1970) presented an approximate method for the analysis of distribution of load amongst the shear walls of a three-dimensional multi-storey building subjected to bending and torsion. The method is based on the continuous connection technique. It is particularly aimed at buildings with uniformity of structure throughout the height. They assumed that bending is the dominant overall mode of behaviour of the structure. The effects of axial deformations in the vertical members were included, as well as the effect of the bending stiffness of the floor slabs in the coupling action between wall assemblies. The method included the effect of bending of beams caused by torsion of coupled core assemblies. Comparison between the results from the method and results measured on a model was given.

Also in 1970 Stafford Smith proposed a modified beam method for analysing symmetrical interconnected shear walls. He assumed that an interconnected wall structure can be represented by the wide column frame, then a second equivalent frame is chosen to have identical behaviour. In this second frame the beams between the wide columns were replaced by beams which are flexible over their whole span between column axes. After obtaining the second moments of area of the replacement beam from a simple formula, any standard computer plane frame work program can be used to analyse the structure. The method is restricted in use to the analysis of cross walls which at all floors are symmetrical about their vertical central section.

In his paper "An elasto-plastic analysis of coupled shear walls" Paulay in 1970 showed that the laminar analysis can be readily extended to assess the elasto-plastic behaviour of coupled shear wall structures at various stages of cracking. He used a step-by-step procedure to trace the post-elastic performance of the structure and showed the need to check ductility requirements in assessing the ultimate strength of coupled shear walls. Pauly also showed that the demand for laminar ductility is affected by the relative stiffness of the coupled shear walls and that the ultimate load on a coupled shear wall structure needs to be compared with the strength of its foundations.

The vibrations of asymmetrical multi-storey shear wall buildings were investigated by Irwin (1971). In his paper he outlined a method which incorporates the use of the continuous connexion technique in the distribution of the overall lateral loads which may cause bending and torsion of the building amongst the various load bearing elements of a complete shear wall building. He extended the method for the use in the analysis of three-dimensional multi-storey shear wall buildings which are subject to dynamic forces. These dynamic forces may cause rotational and translational vibrations of the structure. He gave an example to illustrate the application of the method on a typical apartment shear wall structure.

In 1973 Yuzugullu and Schnobrich described a numerical procedure for the determination of the behaviour of a shear wall and frame system. They used the finite element method to predict the behaviour of the shear wall and frame system well into the cracked state. The comparison between the experimental and analytical results shows a big difference after the first stages of cracking.

Jacob Gluck in 1973 presented an elasto-plastic analysis of coupled shear walls in which plastic hinges at the ends of the coupling beams may develop only on part of the height of the wall. The analysis was given for an upper triangle lateral load. He presented some charts to determine the ultimate lateral load as well as the design parameters.

Irwin in 1975 outlined a simple approximate method for the static analysis of multi-storey shear wall buildings subjected to any pattern of lateral loading causing bending and torsion of the structure. The method included the effect of the in-plane deformation of floor slabs. He illustrated that if the in-plane flexibility of floor slabs is ignored in the analysis of a slender high-rise building, load distribution, deflection and stress patterns may deviate considerably from an accurate solution. He also used the method to determine the dynamic response of such buildings.

In his paper "Design aspects of shear walls for seismic areas", Pauly (1975) considered several aspects of the behaviour of tall as well as squat shear walls. He discussed the problems of brittle and ductile failure modes, diagonal tension, construction joints, alternating plasticity, sliding shear, stiffness degradation and strength loss under reversed cyclic loading. He examined in some detail the behaviour of coupled shear walls and reviewed the problems related to the components of the structure.

In 1976 Nayar and Coull presented an elasto-plastic linear stepwise analysis of coupled shear walls, based on the wide-column frame analogy. The analysis which they performed enables the behaviour of the structure to be traced from working to ultimate load, and allows the spread of plastification to be assessed through the load range. They traced as well the ductility requirement of each plastic hinge from its inception for every load increment up to collapse.

Coull and Mukherjee in their paper "Natural vibrations of shear wall buildings on flexible supports" (1978) considered a shear wall building as an assembly of plane and curvilinear shear walls tied together by floor slabs to act as a composite unit. The analysis is based as well on the continuous medium approach and the governing dynamic equations and boundary conditions were derived from energy principles using Vlasov's theory of thin-walled beams. They adopted the distributed mass technique in their analysis and the numerical solution of the dynamic equations was achieved by employing the Ritz-Galerkin technique, yielding both natural frequencies and mode shapes.

3.3 Finite element method

The emergence of the 'finite element' method as a powerful approximate numerical technique for solving problems in engineering science has made possible the analysis of quite complex structures. The basis of the method is to be found in the theory of the calculus of variations and in particular in the so-called direct methods of the calculus of variations. One can also visualise the method as an extension of matrix methods of structural analysis.

In general the basic concept of the finite element method is the idealization of the actual continuum as an assemblage of elements connected at a finite number of joints or nodal points. For solid continua it was found that the displacement method of structural analysis is most convenient for treating finite element idealization. Using the displacement approach the analysis may be carried out in the following steps:

1. Idealization: Dividing the continuum into a system of appropriately shaped finite elements.
2. Element analysis: Evaluating the stiffness matrices which relate the forces developed at the element nodal points to the corresponding element displacement.

3. Assembly of elements: Evaluating the nodal stiffness matrix for the complete structure by superimposing appropriate element stiffness.

Displacement analysis: Solving the nodal equilibrium equations (expressed by means of the structure stiffness matrix) for the nodal displacements resulting from applied nodal forces.

Stress analysis: Computing the element stresses resulting from the computed nodal displacements, making use of element stiffness matrices.

The following describes the details of a generalized finite element formulation and applies the method to the two cases of:

1. plane stress coupled shear walls
2. slabs coupling shear walls.

3.4 Finite element formulation for plane stress coupled shear walls

3.4.1 Discretization of continuum (idealization)

The first step involved in the finite element analysis is to divide the continuum by fictitious lines or surfaces into a number of discrete elements. In reality these elements are connected together along their common boundaries but in finite element solution the elements are assumed to be connected at the modes only. In three-dimensional cases the elements are prisms or cuboids and in two-dimensions are triangles or quadrilaterals. For the case of the plane stress coupled wall problem a survey of triangular and rectangular elements was carried out and the rectangular elements described below chosen. Figure 3.1 shows a particular mesh configuration of the rectangular elements but the problem was programmed such that the number, size and arrangement of elements could be readily altered in order to best represent any wall configuration and achieve best convergence of solution.

3.4.2 Finite element characteristics (element analysis)

To evaluate the element characteristics the two-dimensional rectangular element i. j, k, l, which is shown in Figure 3.2 is used for the illustration.

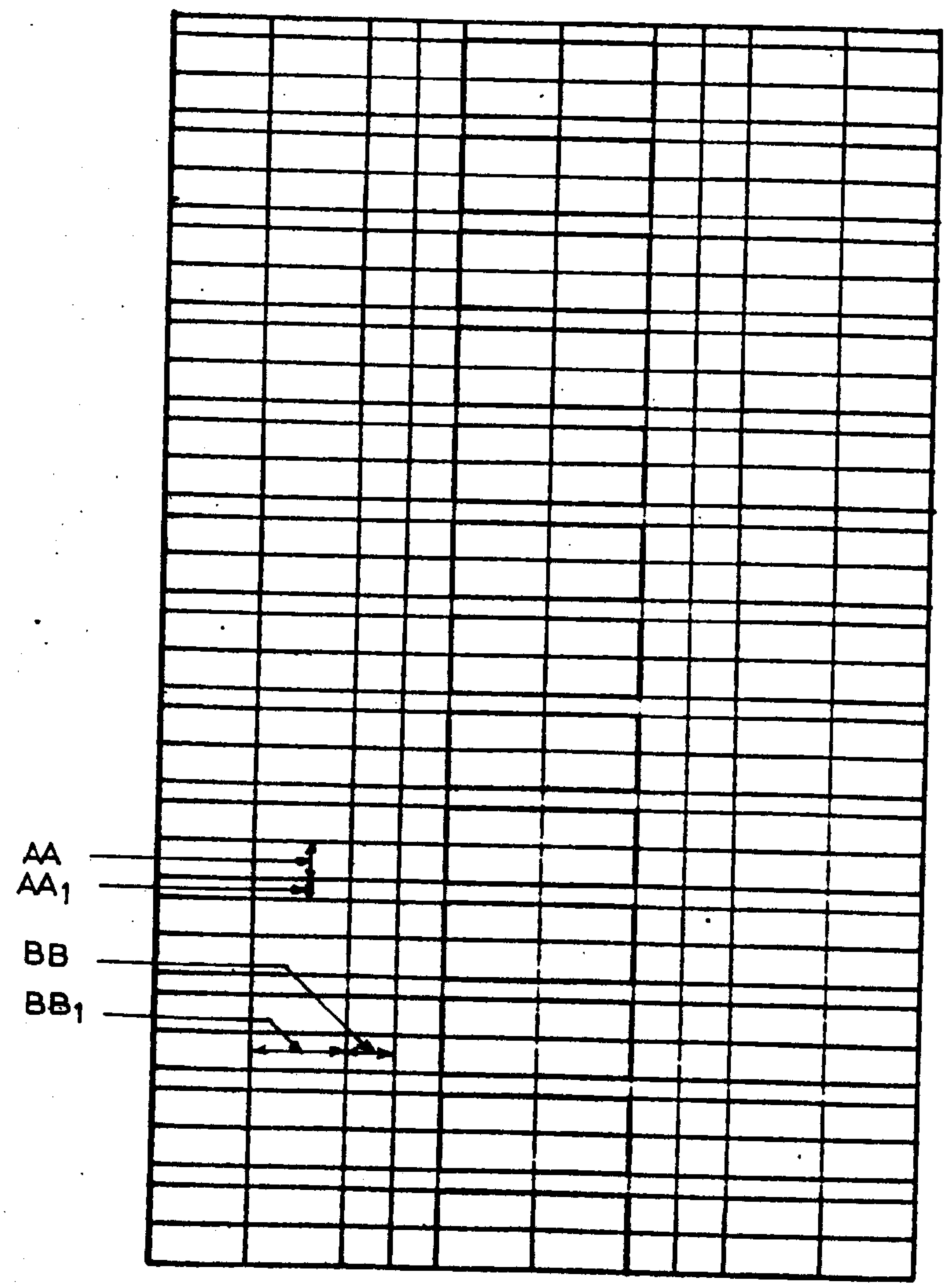


FIGURE 3.1 IDEALISATION OF SHEAR WALLS

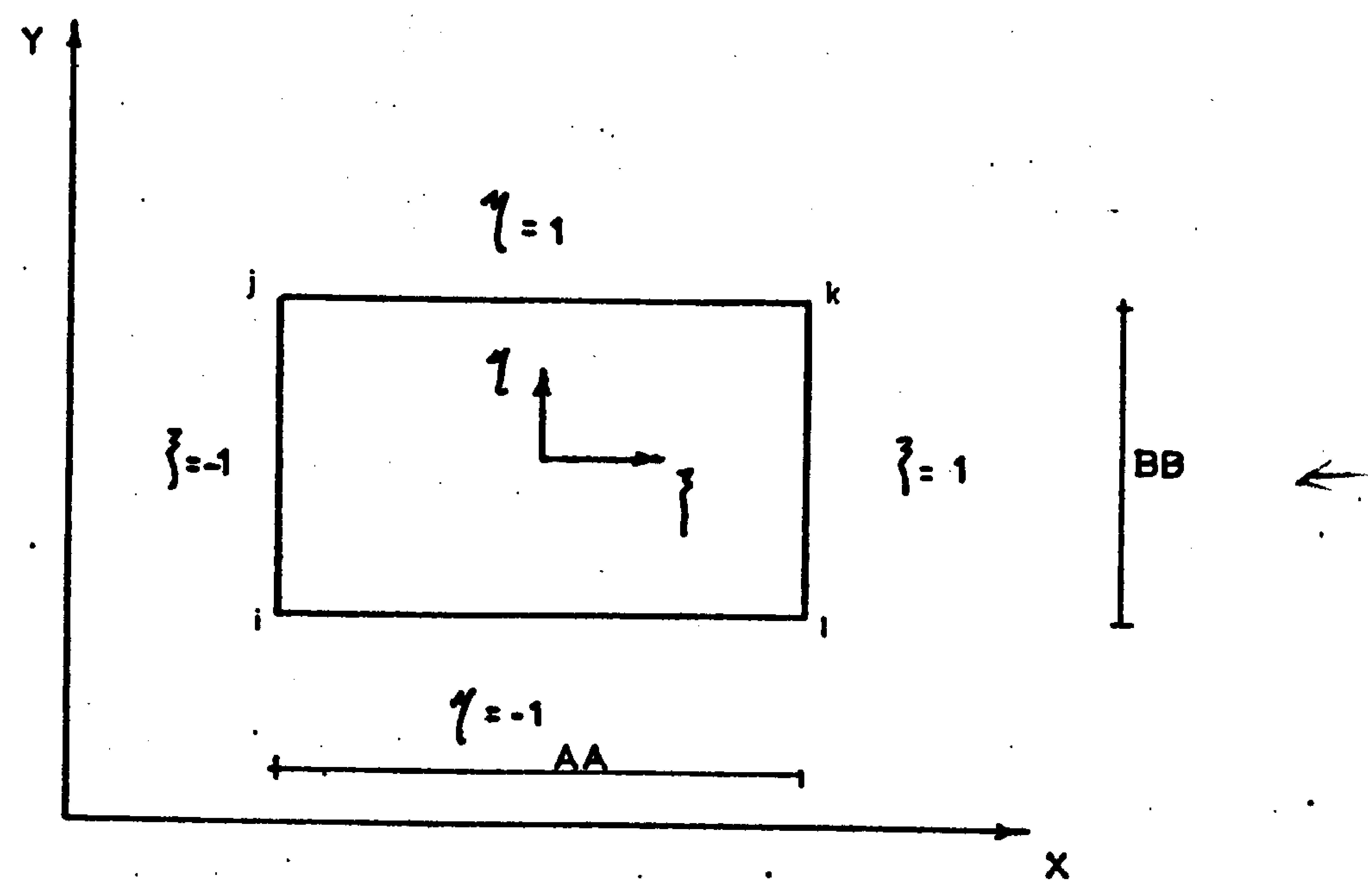


FIGURE 3.2 TWO-DIMENSIONAL RECTANGULAR ELEMENT

3.4.2a Selection of displacement functions

A displacement function is chosen to define the state of displacement within each 'finite element'. It is usually most convenient to make the functions dependent on nodal values placed on the element boundary. The choice of this function should satisfy compatibility across the element boundaries. The shape function used in the displacement formulation of elasticity problems should, in general, satisfy:

(i) the continuity of the unknown only has to occur between elements.

(ii) the function has to allow any arbitrary linear form to be taken so that the constant strain criterion could be observed.

The displacement vector for an element i, j, k, l could be written as:

$$\{f\} = \left[N_i, N_j, N_k, N_l\right] \begin{bmatrix} \delta i \\ \delta j \\ \delta k \\ \delta l \end{bmatrix}$$

i.e.

$$\{f\} = \left[\, N \,\right] \{\delta\} \qquad (3.1)$$

where $\{f\}$ is the displacement vector

$[N]$ is the shape function matrix

$\{\delta\}$ is a listing of nodal displacements for a particular element. For a plane stress elasticity problem, where all displacements are in the plane, the element has two degrees of freedom at each node as shown in Figure 3.2.

The shape function should be chosen so that at a node it has a value of unity when the coordinates of that node are substituted, and zero when coordinates of other nodes are inserted. To simplify the specification of the shape function the coordinates of points within the rectangular are expressed in the local coordinates system defined by coordinates (ζ, η) of the intersection of lines which cut opposite sides of the rectangular element in equal proportions.

For a rectangular element with function values specified at corner nodes, linear shape function completely describes the variation of the function within the element. For the rectangular elements with four nodes at the corner the shape functions are given by:

$$\left.\begin{aligned} N_i &= \frac{1}{4}(1-\zeta)(1-\eta) \\ N_j &= \frac{1}{4}(1-\zeta)(1+\eta) \\ N_k &= \frac{1}{4}(1+\zeta)(1+\eta) \\ N_l &= \frac{1}{4}(1+\zeta)(1-\eta) \end{aligned}\right\} \tag{3.2}$$

It is convenient to use normalized coordinates such that:

ij is the line $\zeta = -1$, kl is the line $\zeta = +1$

jk is the line $\eta = +1$, il is the line $\eta = -1$

The local (ζ, η) coordinates Figure 3.2 are then linearly related to the global (x, y) coordinates.

3.4.2b Strains

At any point within the element the total strain can be defined by its components which contribute to internal work. The various components of strain can be obtained by differentiation of the displacement function. The strain is then given by:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix}$$

$$\{\varepsilon\} = [\,B\,]\,\{\delta\} \tag{3.3}$$

where $\{\varepsilon\}$ is the strain vector

$[\,B\,]$ is a matrix relating strains and displacements and it can be obtained once the shape functions are determined.

u,v are the components of displacements in x,y directions.

3.4.2c Elasticity matrix (stresses)

The stresses $\{\sigma\}$ are calculated by multiplying the strain by an elasticity matrix $[D]$ which contains the material properties (Young's Modulus E and Poisson's Ratio ν). The stress is then given by:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} = \frac{E}{(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

i.e. $$\{\sigma\} = [D]\,\{\varepsilon\} \tag{3.4}$$

The material within the element boundaries may be subjected to initial strains $\{\varepsilon_0\}$ so that the stress will be caused by difference between the actual and initial strains. It is also convenient to assume that at the outset of analysis the body is stressed by some known system at initial residual stresses $\{\sigma_0\}$ which can simply be added on to the general definition. Thus the relation between the stresses and strains in (3.4) can be written as:

$$\{\sigma\} = [D]\ (\{\varepsilon\} - \{\varepsilon_0\}) + \{\sigma_0\} \qquad (3.4a)$$

3.4.2d Equivalent nodal forces and element stiffness matrix

Once the strain and stress matrices have been derived one can formulate the element stiffness matrix $[kM]$ and determine the nodal forces $\{F\}$ due to: loads $\{P\}$; initial strains $\{\varepsilon_0\}$ and initial stresses $\{\sigma_0\}$. This can be done using the method of virtual work. During any virtual displacement imposed on the element the total external work done by the nodal loads must equal the total internal work done by the stresses.

If one imposes an arbitrary virtual nodal displacement $\{\delta^*\}$ at the nodes then:

The displacement in the element is given by:

$$\{f^*\} = [N]\ \{\delta^*\}$$

and the strain is equal to:

$$\{\varepsilon^*\} = [B]\ \{\delta^*\}$$

The external work done by the nodal forces is given by:

$$\{\delta^*\}\ \{F\} \qquad (3.5)$$

The internal work done by the stresses and distributed forces per unit volume is:

$$\{\varepsilon^*\}\ \{\sigma\} - \{f^*\}^T\{P\}$$

or

$$\{\delta^*\}^T\ ([B]^T\{\sigma\} - [N]^T\{P\}) \qquad (3.6)$$

Equating the internal and external work:

$$\therefore \quad [\delta^*]^T\{F\} = \{\delta^*\}^T \left(\int [B]^T\{\sigma\}\, d(vol) - \int [N]^T\{P\}\, d(vol)\right) \qquad (3.7)$$

On substituting equations (3.3), (3.4a) one obtains:

$$\{F\} = \left(\int [B]^T[D]\,[B]\, d(vol)\{\delta\} - \int [B]^T[D]\{\varepsilon_0\}\, d(vol) + \int [B]^T\{\sigma_0\}\, d(vol) - \int [N]^T\{P\}\, d(vol)\right) \qquad (3.8)$$

The stiffness matrix is given by:

$$[kM] = \int [B]^T[D]\,[B]\, d(vol) \qquad (3.9)$$

The nodal forces due to distributed loads are:

$$\{F\}_p = -\int [N]^T\{P\}\, d(vol) \qquad (3.10)$$

The nodal forces due to initial strains and initial stresses are:

$$\{F\}_{\varepsilon_0} = -\int [B]^T[D]\,\{\varepsilon_0\}\, d(vol) \qquad (3.11)$$

and

$$\{F\}_{\sigma_0} = \int [B]^T\{\sigma_0\}\, d(vol) \qquad (3.12)$$

In the case that there are external concentrated forces at the nodes this can be added to the consideration of equilibrium at nodes.

3.4.2e Assembly and analysis of a structure

Once the conditions of overall equilibrium within the element are satisfied it is necessary to establish equilibrium conditions at the nodes of the structure. The only unknown in the resulting equations will be the displacements. Knowing the displacements, the internal forces in the element and the stresses can be calculated. The method of assembly used in this work will be discussed in Appendix II which explains the computer program.

The nodal displacement vector and the nodal forces vector are related by the equation:

$$\{F\} = [k]\ \{\delta\} \qquad (3.13)$$

where $[k]$ is the assembled structure stiffness matrix, so the nodal displacements $\{\delta\}$ can be determined by solving the above equation. Then, using equations (3.3) and (3.4) the strains and stresses can be determined.

3.5 Finite element formulation for slabs coupling shear walls

3.5.1 Discretization of continuum and selection of displacement functions

To analyse the slabs coupling shear walls again rectangular elements were chosen after tests for convergence, and a programme compiled such that any desired sizes and number of elements could be catered for to study both the overall and detailed behaviour of the structures. Additionally, the values of modulus of elasticity in adjacent elements could be varied to take account of differences between the walls and slabs. In general, the slabs and wall cross-section were divided as shown in Figure 3.3 into two-dimens-:ional non-conforming rectangular elements with three degrees of freedom at each node. The three components at each node are:

1. A displacement w_n in the z direction
2. A rotation $(\theta_x)_n$ about the x axis
3. A rotation $(\theta_y)_n$ about the y axis.

Figure 3.4 shows a typical rectangular plate element with forces and corresponding displacements indicated. The slopes of w and rotation are identical except for the sign. The nodal displacements can then be written:

$$\{\delta_i\} = \begin{Bmatrix} w_i \\ \theta_{x_i} \\ \theta_{y_i} \end{Bmatrix} = \begin{Bmatrix} w_i \\ -(\frac{\partial w}{\partial y})_i \\ (\frac{\partial w}{\partial x})_i \end{Bmatrix} \qquad (3.14)$$

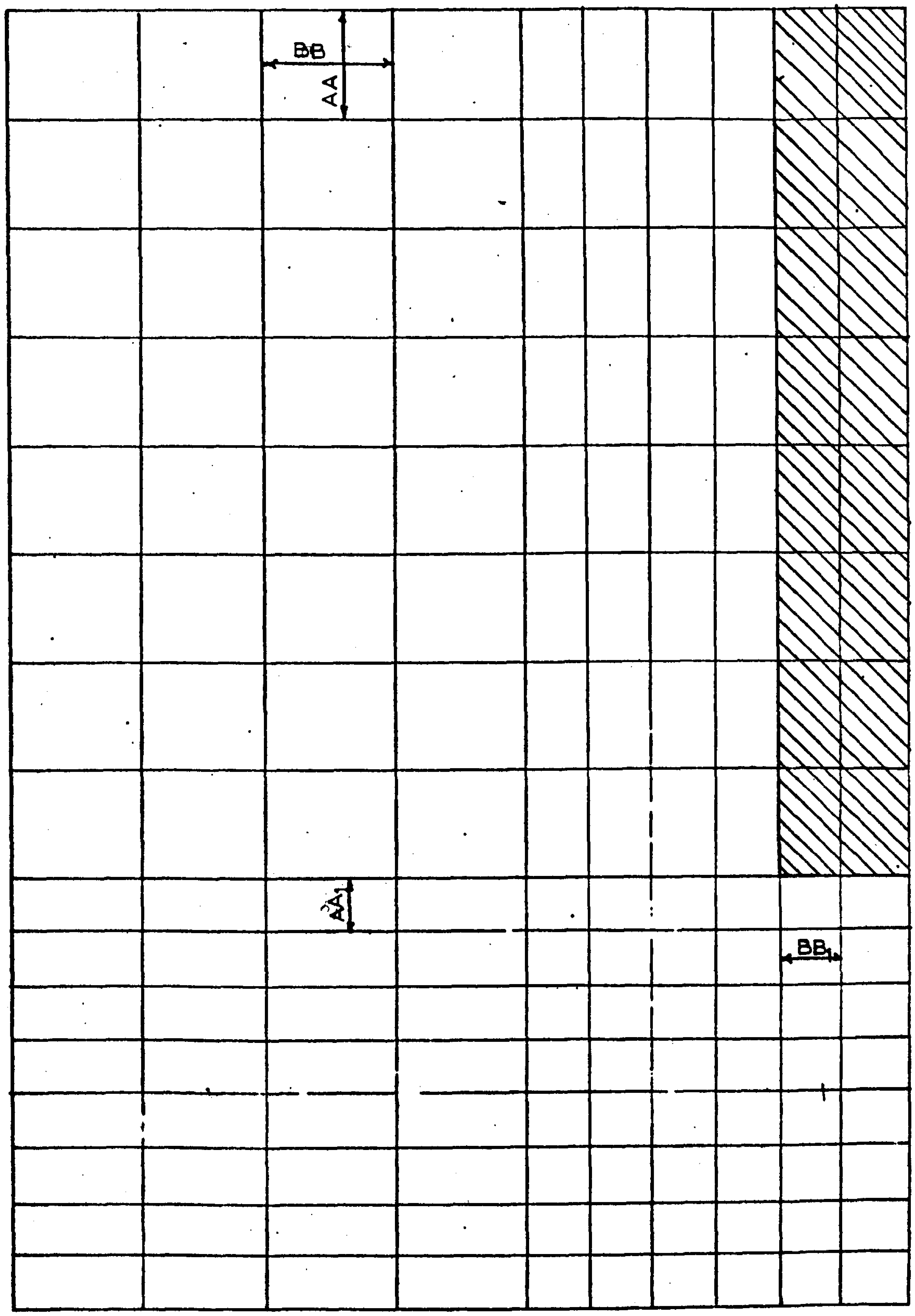


FIGURE 3.3 IDEALISATION OF 1/4 OF SLAB COUPLING A PAIR OF WALLS

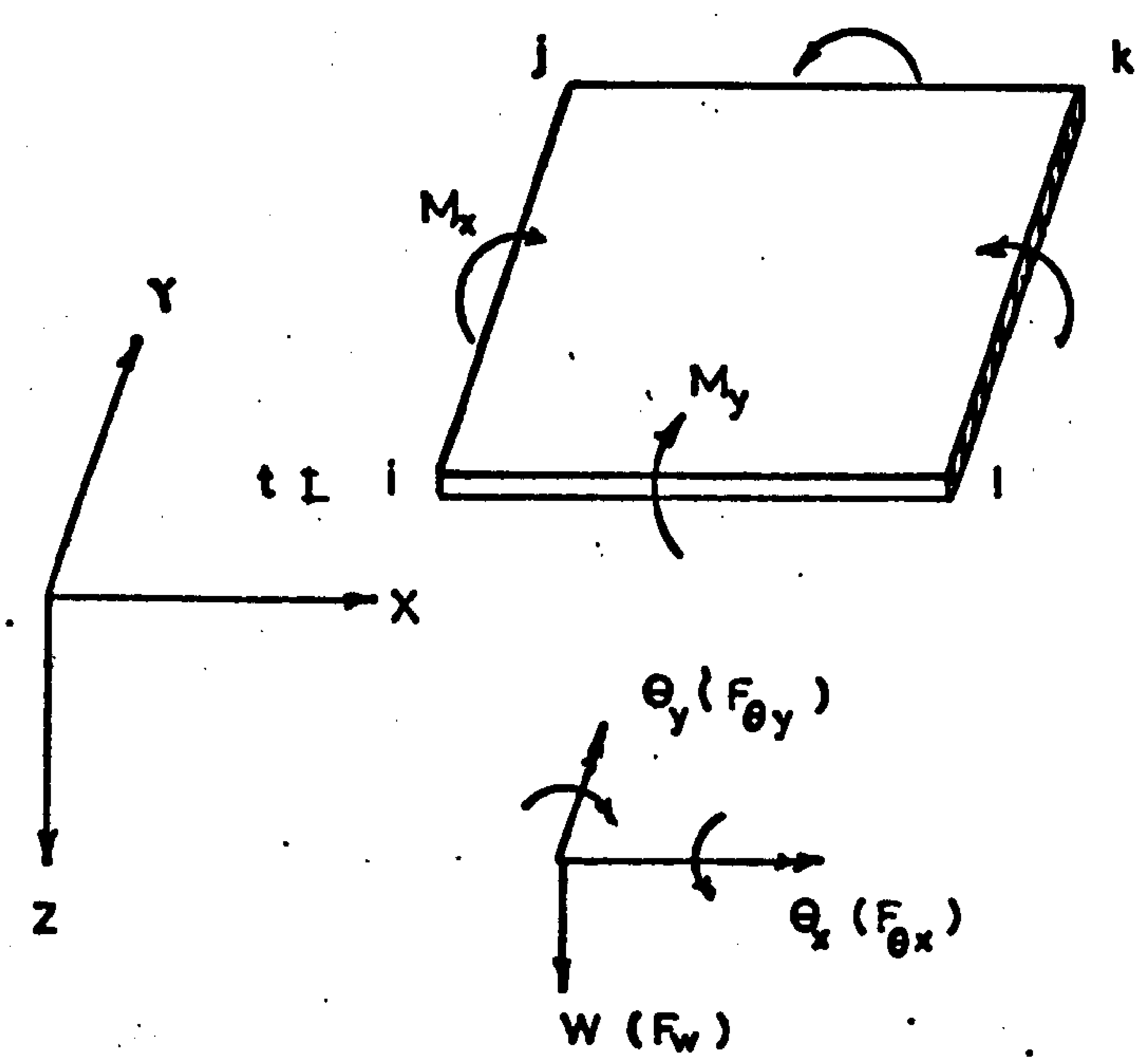


FIGURE 3.4 TYPICAL RECTANGULAR PLATE ELEMENT

In this case a polynomial expression is conveniently used to define the shape functions in terms of twelve parameters. Certain terms must be omitted from a complete fourth-order polynomial (Zienkiewicz (1971)) and writing

$$w = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \alpha_5 xy + \alpha_6 y^2 + \alpha_7 x^3 + \alpha_8 x^2 y + \alpha_9 xy^2 + \alpha_{10} y^3 + \alpha_{11} x^3 y + \alpha_{12} xy^3 \quad (3.15)$$

has certain advantages. In particular along any x = constant or y = constant line, the displacement w will vary as a cubic. The element boundaries or interfaces are composed of such lines. As a cubic is uniquely defined by four constants, the two end values of slopes and displacements at the ends of the boundaries will therefore define the displacements along this boundary uniquely. As such end values are common to adjacent elements, continuity of w will be imposed all along any interface.

One can see that the gradient of w normal to any of the boundaries varies along it in a cubic way. As on such lines only two values of the normal slope are defined, the cubic is not specified uniquely and, in general, a discontinuity of normal slope will occur. The function is thus non-conforming.

The constants α_1 to α_{12} can be evaluated by writing down the twelve simultaneous equations linking the values of w and its slopes at the nodes when the coordinates take up their appropriate values. One can write in matrix form:

$$\{\delta\} = [C]\{\alpha\} \quad (3.16)$$

where $[C]$ is a 12 by 12 matrix depending on nodal coordinates and $\{\alpha\}$ a vector of twelve unknown constants.

The displacement vector for an element i, j, k, l could be written as in equation (3.1):

$$\{f\} = w = [N]\{\delta\} = [P][C]^{-1}\{\delta\} \qquad (3.17)$$

where $[P] = (1, x, y, x^2, xy, y^2, x^3, x^2y, xy^2, y^3, x^3y, xy^3)$

3.5.2 Strains ([B] matrix)

The form of the matrix $[B]$ which relates the strains and displacements is obtained directly from equation (3.15):

$$\{\varepsilon\} = [Q]\{\alpha\} = [Q][C]^{-1}\{\delta\} \quad \therefore \quad [B] = [Q][C]^{-1} \qquad (3.18)$$

where

$$[Q] = \begin{bmatrix} 0 & 0 & 0 & -2 & 0 & 0 & -6x & -2y & 0 & 0 & -6xy & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & -2x & -6y & 0 & -6xy \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 4x & 4y & 0 & 6x^2 & 6y^2 \end{bmatrix} \qquad (3.19)$$

3.5.3 Elasticity matrix

The elasticity matrix $[D]$ which relates the stresses and the strains for isotropic slab is:

$$[D] = \frac{E\ t^3}{12(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \qquad (3.20)$$

i.e. $\{\sigma\} = [D]\{\varepsilon\}$

The formulation of the element stiffness matrix $[kM]$, the determination of the nodal forces $\{F\}$ as well as the assembly and the analysis of a structure is the same as the case of the plane stress element.

3.6 Analysis of complete multi-storey shearwall structures

In order to assess the loads which are carried by the different forms of load bearing units, such as coupled shearwalls and box cores, due to lateral loadings, it is necessary to consider the effect of such loadings on the complete structure. In order to reduce the complexity of the analysis, one can safely ignore, apart from those cases described in the introduction to this chapter, the out-of-plane bending of walls, torsion of individual columns or beams and deform-:ations in the plane of the floors. The method of analysis which is briefly explained here is based on the continuous medium technique for the analysis of coupled shear walls in which the discrete system of connecting beams is replaced by a continuous medium of equivalent stiffness. The method is explained and discussed in detail by Irwin (1970)

As any set of applied forces and moments on a rigid body may always be resolved into a force and couple at any specified position on the body; the total lateral loading on a building can then be represented by a series of point loads applied at a number of equally spaced selected reference levels.

For any arbitrarily chosen vertical axis the resultant lateral load at level 'i' is positioned at a distance Z from the axis so that the building is subjected to a force P_i and twisting moment $P_i Z$ as shown in Figure 3.5. The forces must be restricted by each load bearing unit 'k' at level 'i'

i.e.

$$P_i = \Sigma P_{ik} \qquad (3.21)$$

$$T_i = P_i Z = \sum_{k=1}^{m} P_{ik} Z_k + \sum_{k=1}^{m} T_{ik} \qquad (3.22)$$

where P_{ik} and T_{ik} are the load and twisting moments restricted by unit 'k' at distance Z_k from the chosen vertical axis;

m is the number of load bearing units.

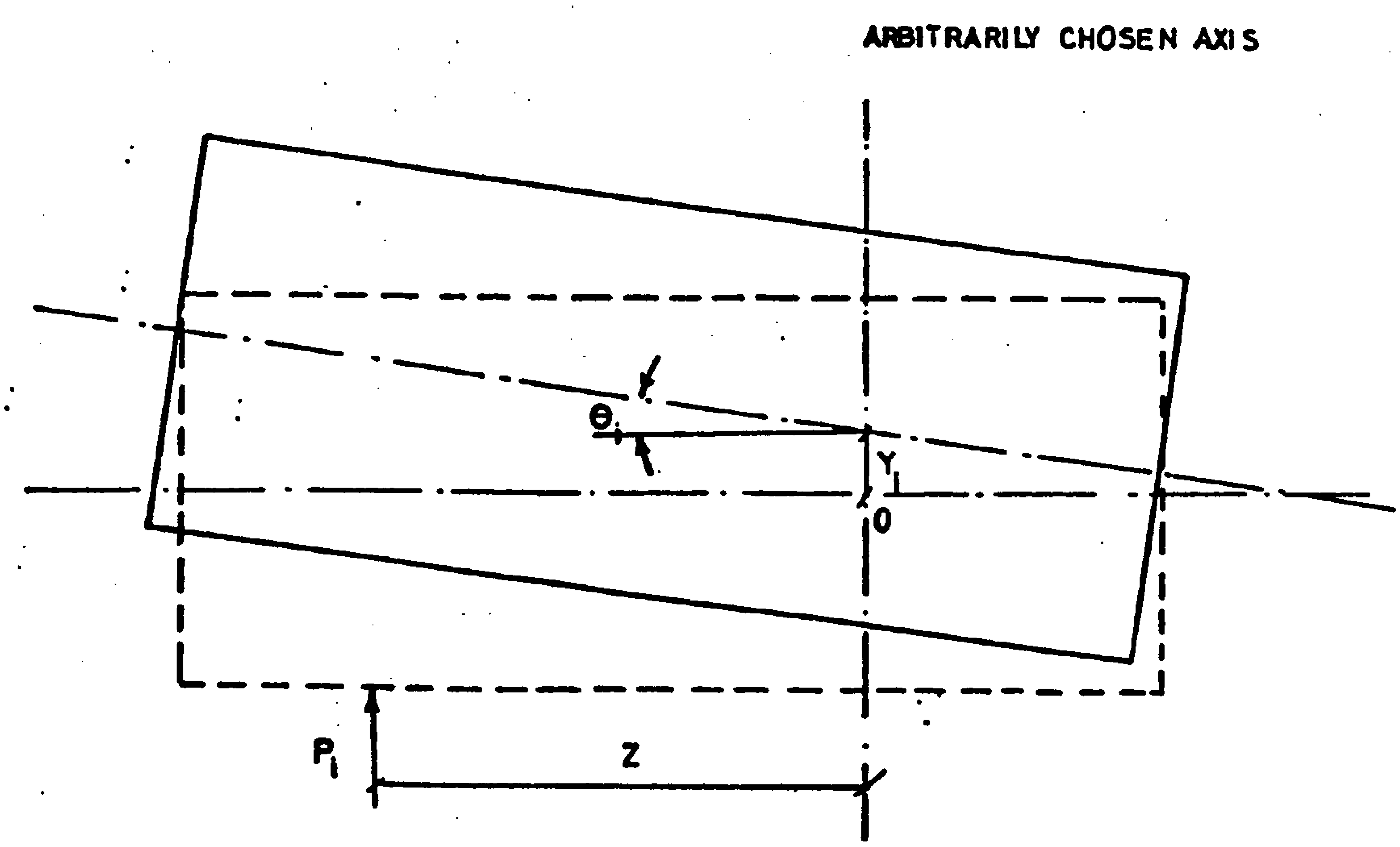


FIGURE 3.5 DISPLACEMENT OF FLOOR SLAB

For the k^{th} load bearing unit the load deflection relationship can be expressed in the matrix form:

$$\{Y\}_k = [F]_k \{P\}_k \qquad (3.23)$$

where $\{Y\}_k$ and $\{P\}_k$ are column vectors of deflections

Y_{ik} and applied loads P_{ik} at the chosen set of reference levels

$[F]_k$ is the square matrix of flexibility coefficients f_{ijk}.

If the floor slabs are assumed infinitely stiff in their own plane, then only rigid body displacements will occur. At any level the total displacement of any unit 'k' at a distance Z_k from the chosen vertical axis can be expressed in the form:

$$\{Y\}_k = \{Y\} + Z_k \{\theta\} = [F]_k \{P\}_k \qquad (3.24)$$

where $\{Y\}$ and $\{\theta\}$ are column vectors of lateral deflections and rotations of the arbitrarily chosen axis as shown in Figure 3.5. The lateral load on load bearing unit 'k' at any level 'i' can be expressed in the form:

$$\{P\}_k = [F]_k^{-1} \left(\{Y\} + Z_k \{\theta\}\right) \qquad (3.25)$$

The equilibrium equations (3.21) and (3.22) can now be written as:

$$[T] = \sum_{k=1}^{m} [F]_k^{-1} (\{y\} + Z_k \{\theta\}) Z_k + \sum_{k=1}^{m} [k]_k [\theta] \qquad (3.26)$$

$$[P] = \sum_{k=1}^{m} [F]_k^{-1} (\{y\} + Z_k \{\theta\}) \qquad (3.27)$$

Where $\{T\}$ and $\{P\}$ are column vectors of the total applied twisting moment and load respectively at each level 'i',

and $[k]_k$ is a square matrix of rotational stiffness coefficients.

The solution of the two equations (3.26) and (3.27) give the structural displacements and rotations about the chosen axis for any system of applied loading causing bending and torsion of the structure. That is:

$$\{Y\} = \left[-\delta_2\ \delta_1^{-1}\ \delta_2 + \delta_3 + \delta_4\right]^{-1}\ \left[-\delta_2\ \delta_1^{-1}\{P\} + \{T\}\right] \tag{3.28}$$

$$\{\theta\} = \left[-(\delta_4 + \delta_3)\delta_2^{-1}\ \delta_1 + \delta_2\right]^{-1}\ \left[-(\delta_4 + \delta_3)\delta_2^{-1}\{P\} + \{T\}\right] \tag{3.29}$$

where

$$\delta_1 = \sum_{k=1}^{m} [F]_k^{-1}$$

$$\delta_2 = \sum_{k=1}^{m} [F]_k^{-1} Z_k$$

$$\delta_3 = \sum_{k=1}^{m} [F]_k^{-1} Z_k^{\,2}$$

$$\delta_4 = \sum_{k=1}^{m} [k]_k$$

Using equation (3.24) and (3.25) the deflection and force vectors for each unit can be determined. The twisting moments at each reference level on load bearing unit 'k' can be obtained from:

$$\{T\}_k = [k]_k\ \{\theta\} \tag{3.30}$$

3.6.1 Analysis of coupled shear walls

The force vector for a specific coupled shear wall bearing unit which is determined using the above method is now applied to equation (3.13) to obtain the nodal displacements. Once the nodal displacement is determined, one can use equations (3.3) and (3.4) to calculate the strains and stresses at all points on the coupled shear wall. If the building consists of several identical coupled shear wall load bearing units (and subject to a symmetrical loading) only the analysis using the finite element method is performed directly without need of the above calculations. In this case the structure can be treated as a two-dimensional coupled shear wall with the total load applied on it.

3.6.2 Analysis of slabs coupling shear walls

Once the forces at the nodes in the intersection between the slabs and the walls is determined from above, these forces are then taken and applied again using equation (3.13) to determine the displacement for all points of the slab. The matrix $[k]$ in this case is the stiffness matrix for the slab. One can then use equations (3.18) and (3.20) to determine the strains and stresses all over the slab.

3.7 Computations and general description of the programs

The numerical work involved in performing the analysis, especially in the case of the finite element analysis, makes the use of an electronic digital computer essential. Computation was carried out on an IBM 360,370 computer and on a Honeywell/66.

The finite element program consists of building blocks in the form of Algol procedures. It is composed of a series of common modules, which may have different uses in different contexts. The main program is segmented as follows:

1. Input of data
2. Build up of node freedom array
3. Build up of element stiffness matrix
4. Formation of total stiffness matrix
5. Formation of the load vector
6. Solution of equations
7. Output of results.

A complete description of the program is given in Appendix II together with a sample program.

The three-dimensional analysis program consists of the following basic sections:

1. Formulation of the structural bending stiffness matrix
2. Formulation of the structural torsional stiffness matrix
3. Evaluation of the overall structural deflections and rotations
4. Evaluation of the deflections and forces of the various load bearing assemblies.

3.8 Dynamic analysis

The method used in the present work for the three-dimensional dynamic analysis of a complete building was first presented by Irwin (1971). The method allows both bending and torsional modes of deformation to be computed together with the periods and natural frequencies of each mode. A brief description of the method is given in this section.

In the case of forced vibrations of the building together with bending and torsional modes of deformation of the system, a set of coupled matrix equations govern the system:

$$\left[\begin{array}{c|c} [M] & \text{related parts} \\ \hline \text{related parts} & [I_\alpha] \end{array}\right] \begin{bmatrix} \ddot{Y}_v \\ \\ \ddot{\theta}_v \end{bmatrix} + \left[\begin{array}{c|c} [F]_Y^1 & \text{related parts} \\ \hline \text{related parts} & [F]_\theta^1 \end{array}\right] \begin{bmatrix} Y_v \\ \\ \theta_v \end{bmatrix} = \begin{bmatrix} P(t) \\ \\ T(t) \end{bmatrix} \tag{3.31}$$

where

$[M]$ is the mass matrix

$[I_\alpha]$ is the mass moment of inertia matrix

If only the independent lateral and torsional modes of vibration are considered then one can isolate the parts of equation (3.31). In addition, for free vibrations of the structure P(t) = 0 and T(t) = 0 then the governing equations become:

$$[M]\ \{\ddot{y}_v\} \ + \ [F]_y^{-1}\ \{y_v\} \ = \ 0 \qquad (3.32)$$

$$[I_\alpha]\ \{\ddot{\theta}_v\} \ + \ [F]_\theta^{-1}\ \{\theta_v\} \ = \ 0 \qquad (3.33)$$

For free vibrations of the system:

$$y_v \ = \ Q \sin (w_n t + u)$$

$$\ddot{y} \ = \ -w_n^2\ Q \sin (w_n t + u)$$

and similar expressions hold for θ where

w_n is the natural circular frequency

t is time

u is the phase angle

Q is the amplitude.

Equations (3.32) and (3.33) can then be written as:

$$(-[M]w_n^2 \ + \ [F]_y^{-1})\ \{e_y\} \ = \ 0 \qquad (3.34)$$

$$(-[I_\alpha]w_n^2 \ + \ [F]_\theta^{-1})\ \{e_\theta\} \ = \ 0 \qquad (3.35)$$

where

$$e \ = \ Q \sin (w_n t \ + \ u)$$

Putting equations (3.34) and (3.35) in the form:

$$([M]^{-1}\ [F]_y^{-1} \ - \ w_n^2\ [I])\ \{e_y\} \ = \ 0 \qquad (3.36)$$

$$([I_\alpha]^{-1}\ [F]_\theta^{-1} \ - \ w_n^2\ [I])\ \{e_\theta\} \ = \ 0 \qquad (3.37)$$

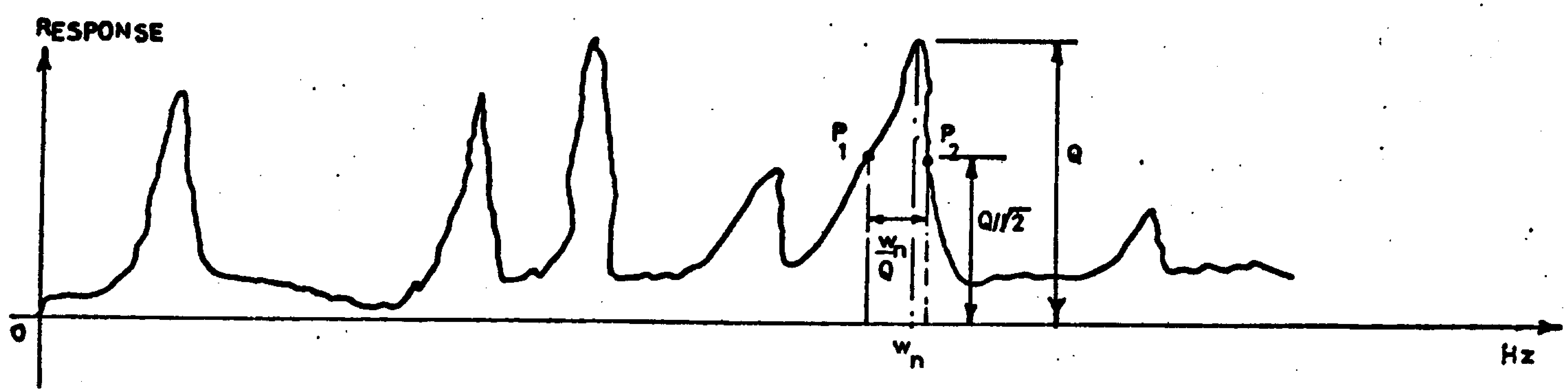


FIGURE 3.6 TYPICAL OUTPUT FROM A SINGLE ACCELEROMETER

enables the eigenvalues and eigenvectors for any number of modes of vibration to be found.

Once the eigenvalues have been computed the natural frequencies and periods can be found from:

$$f = \frac{w_n}{2\Pi} = \frac{1}{T} \tag{3.38}$$

3.8.1 Evaluation of maximum values of response

Three quantities are required to evaluate the maximum values of response for each mode 'n':

a) The ordinate of displacement spectrum S_n at period of mode T_n, from a wind or earthquake (design) spectra.

b) The modal participation factor $\zeta_n = \dfrac{\Sigma e_{in} M_i}{\Sigma e_{in}^2 M_i}$

c) The magnitude of the mode 'n' shape which is found in computation of the eigenvectors.

where e_{in} is the magnitude of the mode shape in the mode 'n' at level 'x_i'.

The S_n value is obtained either from a design spectrum for earthquakes of a selected magnitude (% gravity) allowing for a damping compatible with the form of concrete structure under consider:ation, or from formulae such as those given by the Committee of Structural Steel Producers of American Iron and Steel Institute (1960).

For wind loading similar values of parameters can be found from probability spectra.

The maximum values of response at each level for a given mode are:

$$Y_{in_{max}} = \zeta_n e_{in} S_n \tag{3.39}$$

$$\theta_{in_{max}} = \frac{\zeta_n e_{in} S_n}{R_i} \tag{3.40}$$

The ordinate of displacement spectrum, S_n in equation (3.40) is divided by the radius of gyration of the entire structure, R_i, at each level x_i for compatibility.

3.8.2 Maximum lateral and rotational displacements

From probability studies it is generally accepted that a root mean square superposition of modes yields realistic values:

$$y_{i_{max}} \approx \sqrt{\Sigma\, y^2_{in_{max}}} \tag{3.41}$$

$$\theta_{i_{max}} \approx \sqrt{\theta^2{}_{in_{max}}} \tag{3.42}$$

The maximum probable deflection of a unit at a distance Z_k from the calculated centre of rotation would then be:

$$[y]_{k_{max}} = [y]_{max} + Z_k\,[\theta]_{max} \tag{3.43}$$

3.8.3 Estimation of damping and dynamic modulus of elasticity

The half-power band-width method is used to calculate damping for existing systems. Figure 3.6 shows a typical output from a single accelerometer, obtained by forcing a structure to vibrate at a constant frequency, measuring the response and stepping frequency until sufficient data has been recorded.

The half-power points P_1 and P_2 are first evaluated as shown in Figure 3.6. The frequencies corresponding to these points i.e. w_1 and w_2 can be related to the critical damping using the relation:ship $(w_2 - w_1) = w_c/Q$.

As $Q = \frac{1}{2\gamma} = \frac{\sqrt{km}}{d}$ where $\gamma = \frac{d}{d_{crit}}$

$\therefore$ knowing $\frac{w_n}{Q}$, Q and w_n

$$\therefore \quad (w_2 - w_1) = \frac{w_c}{Q} = \frac{w_c}{1/(2\gamma)}$$

i.e.
$$\gamma = \frac{(w_2 - w_1)}{2w_c} \qquad (3.44)$$

where k = stiffness $\qquad w$ = frequency

d = damping $\qquad P_i$ = half-power points

Using this method for a system such as a uniform beam or single degree of freedom mass one can determine the natural frequencies. Then using the appropriate exact equation for finding the natural frequencies of the system the dynamic modulus of elasticity of the structural material at each natural frequency can be calculated.

For example for a uniform cantilever as shown in Figure 3.7 the dynamic modulus of elasticity can be determined from:

$$w_1 = \frac{3.516}{l^2} \left(\frac{EI}{\rho A}\right)^{\frac{1}{2}} \quad \text{for the first mode}$$

$$w_2 = \frac{22.03}{l^2} \left(\frac{EI}{\rho A}\right)^{\frac{1}{2}} \quad \text{for the second mode} \qquad (3.45)$$

$$w_3 = \frac{61.70}{l^2} \left(\frac{EI}{\rho A}\right)^{\frac{1}{2}} \quad \text{for the third mode}$$

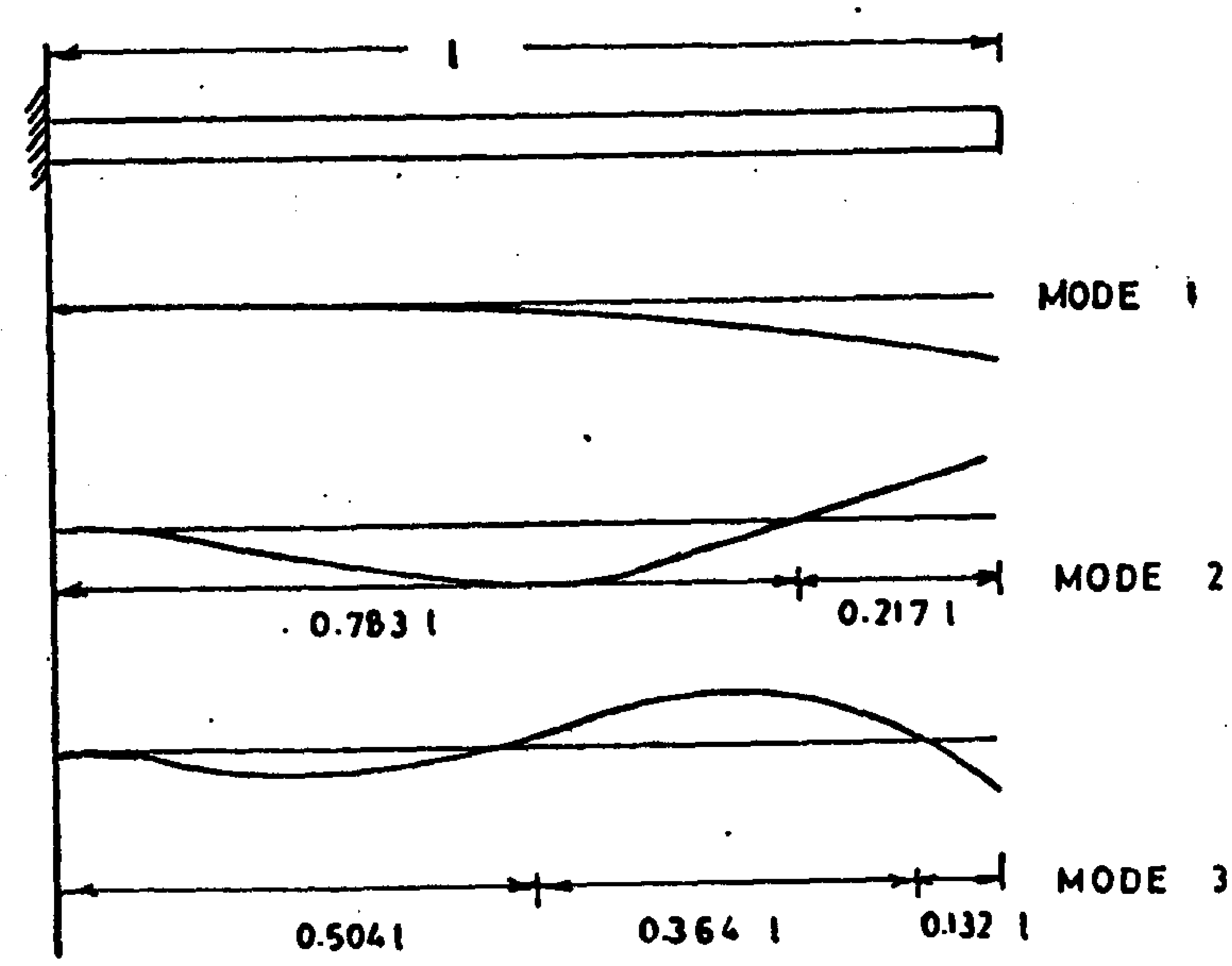


FIGURE 3.7

CHAPTER FOUR

EFFECTIVE WIDTH OF SLABS CONNECTING SHEAR WALLS

4.1 Introduction and past work

As shown in the previous chapters shear walls are used in multi-storey buildings to efficiently resist lateral and gravity loads. In a coupled shear wall structure the external lateral load generates flexure, shear and axial load in the walls in addition to compression and bending with high shear stresses in the coupling slabs. Due to bending stiffness of the slabs the walls do not act as independent simple cantilevers. In this way, depending on the plan configuration and dimensions, the slabs, in conjunction with the shear walls, provide stiffer and stronger structures to resist the lateral loads. Therefore, to achieve a more efficient and economical building of this type it is important to be able to assess accurately the stiffness or the effective width of the slabs which connect the shear walls. Little work has been done or published so far to help the designer in a rapid evaluation of the effective width of slabs coupling shear walls.

In 1969 Qadeer and Stafford Smith described a theoretical and experimental investigation of the bending stiffness of slabs due to the parallel rotation of pairs of equal plane walls. In their theoretical analysis they used the finite difference technique. The experimental investigations were done using a model comprising of two heavy steel plate walls coupled to an asbestos cement sheet slab. They presented curves to enable the 'effective width' for the connecting slab coupling a pair of shear walls to be found and to illustrate the influence of the different parameters on the stiffness of the slab against rotation. In 1974 the same authors gave information on the distribution of bending and twisting moments and shear forces through the slab.

Coull and El Hag (1975) investigated, in their experimental work, the influence of the dimensions and shape of the walls, wall spacing and slab dimensions on the stiffness and the effective width of the coupling slabs. In their small scale models they formed the walls from steel plates and the slabs from perspex sheet. The study was purely experimental and a limited number of tests were used to draw the curves they presented.

In 1977 Tso and Mahmoud used a triangular bending element in their finite element method to obtain the stiffness of a slab coupled shear wall system. They presented design curves in terms of the effective width of the slab between the shear walls. These curves were given for a number of wall system configurations. Comparison of their results were made with the experimental results of Coull and El Hag and Qadeer and Stafford Smith's theoretical results, some differences being noted.

4.2 Formulation of problem and theoretical analysis

If a s shear wall building consists of shear walls and floor slabs as shown in Figure 4.1, two situations can cause interaction between the walls and the floor slabs:

1. When the building is subjected to lateral horizontal loadings which tend to axially deform the building and rotate the vertical walls, as shown in Figure 4.2a, the floor slabs react and resist the bending and axial deformations of the walls in addition to their strut action to maintain the wall spacing.

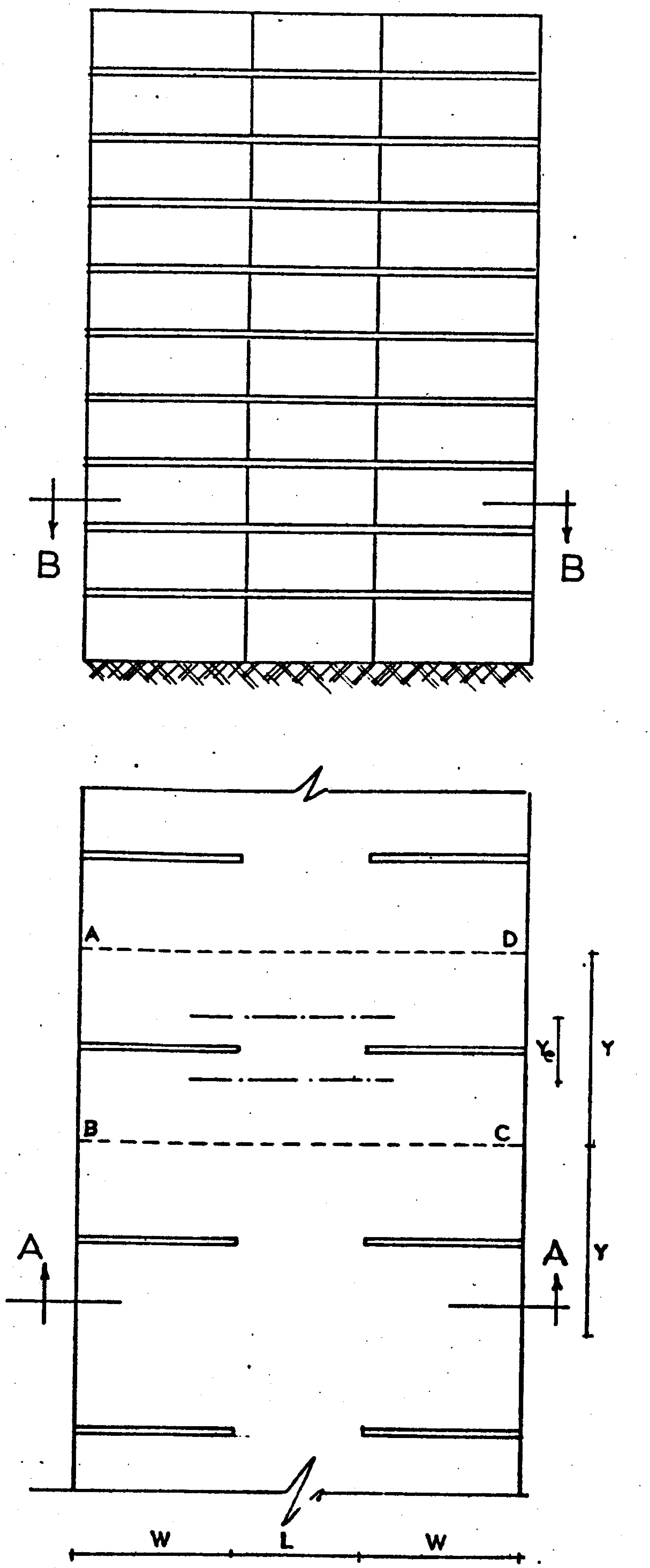


FIGURE 4.1 SHEAR WALLS COUPLED WITH SLABS STRUCTURE

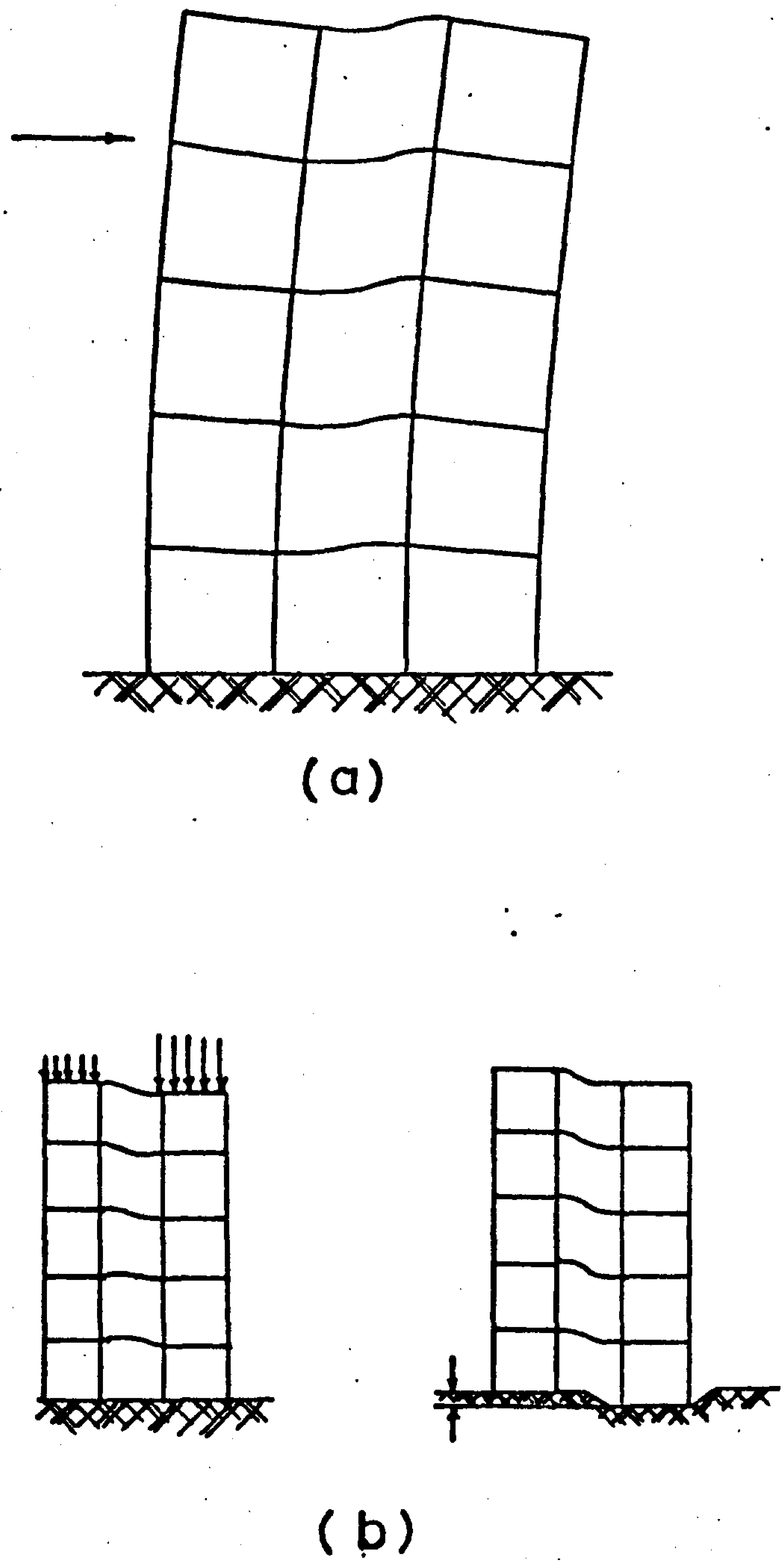


FIGURE 4.2 DEFORMATION OF CROSS WALL STRUCTURE AND SLABS UNDER VARIOUS CONDITIONS

2. If there is a relative vertical movement of the walls due to either differences in the vertical loadings on the walls or relative settlement or temperature expansion of the walls, the action of the floor slabs coupling the walls is as shown in Figure 4.2b.

Assuming the slab is rigidly attached to the walls the effect of the rotation of the walls caused by the lateral load is similar to that due to relative vertical movement as shown in Figures 4.3a,b. The stiffness and effective width of the floor slabs can, therefore, be obtained in two ways.

4.2a The method used by Qadeer and Stafford Smith (1969)

Referring to Figure 4.3a and using the moment area diagram to determine the relation between the moment M and the angle of rotation θ due to the rotation of the walls, let the deflection δ equal the moment of area ~~abo~~ about 0 then:

$$\delta = \frac{M}{EI} \frac{L^3}{12(L + w)} \tag{4.1}$$

and

$$\theta = \frac{2\delta}{L + w} \tag{4.2}$$

$$\therefore \quad M = \frac{6EI(L + w)^2}{L^3} \theta \tag{4.3}$$

where EI is the equivalent beam stiffness of the connecting slabs. The stiffness of the slab is then:

$$K = \frac{M}{D\theta} \tag{4.4}$$

where D is the stiffness of slab per unit width

$$= Et^3/12(1 - \nu^2)$$

E and ν are the modulus of elasticity and Poisson's ratio.

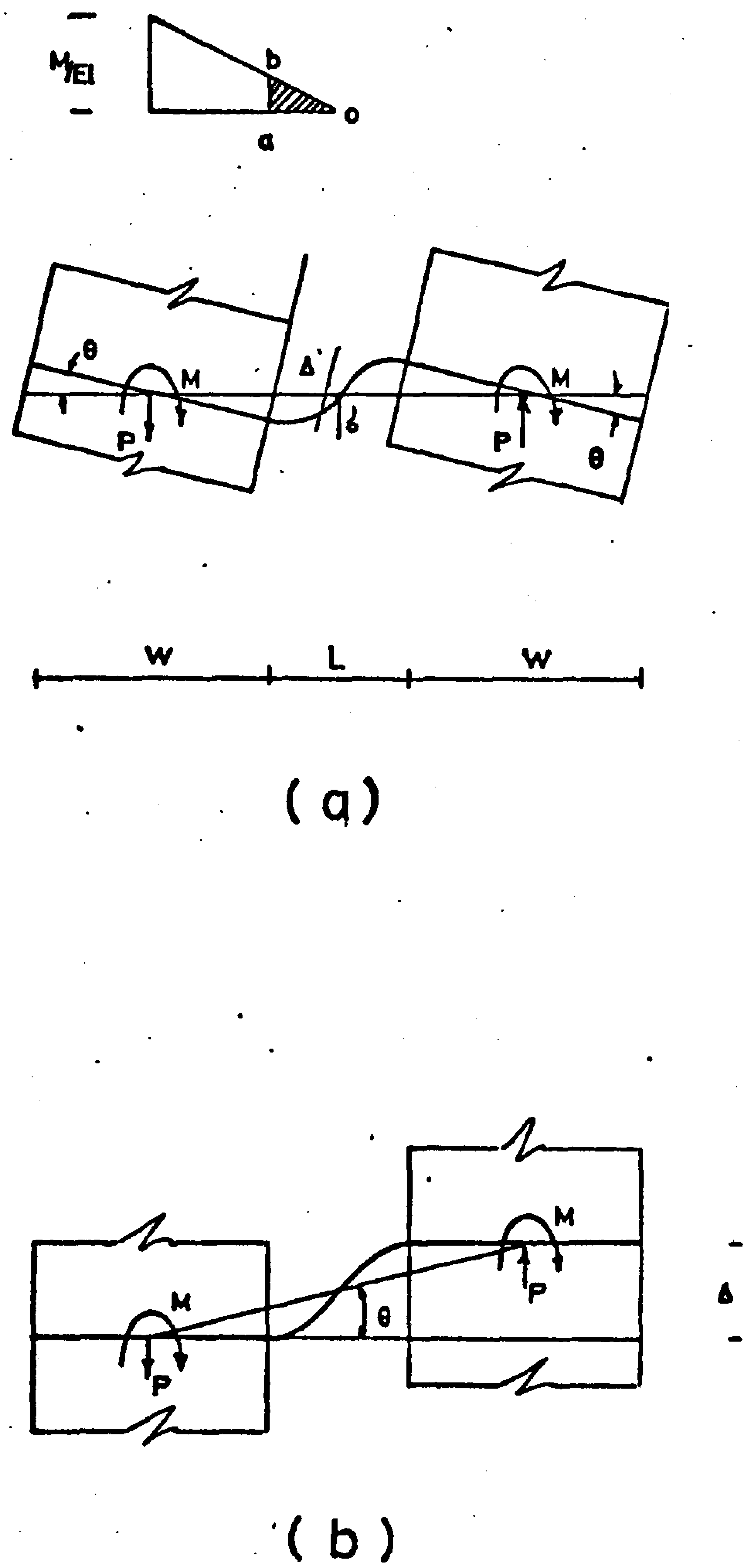


FIGURE 4.3 SLAB DEFORMATION RESULTING FROM: (a) WALL ROTATION (b) RELATIVE VERTICAL MOVEMENT

The ratio of the effective width Y_e to the full width of slab is then given by:

$$Y_e/Y = \frac{KL^2/(L+w)^2}{6(1-\nu^2)(Y/L)} \tag{4.5}$$

4.2b <u>The method used in the present work</u>

In this method the simple cantilever theory is used to determine directly the effective width and the stiffness of the slabs coupling shear walls. Referring to Figure 4.4 the deflection for the equivalent cantilever which represents half the slab is given by:

$$\Delta/2 = \frac{1}{3}\frac{P(L/2)^3}{EI} \tag{4.6}$$

where I is the moment of inertia for the equivalent beam = $\dfrac{Y_e\ t^3}{12}$

t is the slab thickness

Y_e is the effective width of the slab.

Then

$$\Delta = \frac{P\ L^3}{E\ Y_e\ t^3} \tag{4.7}$$

Therefore, the effective width is given by:

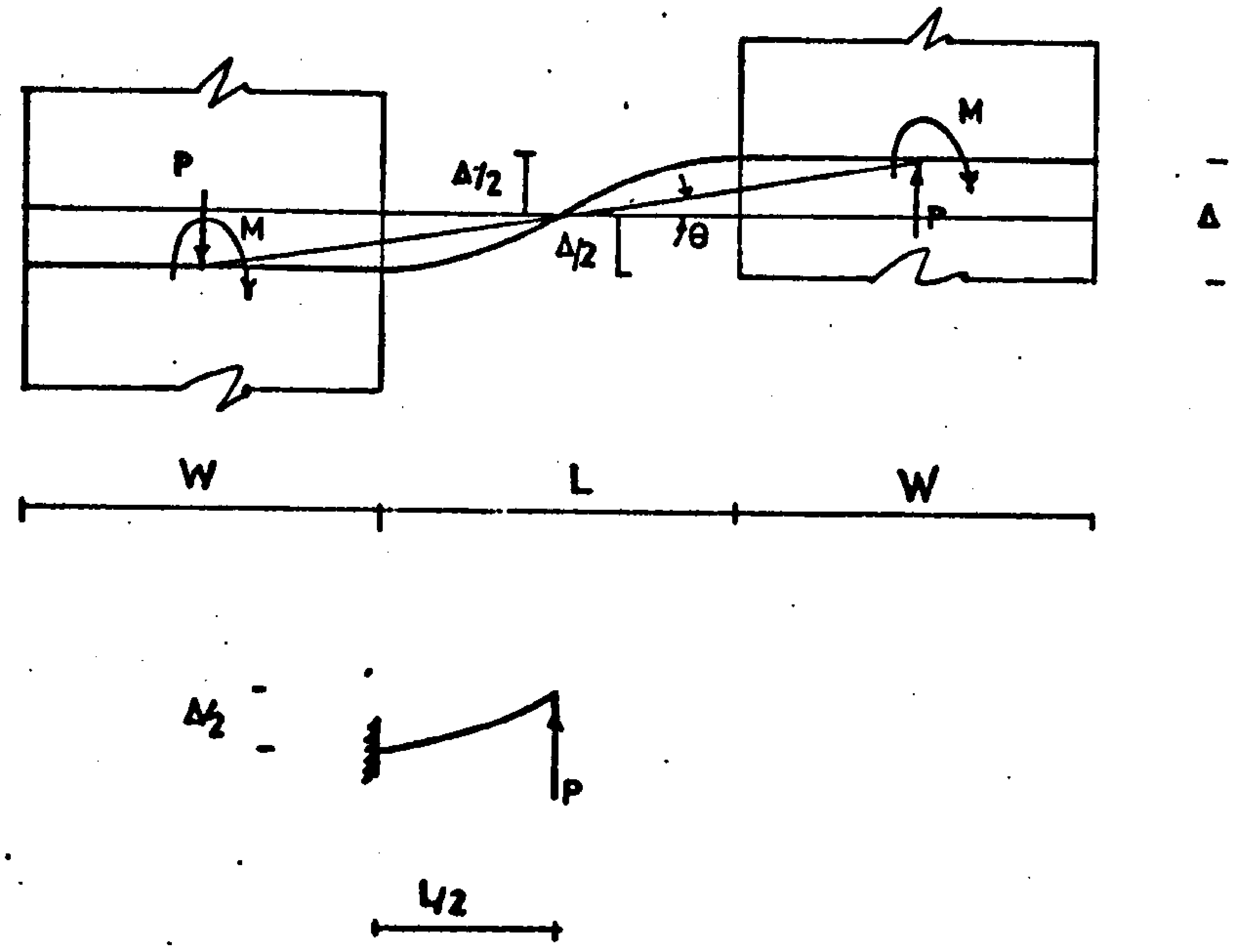
$$Y_e = \frac{P\ L^3}{E\ t^3\ \Delta} \tag{4.8}$$

where P is the force on one wall

Δ is the total relative vertical displacement between the two walls as shown in Figure 4.4

L is the wall opening distance.

The stiffness of the floor slab can be defined in two ways; the first is to define the stiffness by the relationship between the angle of rotation θ and the moment M as given by equation (4.4).



<u>FIGURE 4.4</u>   <u>CANTILEVER REPRESENTATION OF SLAB</u>

In this case no vertical movement of the walls is allowed. The second is to define the stiffness of the floor slab as the vertical force on one wall required to produce a relative displacement Δ equal to unity between the points of the slab attached to the right and left walls. In this situation no rotation of the shear wall is permissible. So, using equation (4.8) and letting Δ equal unity:

$$K_1 = \frac{Y_e E t^3}{l^3} \qquad (4.9)$$

K_1 can be determined directly by imposing a unit relative displacement between the walls. The effective width Y_e can then be found from:

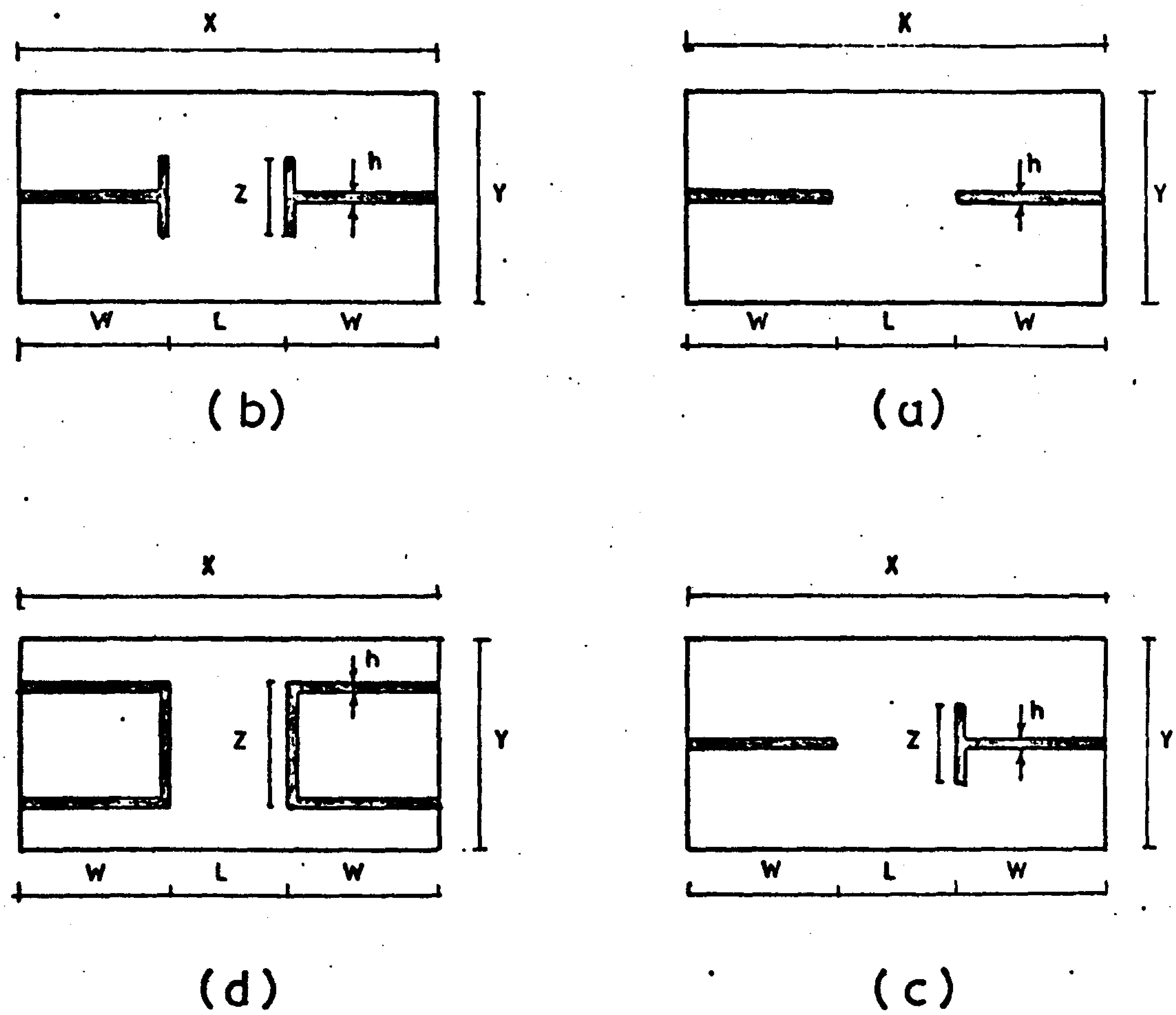
$$Y_e = \frac{K_1 l^3}{E t^3} \qquad (4.10)$$

instead of equation (4.8).

In the present work the both methods are used to estimate the stiffness of the slab coupling the shear walls. The values obtained from the first method can be compared to these obtained by Qadeer and Stafford Smith and the experimental values obtained by Coull and El Hag.

The finite element displacement method was used to determine the force required to impose a unit relative displacement between the walls. The rectangular element for thin plate in bending described in the previous chapter was used in performing the idealization of the slab. The slab was assumed homogeneous, isotropic and linearly elastic. All the nodes had three free degrees of freedom except those representing the walls, which had only one free degree of freedom which represents the vertical displacements.

Four types of wall configurations were studied and Figure 4.5 shows these configurations. Advantage was taken in the cases where



(a) Plane walls

(b) T walls

(c) Plane and T walls

(d) Box cores

FIGURE 4.5 SLAB COUPLED SHEAR WALL CONFIGURATIONS there is symmetry and anti-symmetry to reduce the computational time and effort. Figure 4.6 shows a typical plane wall and slab configuration with the finite element idealization superimposed.

4.3 Discussion of results

The four configurations shown in Figure 4.5 are studied together with the configuration of an end panel for the core of the plane walls. The symbols chosen to represent the overall dimensions of the slab and walls are shown in Figure 4.5. Curves are given to determine the effective width of the slabs as a ratio of the complete slab width and to show the variation of the stiff-:ness due to the various parameters. In Figure 4.7 a comparison between the present method and the results obtained by Qadeer and Stafford Smith (1969) and Tso and Mahmoud (1977) is given for the case when the thickness of the wall is equal to zero. It is clear from this Figure that the finite element methods give higher values for Y_e/Y than those obtained by the finite difference technique.

4.3.1 Slab and plane wall configuration

The slab and plane shear wall structure is the most common type in the construction of coupled shear wall tall buildings. The design curves presented in Figures 4.8 and 4.9 are for a wall thickness h = 25 cm and slab thickness of 20 cm for different values of L/X and Y/X ratios. Figures 4.10 to 4.12 show the effect of the various parameters on the stiffness of the slab. It is obvious that the stiffness of slab is significant for values of L/X less than about 0.3, and the influence of slab stiffness increases with increase of the ratio Y/X. For an end bay with wall thickness equal to that of an interior bay the stiffness of such a slab is found to be 47% of that of an interior panel. This value is compared with the 54%

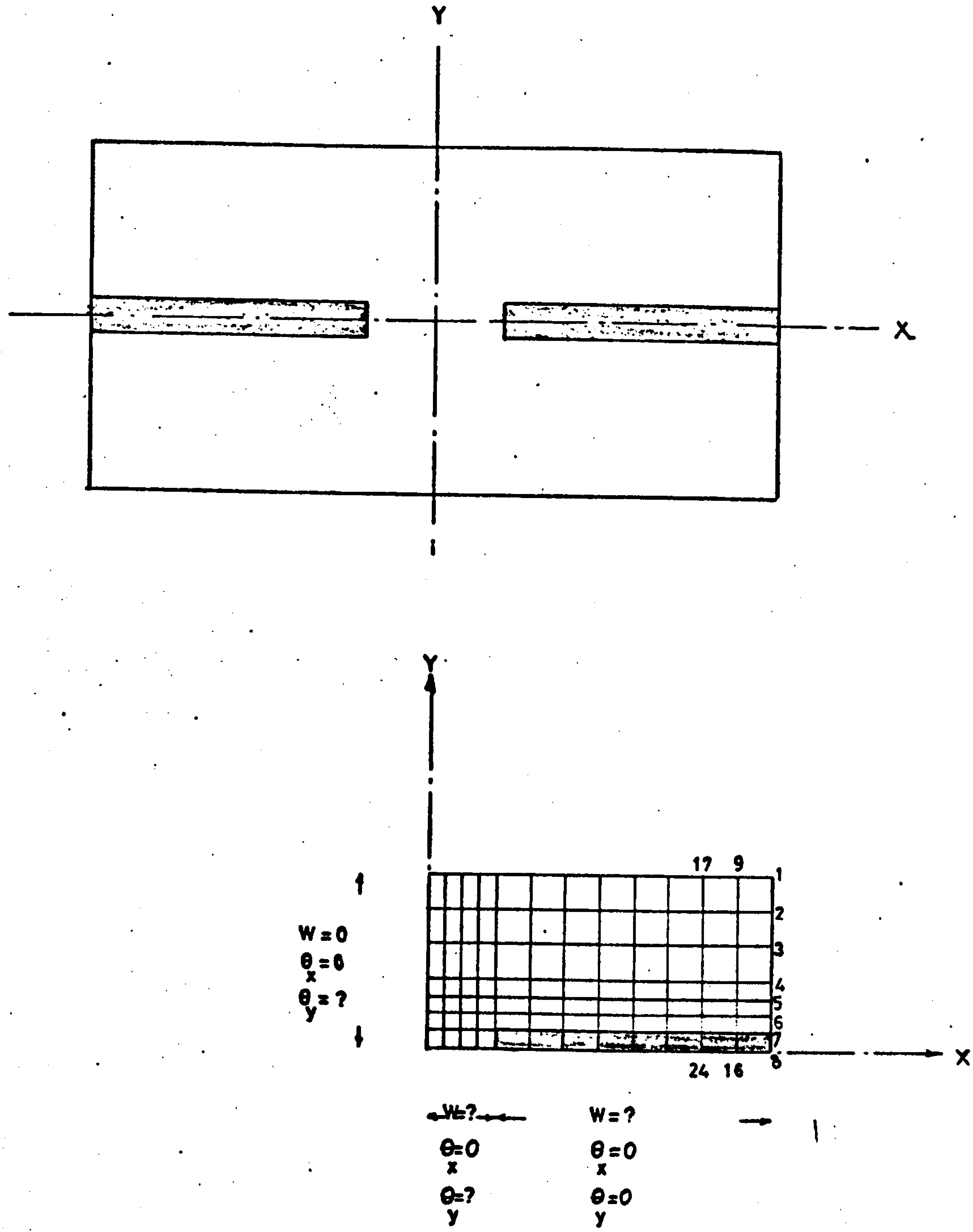


FIGURE 4.6 PLANAR WALLS AND SLAB CONFIGURATION

suggested by Tso and Mahmoud and the 42% suggested by Qadeer and Stafford Smith. A comparison between the bending stresses and twisting stresses is shown in Figure 4.13. The effect of the various walls and slab parameters on the maximum bending stress and deflection is given in Figures 4.14 - 4.17.

4.3.2 Slab and T-section coupled walls configuration

It was found during the analysis of the planar coupled walls that the change in the thickness of the wall leads - specially in the case where L/X < 0.3 - to a significant change in the stiffness and effective width of the slab, so it was obvious that by using T-section coupled walls both the stiffness and the effective width of the slab will increase depending on the width of the flang of the T-section wall, as shown in Figure 4.18. The design curves and the effect of the various dimensions on the stiffness of the slab are shown in Figures 4.19 to 4.24. The finite element method tends to give higher values for the stiffness of the slab than those obtained experimentally by Coull and El Hag. The writer feels that more experimental work is required for this particular case to assess accurately the effect of the local bending deformations of the T-section flange and the effect of wall/slab thickness ratio.

As in all the experimental work done by Coull and El Hag, and by Qadeer and Stafford Smith the walls were represented using rigid steel plates with either asbestos cement sheet or perspex sheet slabs which does not permit the flange deformation due to local bending moments. Figure 4.25 gives a comparison between the bending stresses of the planar coupled shear wall configuration and the T-section configuration.

4.3.3 Slab and box core walls configuration

For this configuration which is shown in Figure 4.5d the calculations have been carried out for slab dimensions Y/X = 0.5 and Z/X ratio = 0.2 and L/X ratio from 0.1 - 0.7. The stiffness of such configuration was found to be less than that of the T-section coupled wall confirguration by less than 2% for L/X between 0.2 - 0.7 and less than 7% for L/X = 0.1. The difference between the stiffness for the box core walls and T-section walls is due to the reduction of the local bending deformation of the flange because of the double walls support as against a single wall in the case of the T-section.

4.3.4 Coupled planar and T-section walls

Figures 4.26 - 4.29 show the results obtained for this type of configuration for flang/slab length ratios Z/X = 0.1 and 0.2 and for Y/X = 0.5 and 0.75. In this situation, as in the case of the coupled T-section configuration, the local bending deformation of the flange may reduce the stiffness and the effective width of the slab. Although in this case the effect will be less than in the case of the coupled T-section. In fact, with the increase of the width of the slab the stiffness of the slab becomes insensitive to the increase of the flange width 'Z' of the T-section wall.

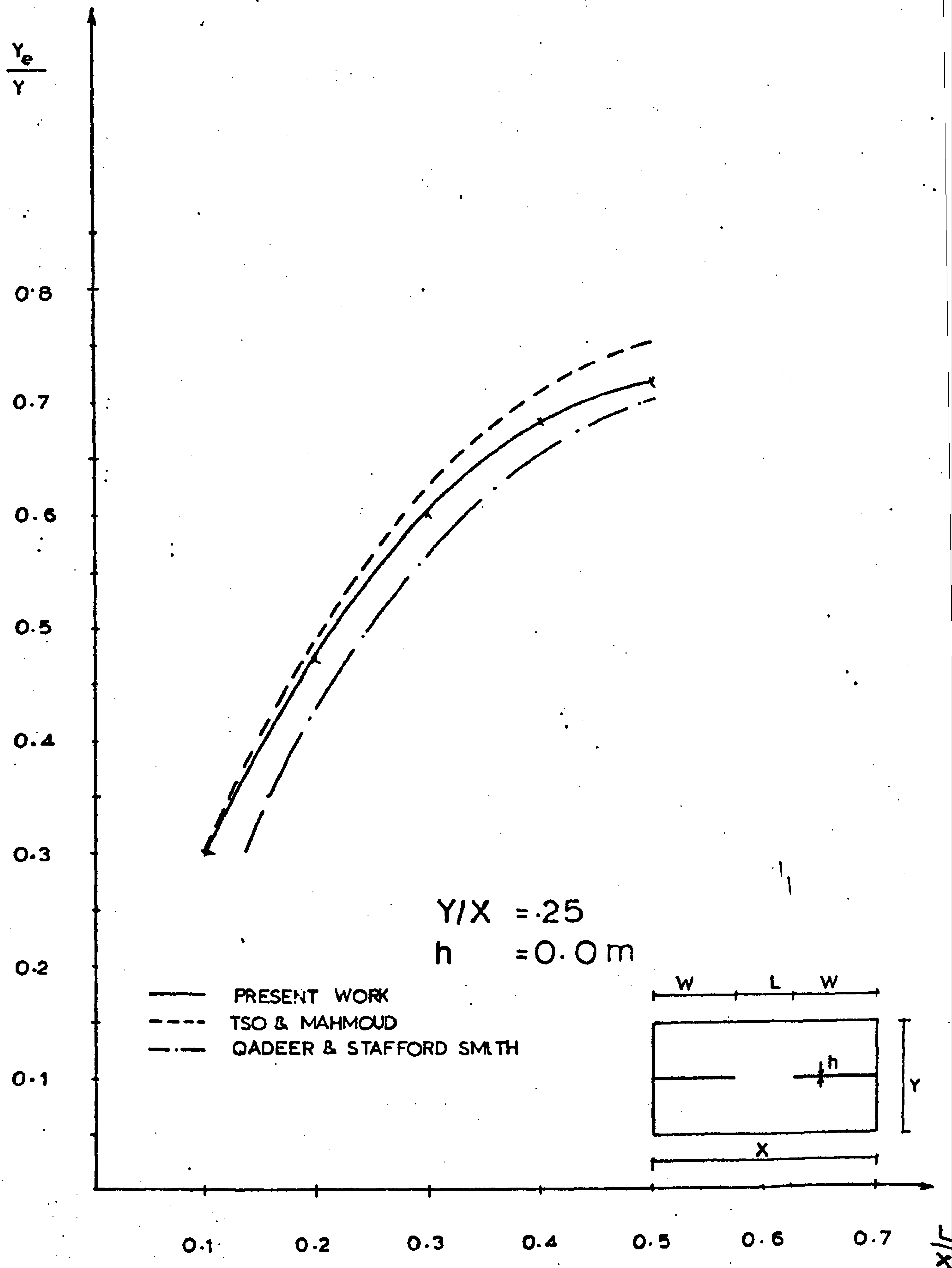


FIGURE 4.7(a) COMPARISON FOR WALL THICKNESS = ZERO

FIGURE 4.7(b) VALUE OF SLAB STIFFNESS NUMBER K FOR Y/X = 0.25

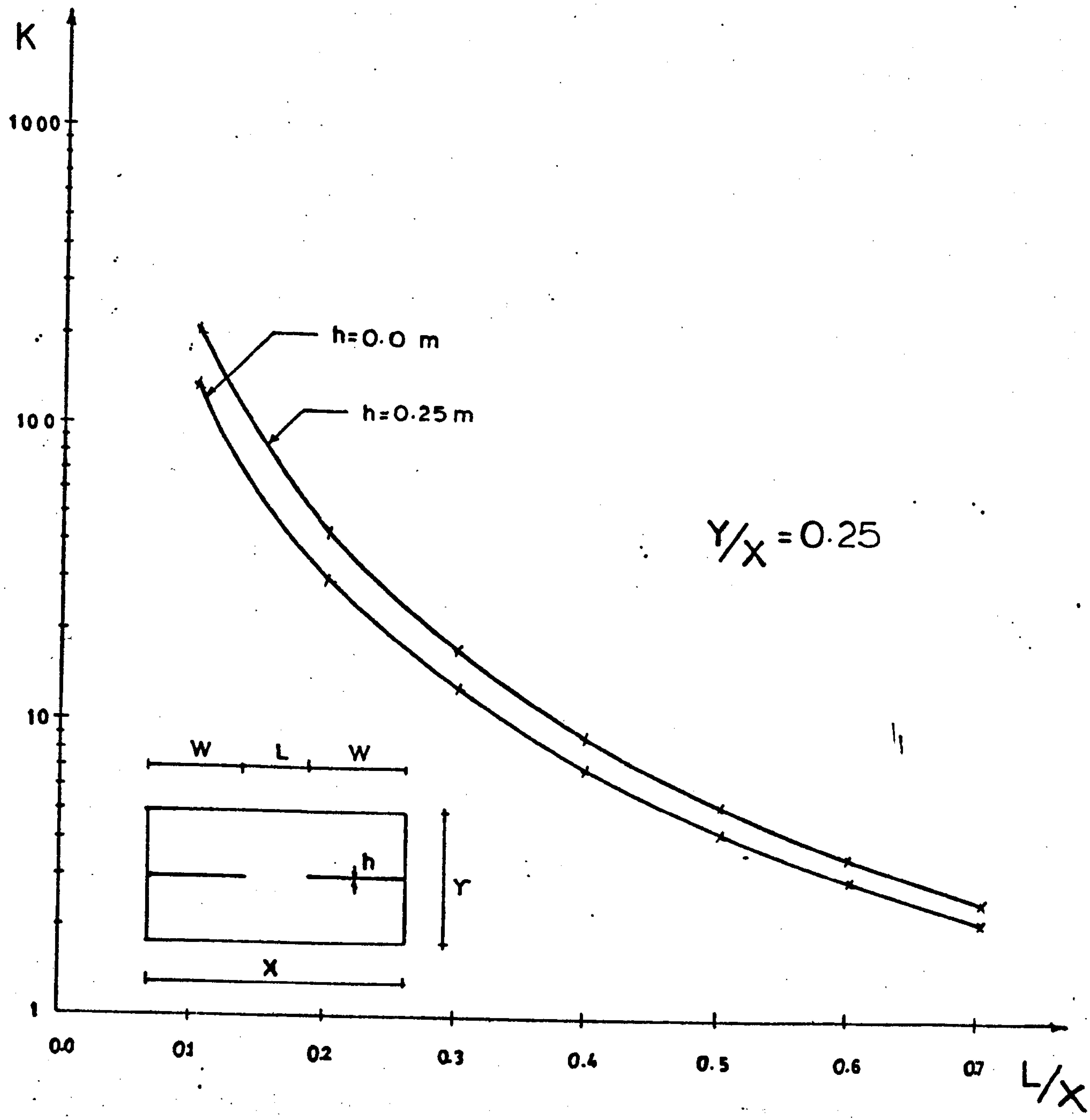


FIGURE 4.7(c) VALUE OF SLAB STIFFNESS NUMBER K

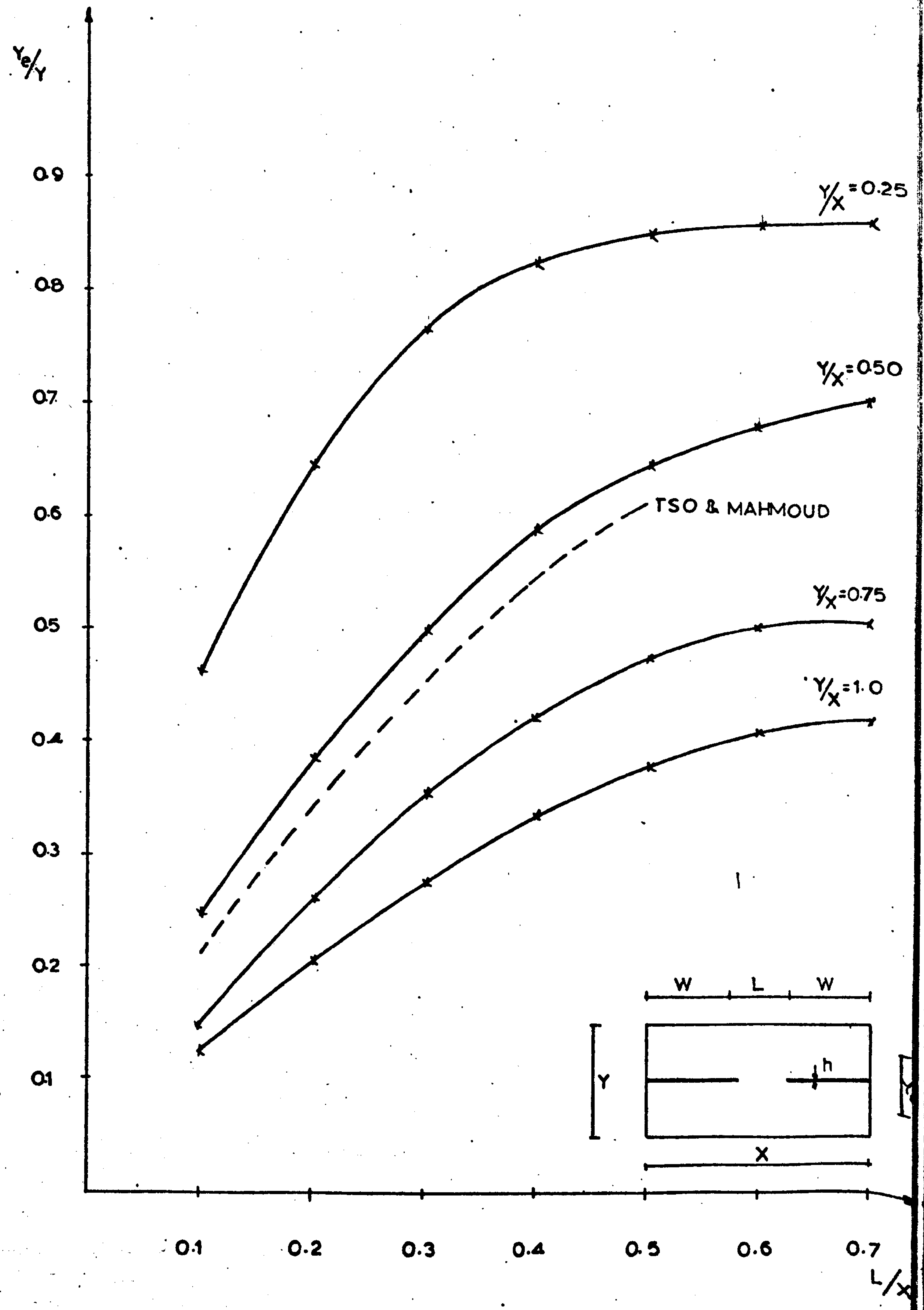


FIGURE 4.8 DESIGN CURVES (VALUE OF Y_e) FOR PLANAR COUPLED WALLS

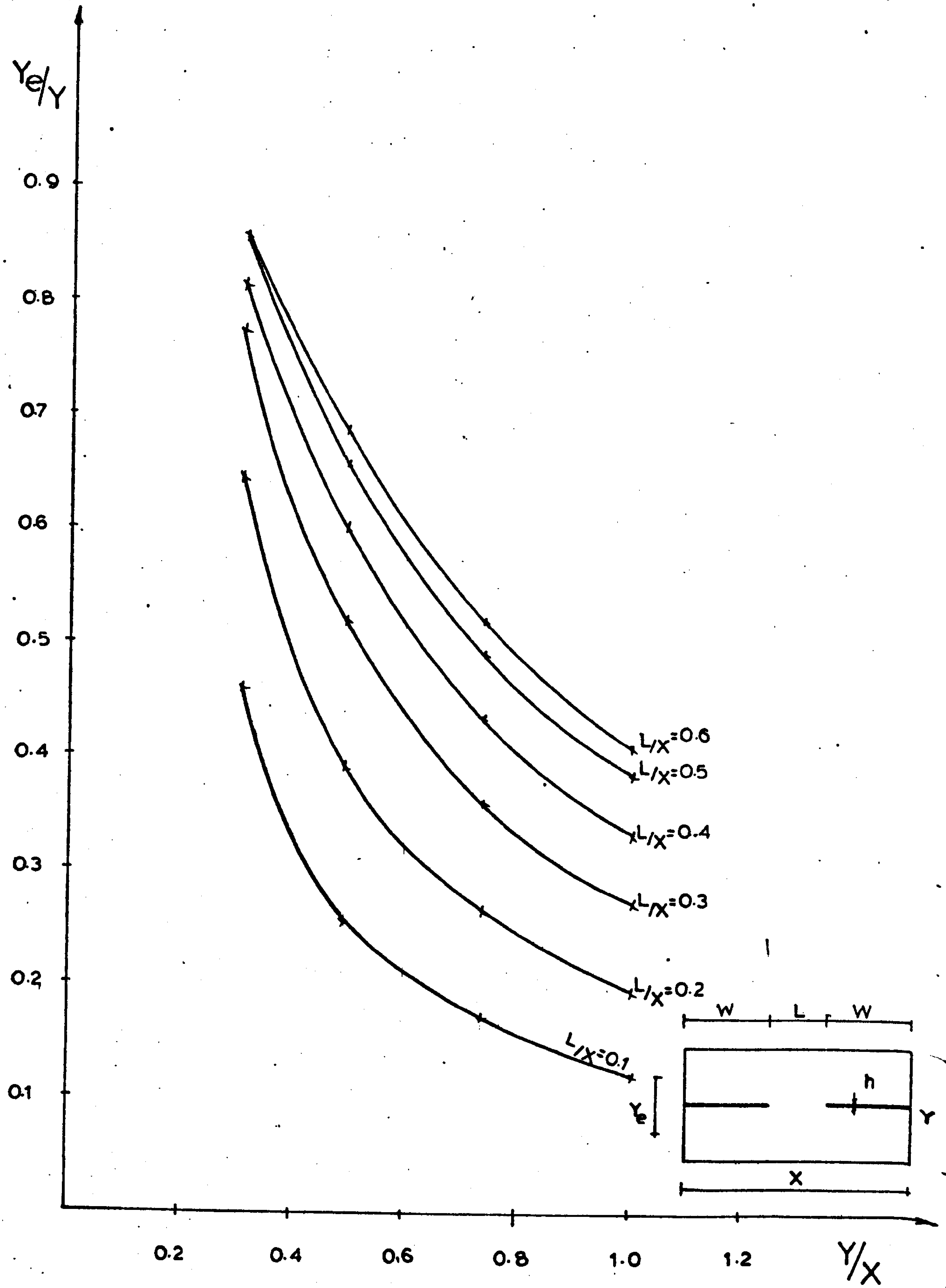


FIGURE 4.9 DESIGN CURVES FOR PLANAR COUPLED WALLS

FIGURE 4.10 VARIATION OF THE STIFFNESS NUMBER K FOR PLANAR WALLS

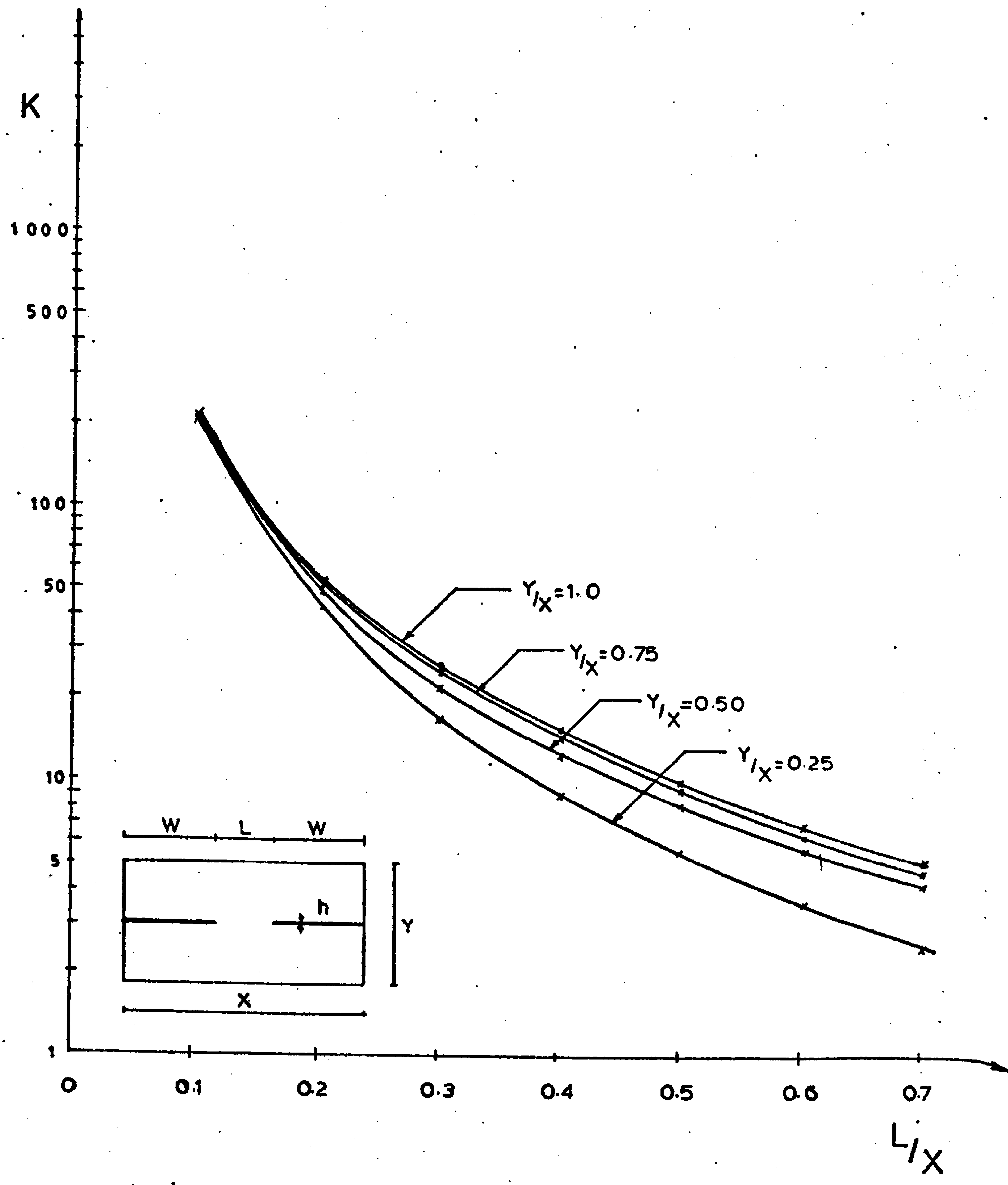


FIGURE 4.11(a) VARIATION OF STIFFNESS NUMBER K FOR PLANAR COUPLED WALLS

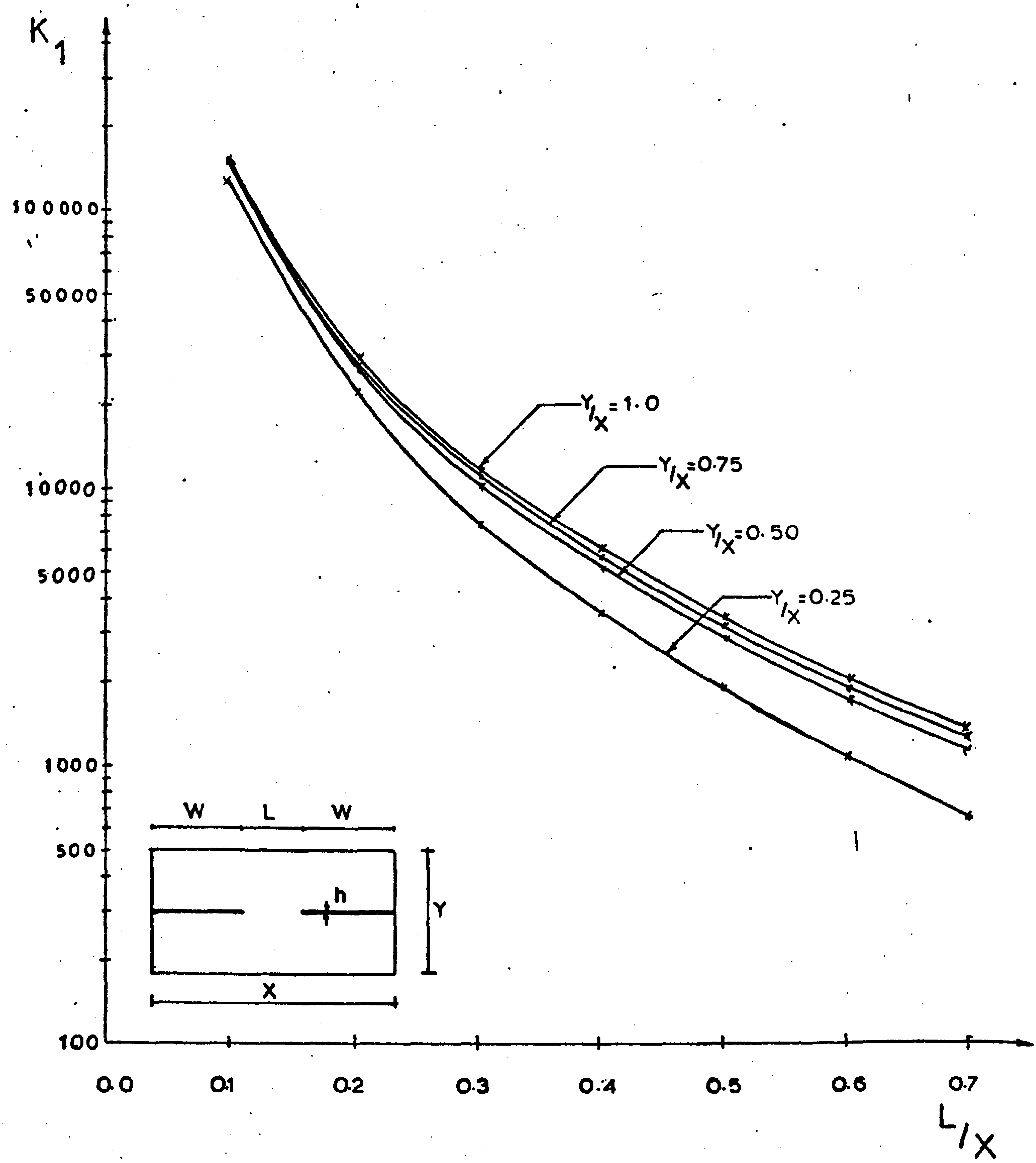


FIGURE 4.11(b) VARIATION OF STIFFNESS NUMBER K_1 FOR PLANAR COUPLED WALLS

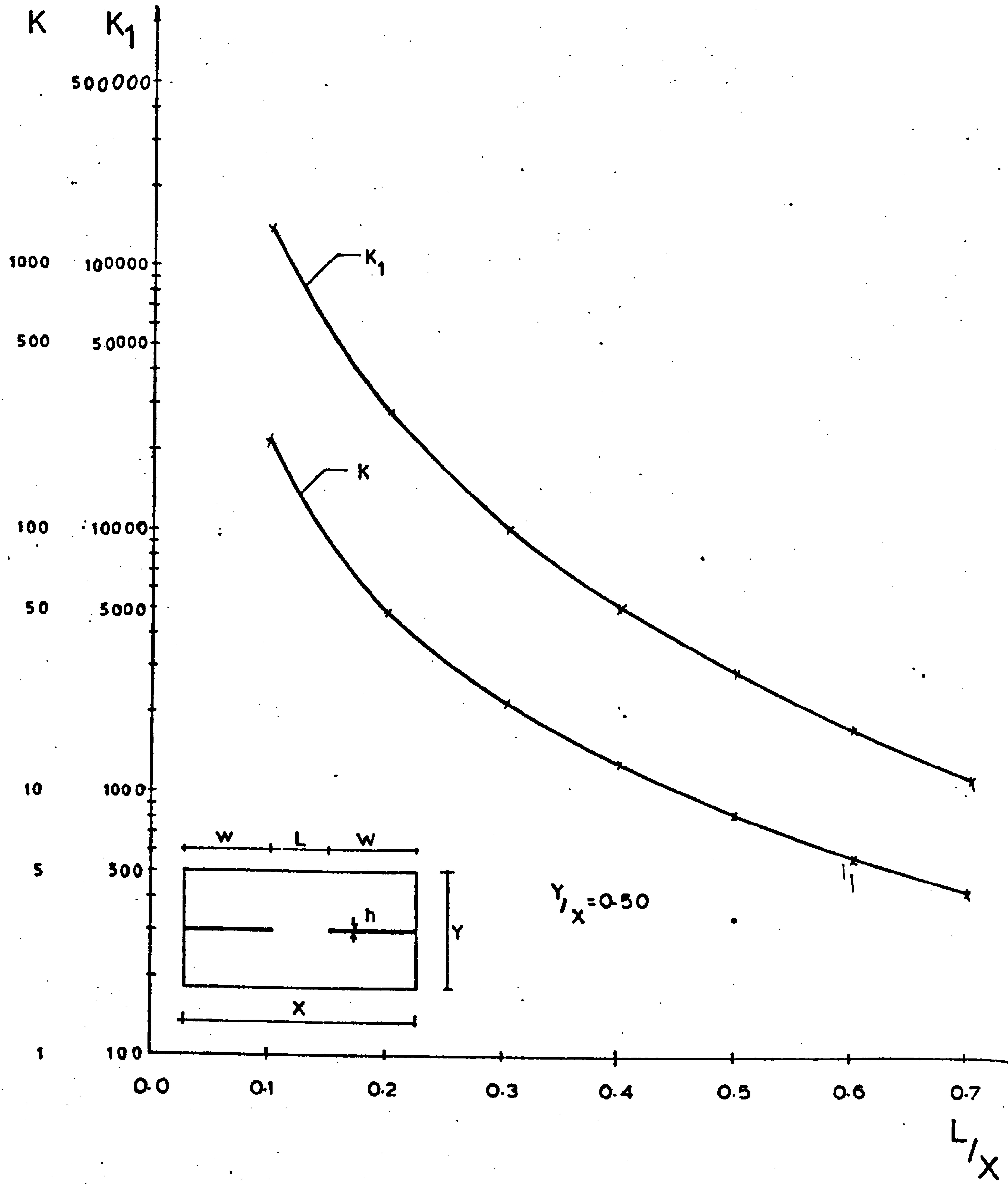


FIGURE 4.11(c) COMPARISON BETWEEN STIFFNESS NUMBER K_1, K

FIGURE 4.12 SLAB STIFFNESS K AS A FUNCTION OF Y/X

FIGURE 4.13 STRESS DISTRIBUTION AT SECTION A-A IN THE X AND Y DIRECTIONS FOR COMMON SCALE

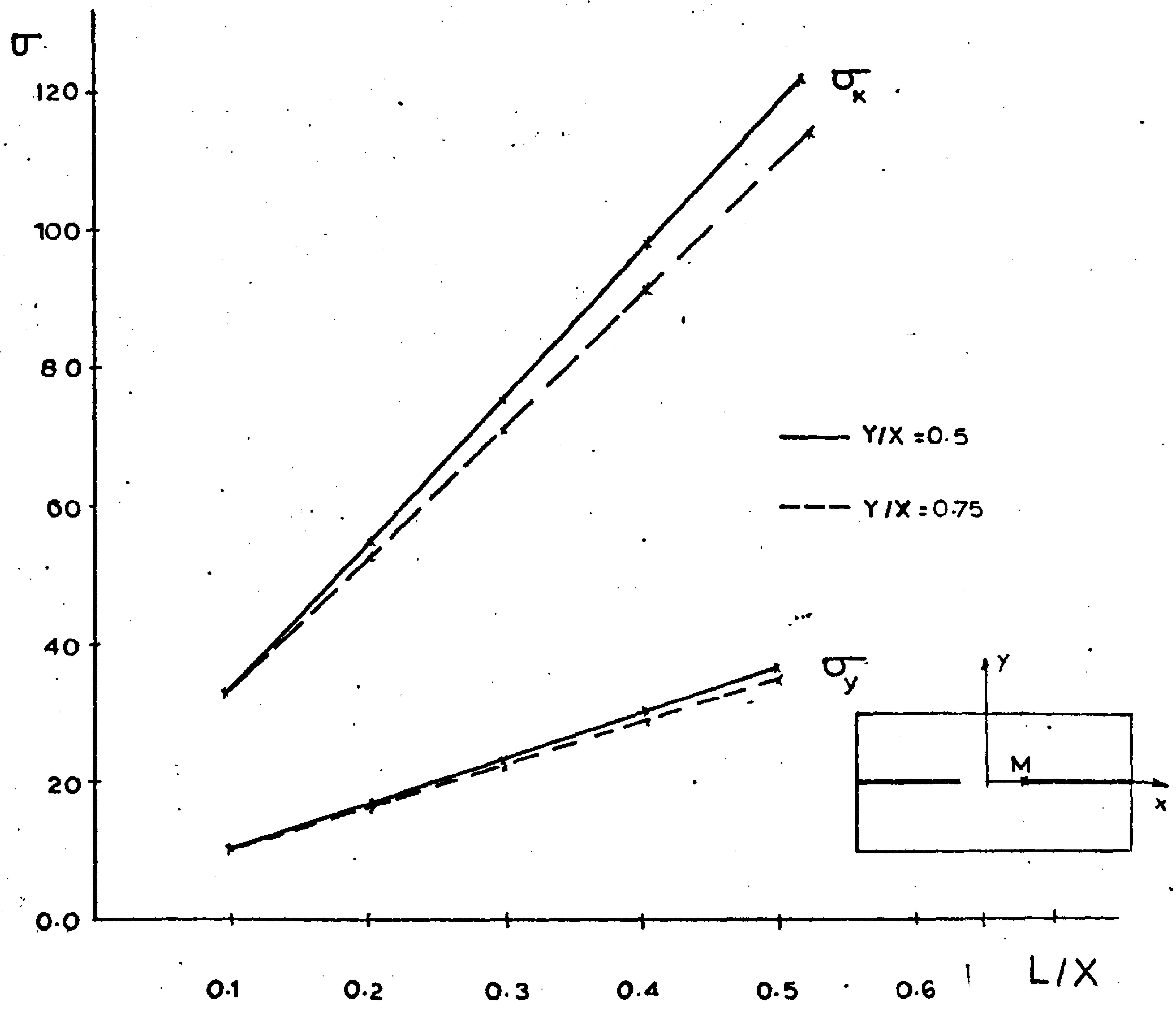


FIGURE 4.14 σ_x AND σ_y AT POINT M FOR COMMON SCALE

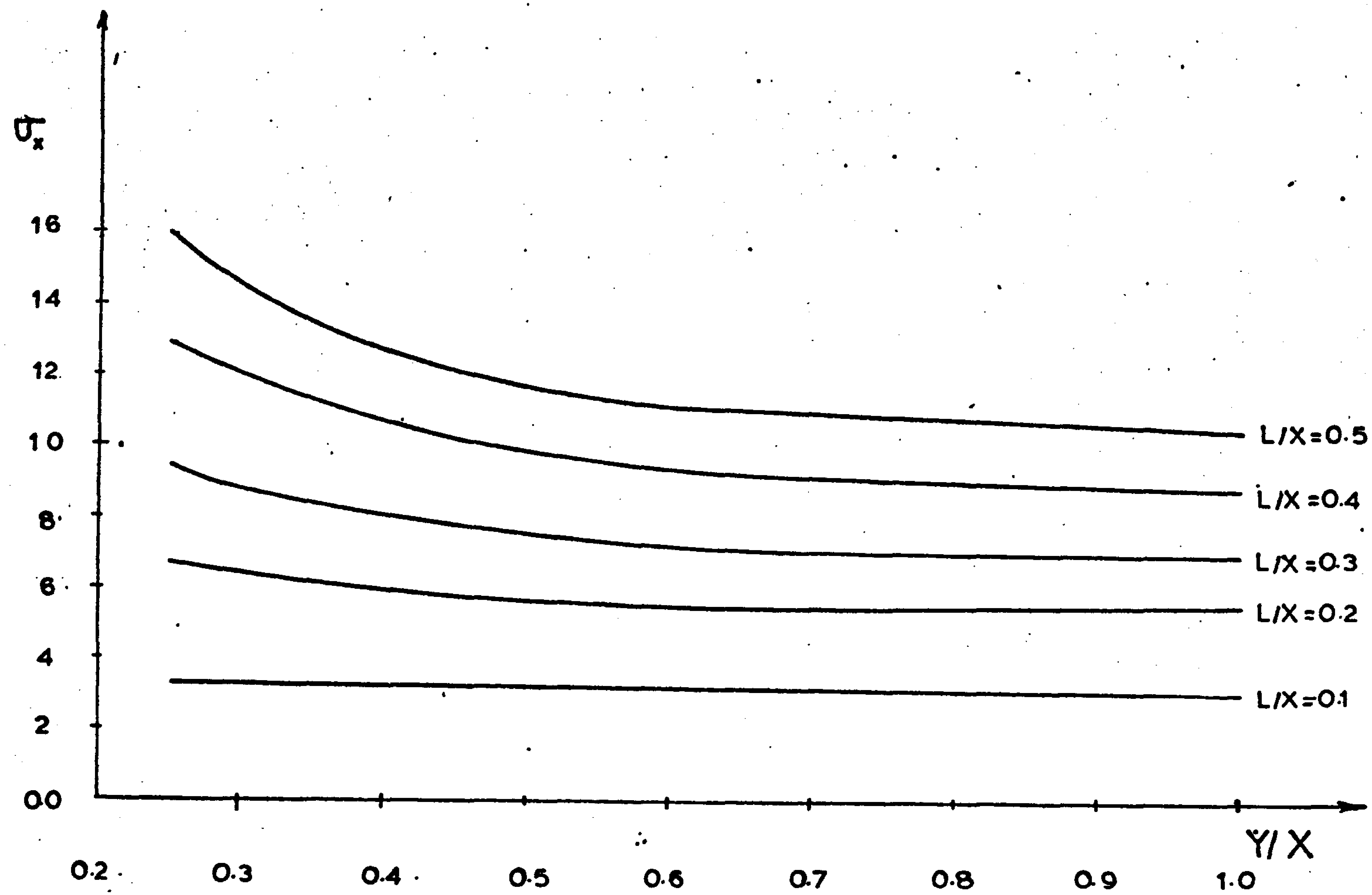


FIGURE 4.15 RELATION BETWEEN THE STRESS AND SLAB WIDTH AT POINT OF MAXIMUM SLAB STRESS

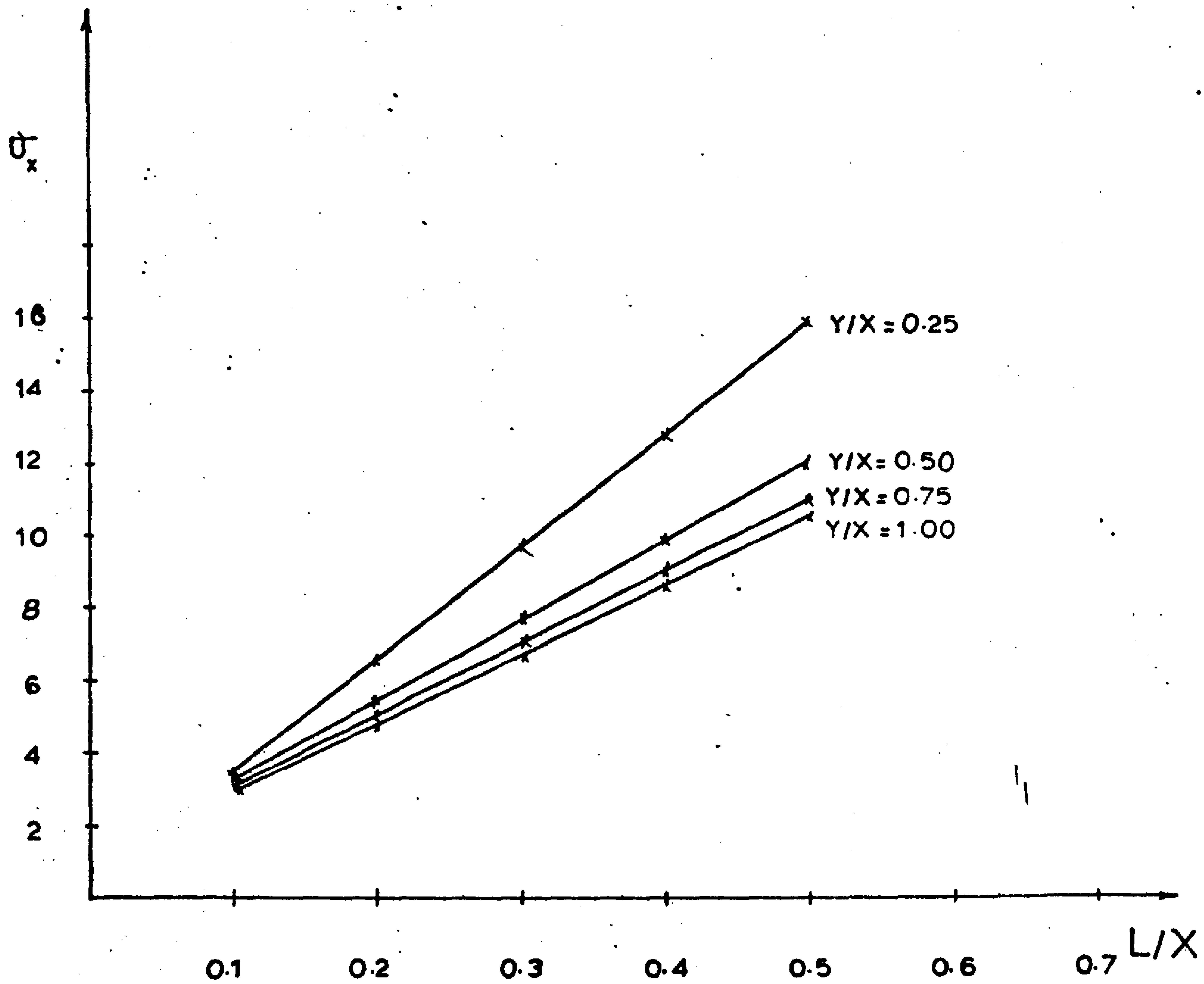


FIGURE 4.16 RELATION BETWEEN STRESS AND WALL OPENINGS AT POINT OF MAXIMUM SLAB STRESS (M)

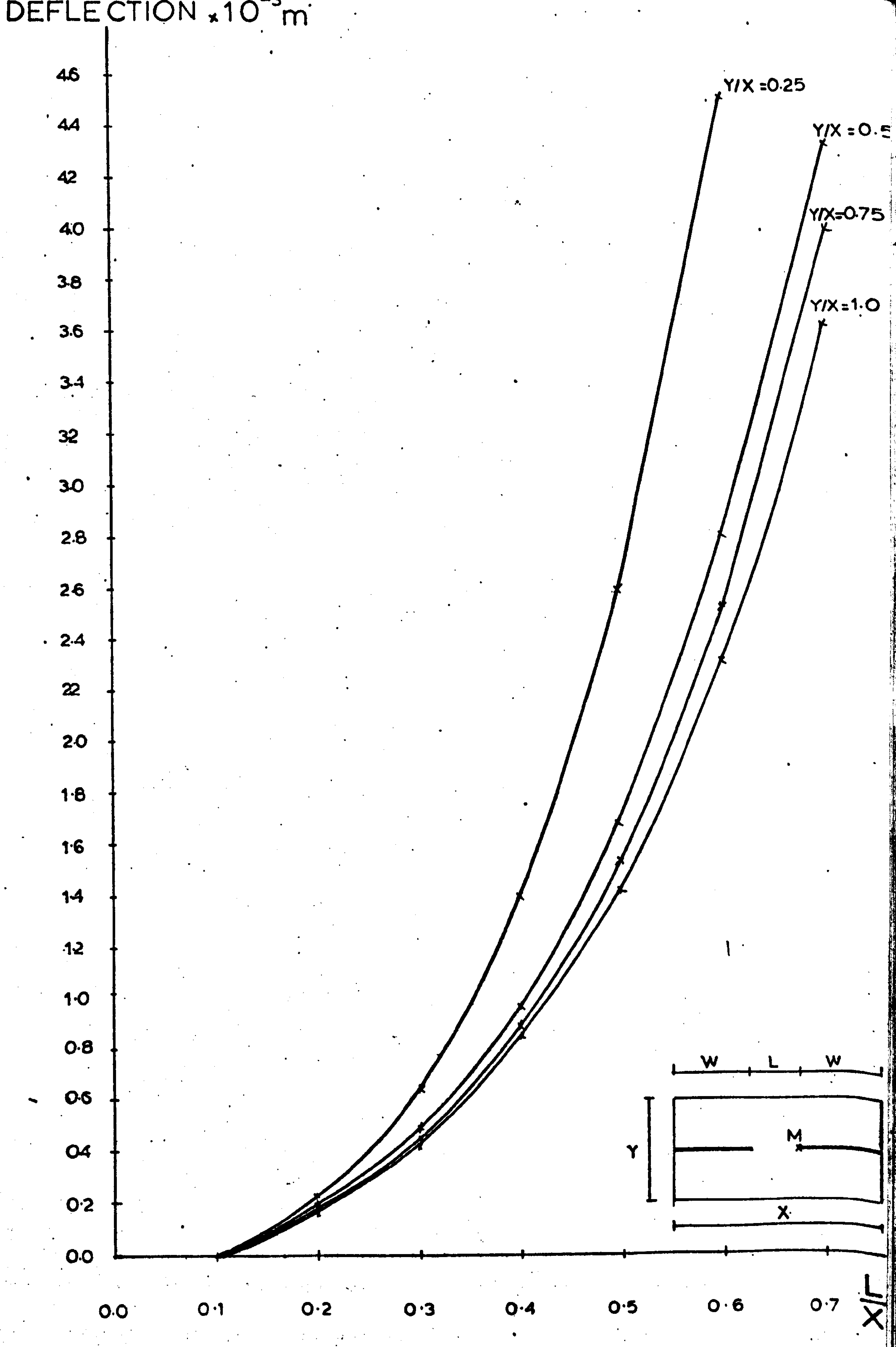


FIGURE 4.17 DEFLECTION AT POINT M FOR VARIOUS SLAB WIDTHS

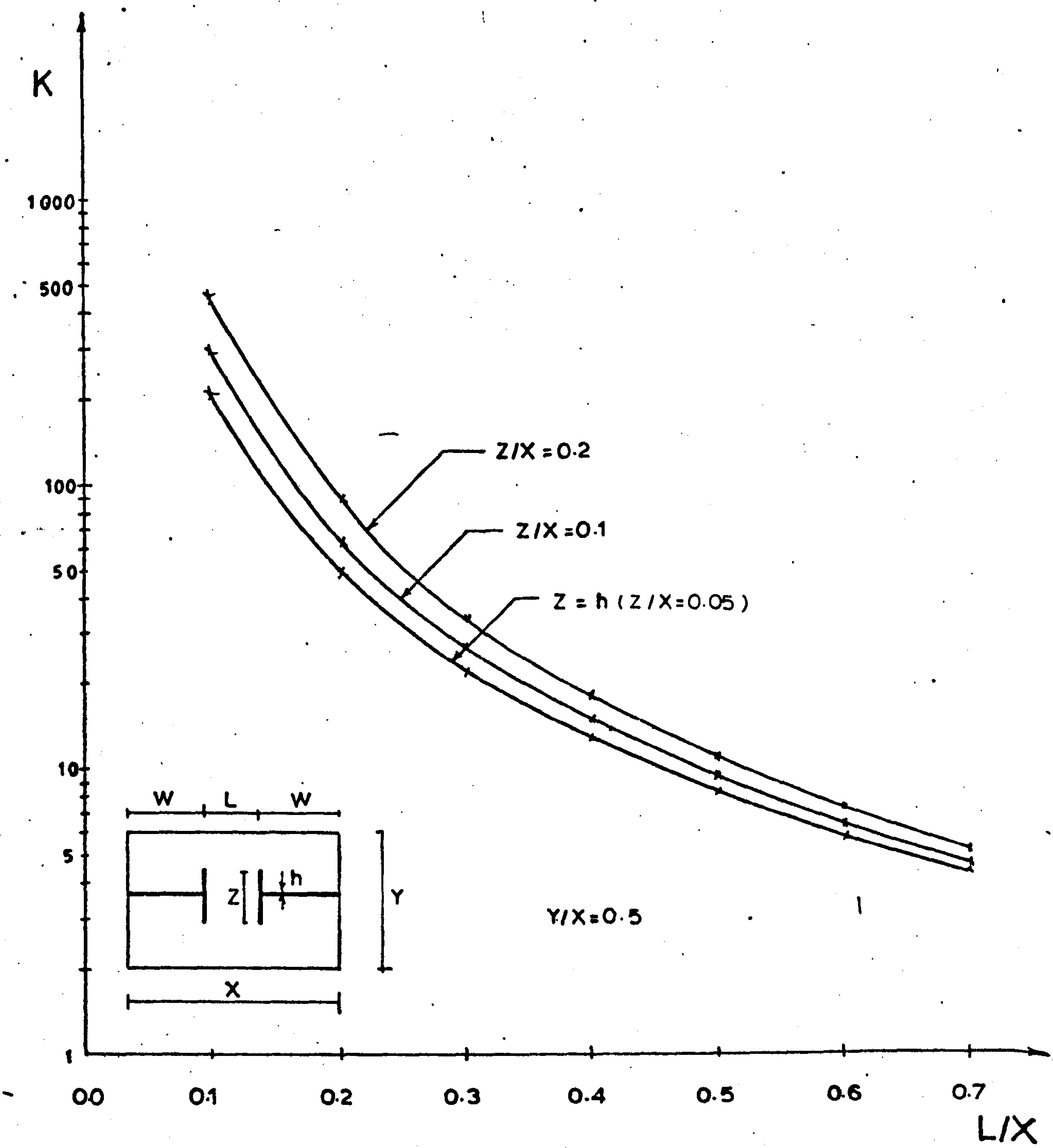


FIGURE 4.18 THE EFFECT OF FLANGE WIDTH ON THE STIFFNESS NUMBER K OF THE SLAB

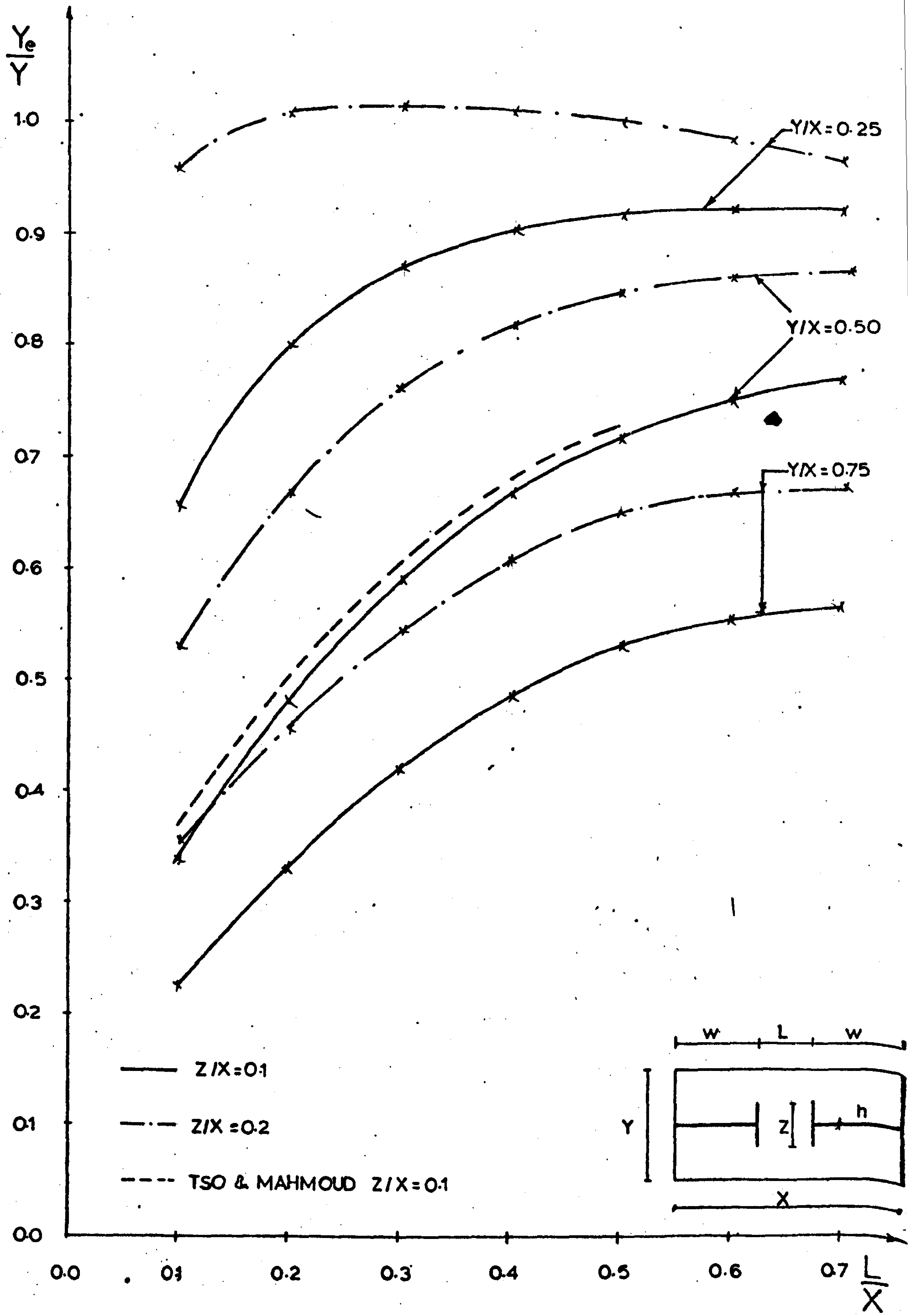


FIGURE 4.19 DESIGN CURVES FOR T-SECTION COUPLED SHEAR WALLS

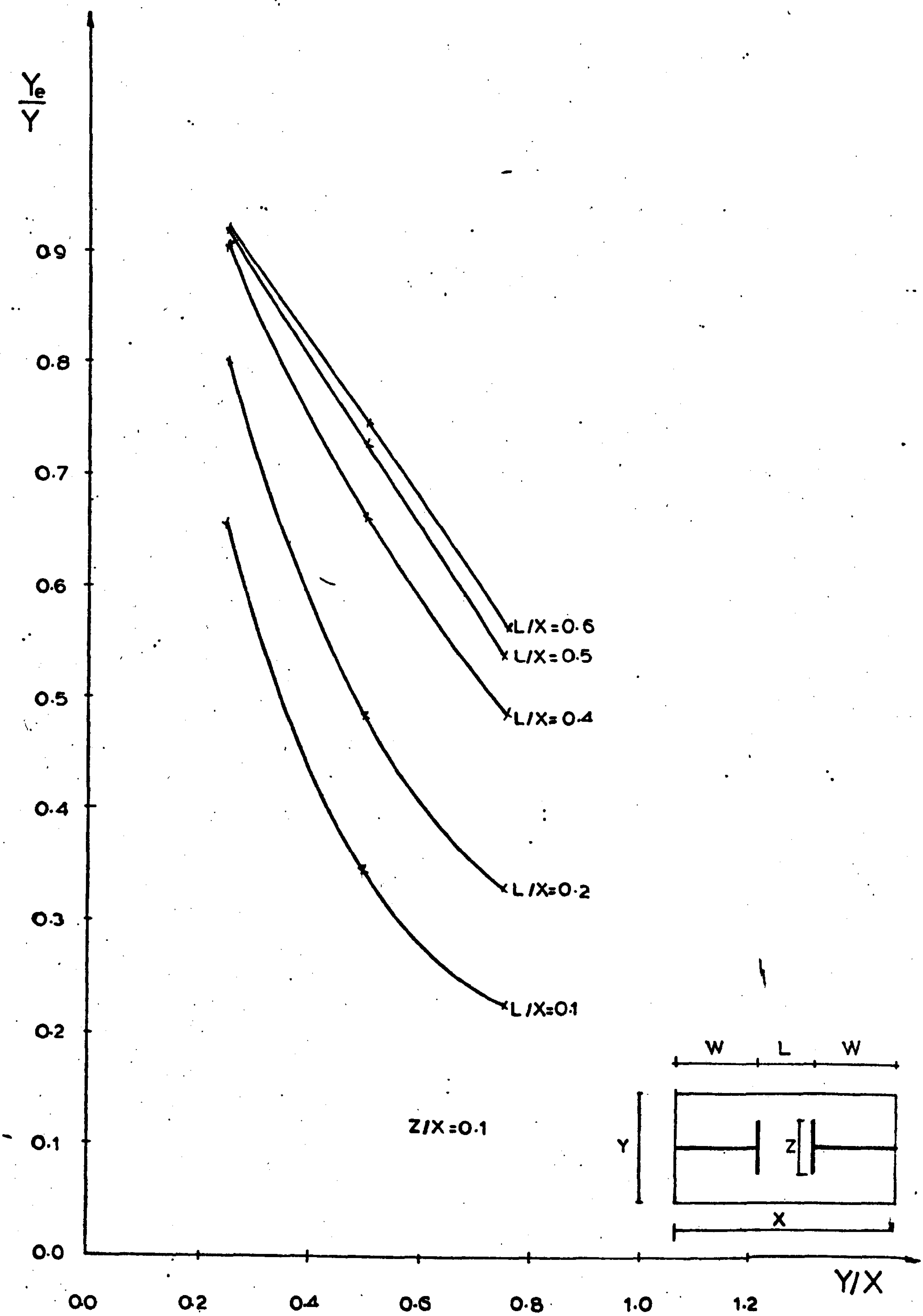


FIGURE 4.20 DESIGN CURVES FOR T-SECTION COUPLED SHEAR WALLS

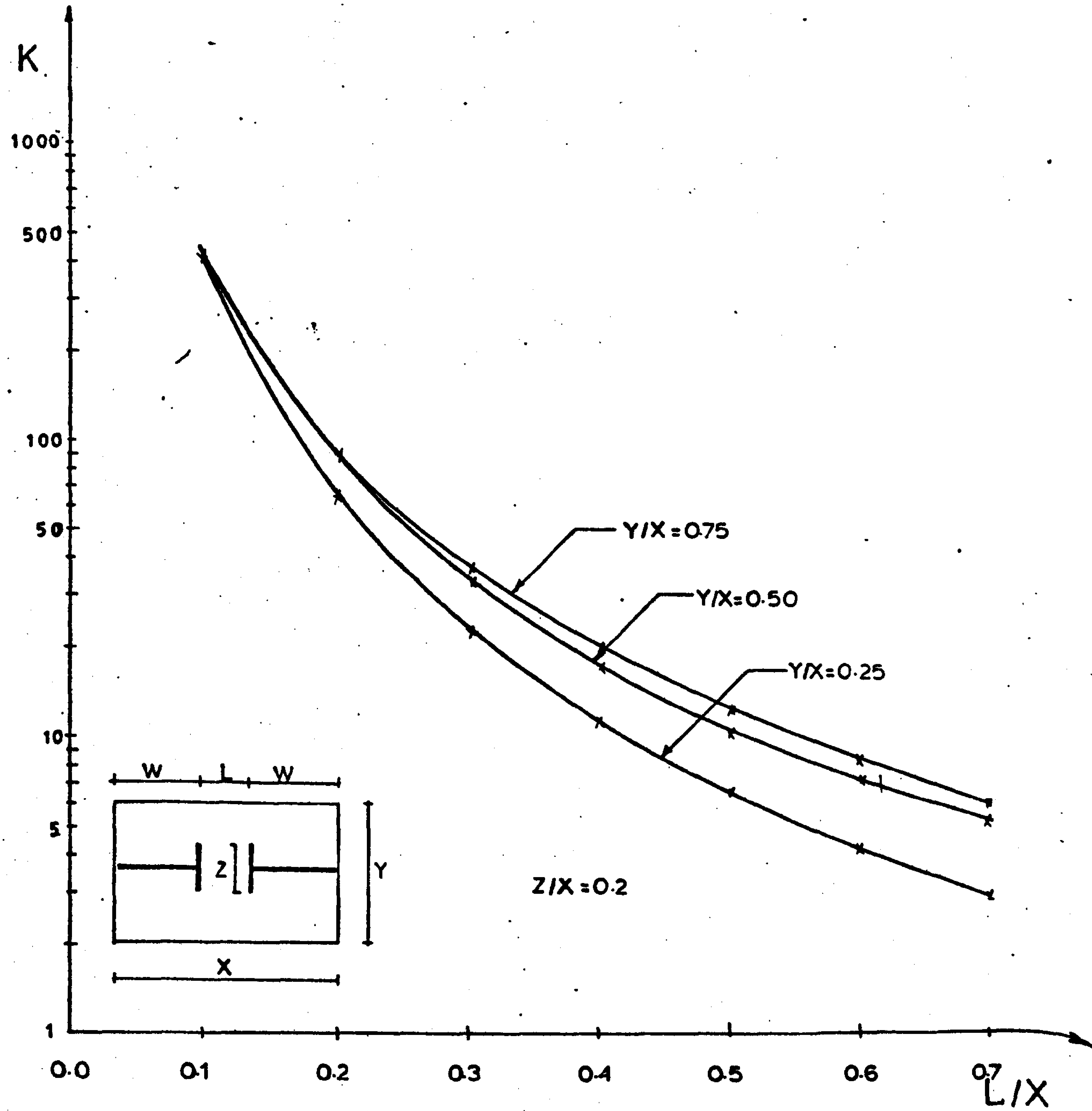


FIGURE 4.21 STIFFNESS NUMBER K OF THE SLAB FOR Z/X = 0.2

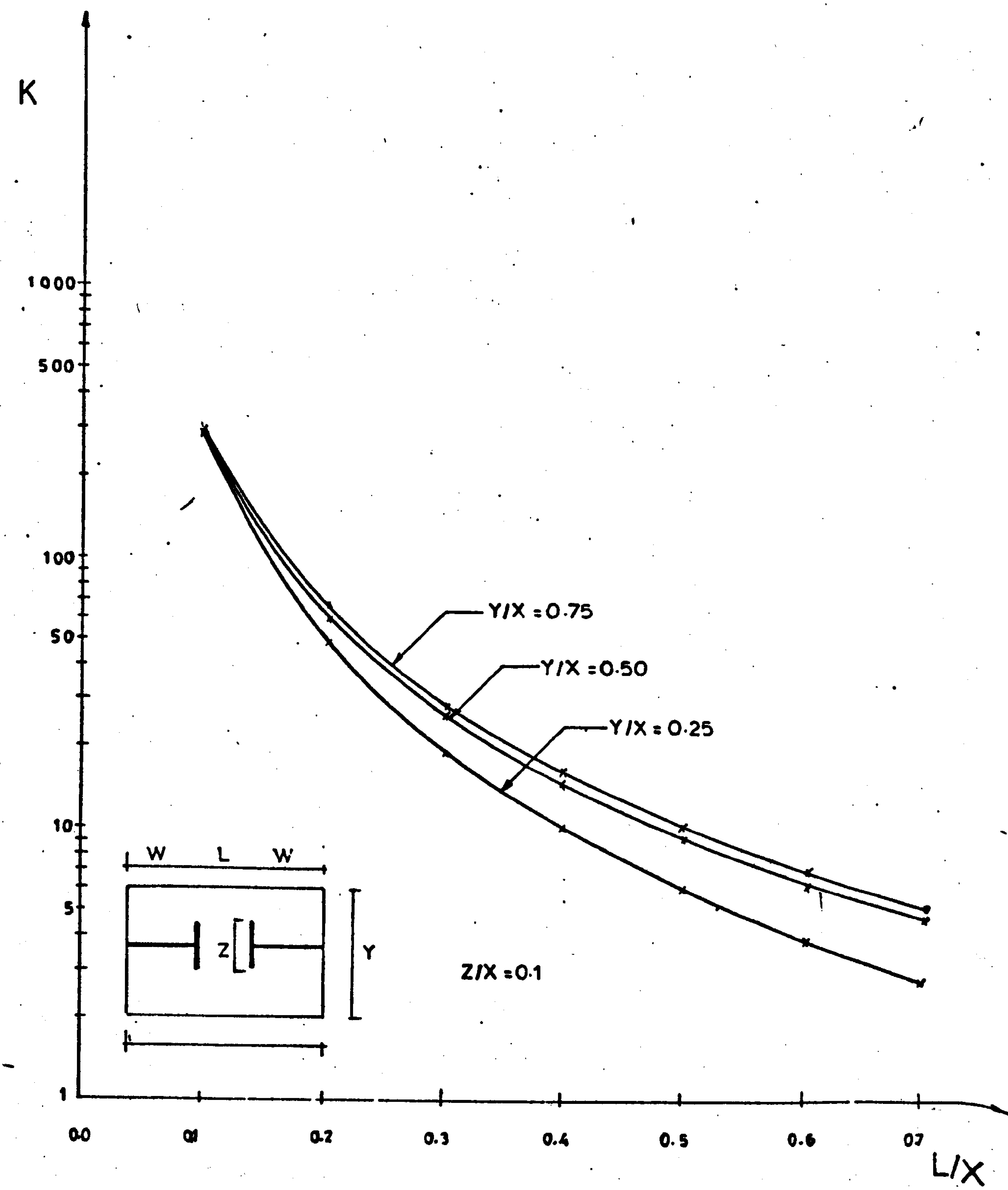


FIGURE 4.22 STIFFNESS NUMBER K OF THE SLAB FOR Z/X = 0.1

FIGURE 4.23 STIFFNESS NUMBER K OF THE SLAB FOR Z/X = 0.1

FIGURE 4.24 COMPARISON BETWEEN THE STIFFNESS NUMBERS K FOR VARIOUS CASES

FIGURE 4.25 EFFECT OF THE FLANGE ON THE STRESS DISTRIBUTION

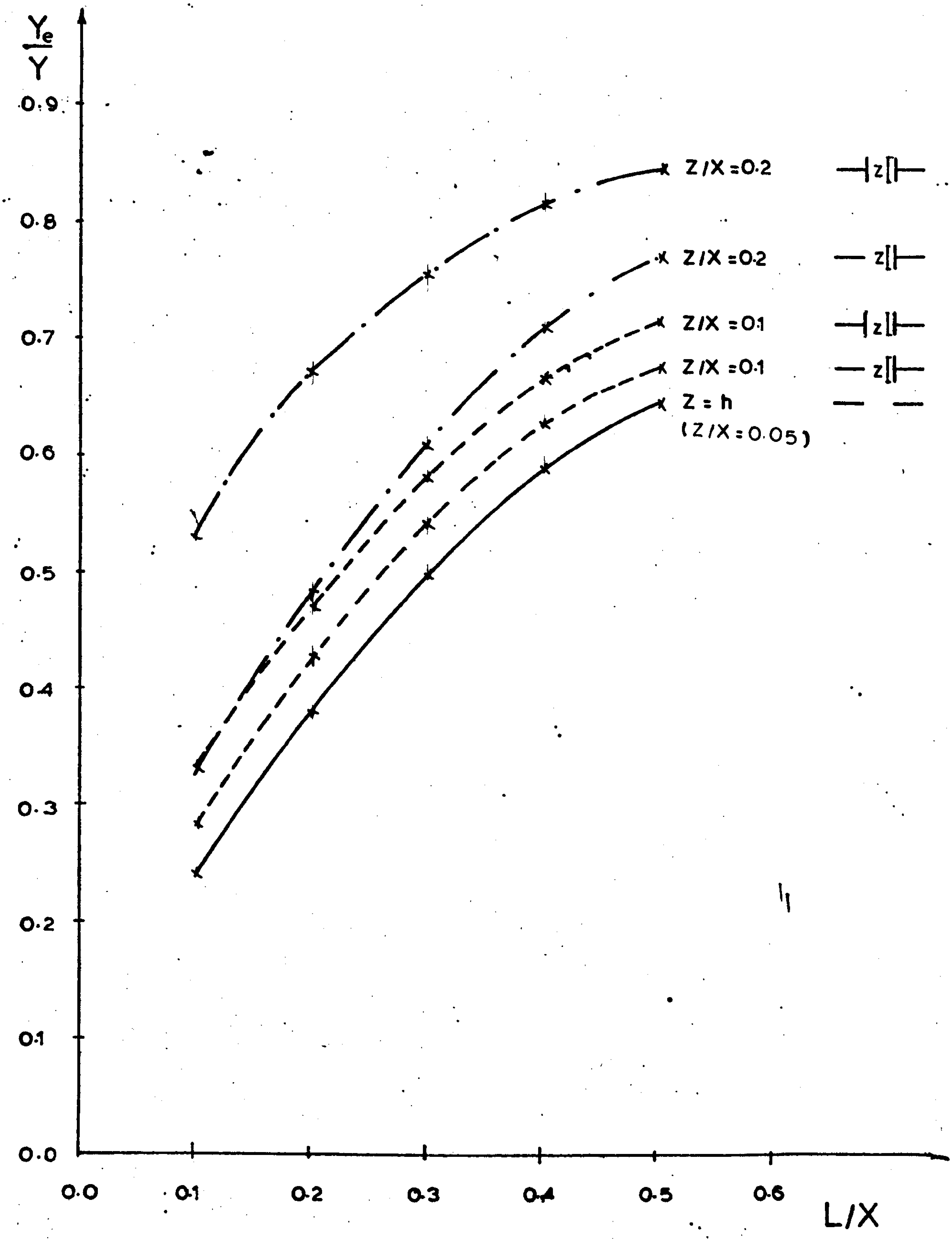


FIGURE 4.26 DESIGN CURVES FOR COUPLED PLANAR AND T-SECTION FOR Y/X = 0.5

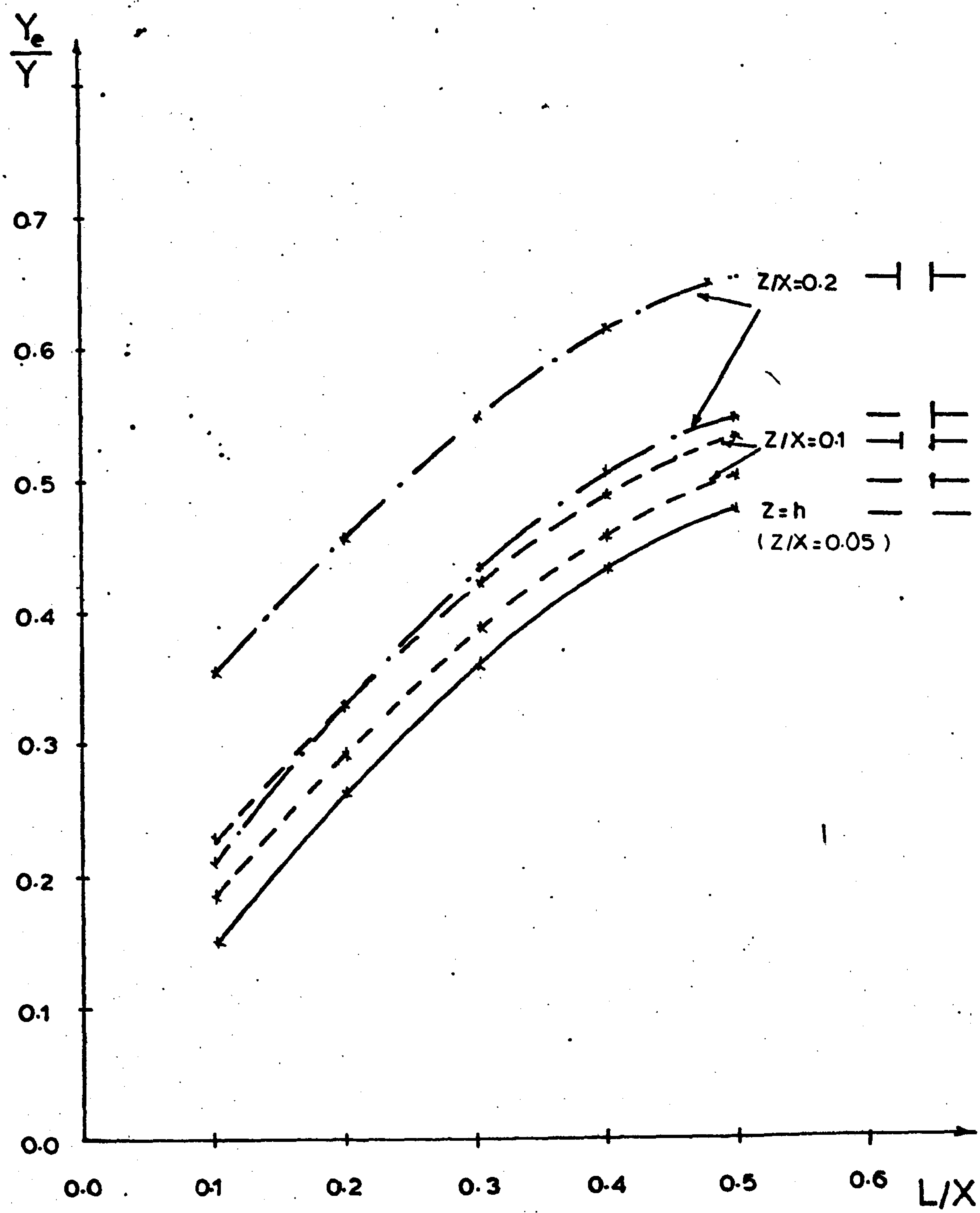


FIGURE 4.27 DESIGN CURVES FOR COUPLED PLANAR AND T-SECTION FOR Y/X = 0.75

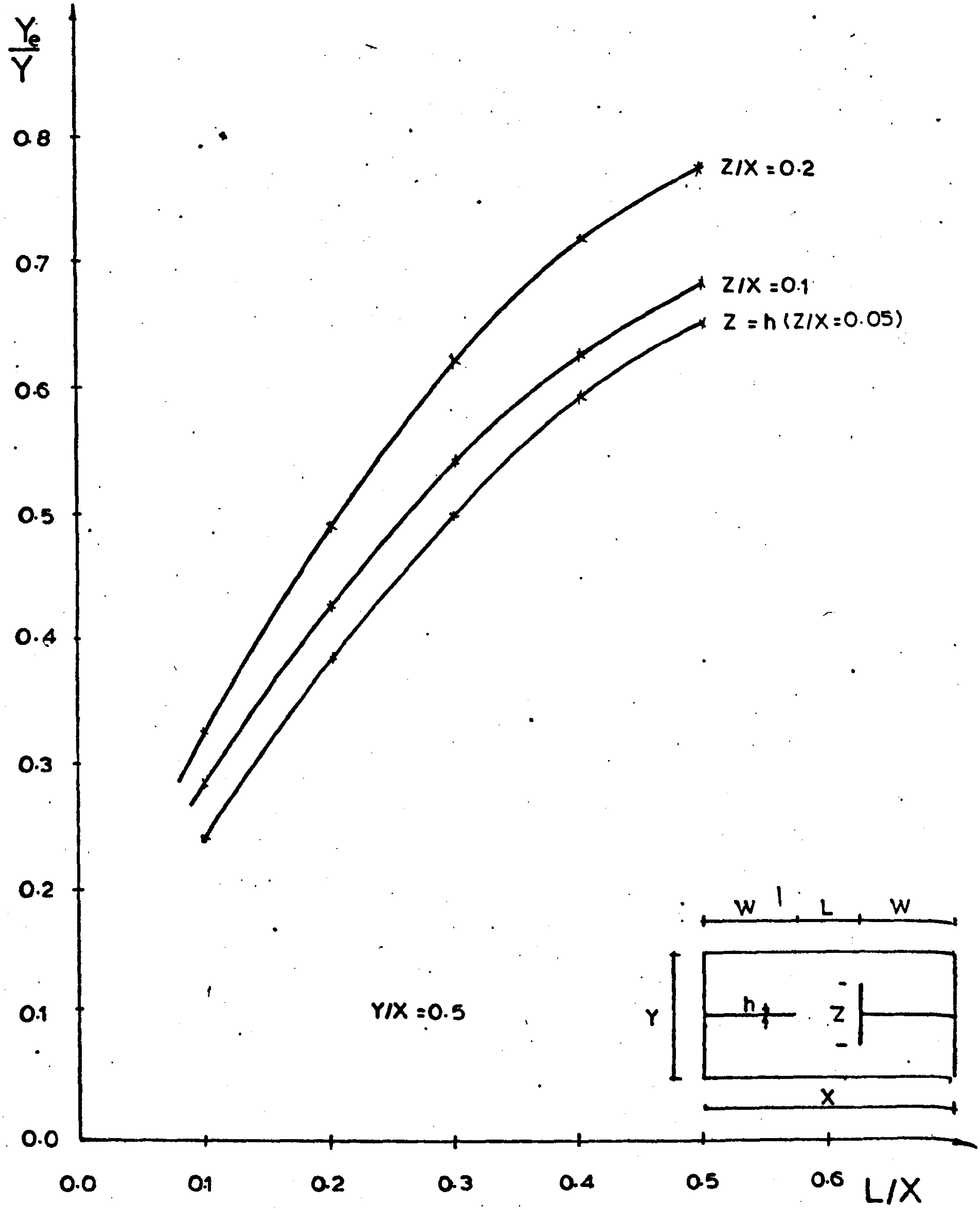


FIGURE 4.28 DESIGN CURVES FOR COUPLED PLANAR AND T-SECTION WALLS

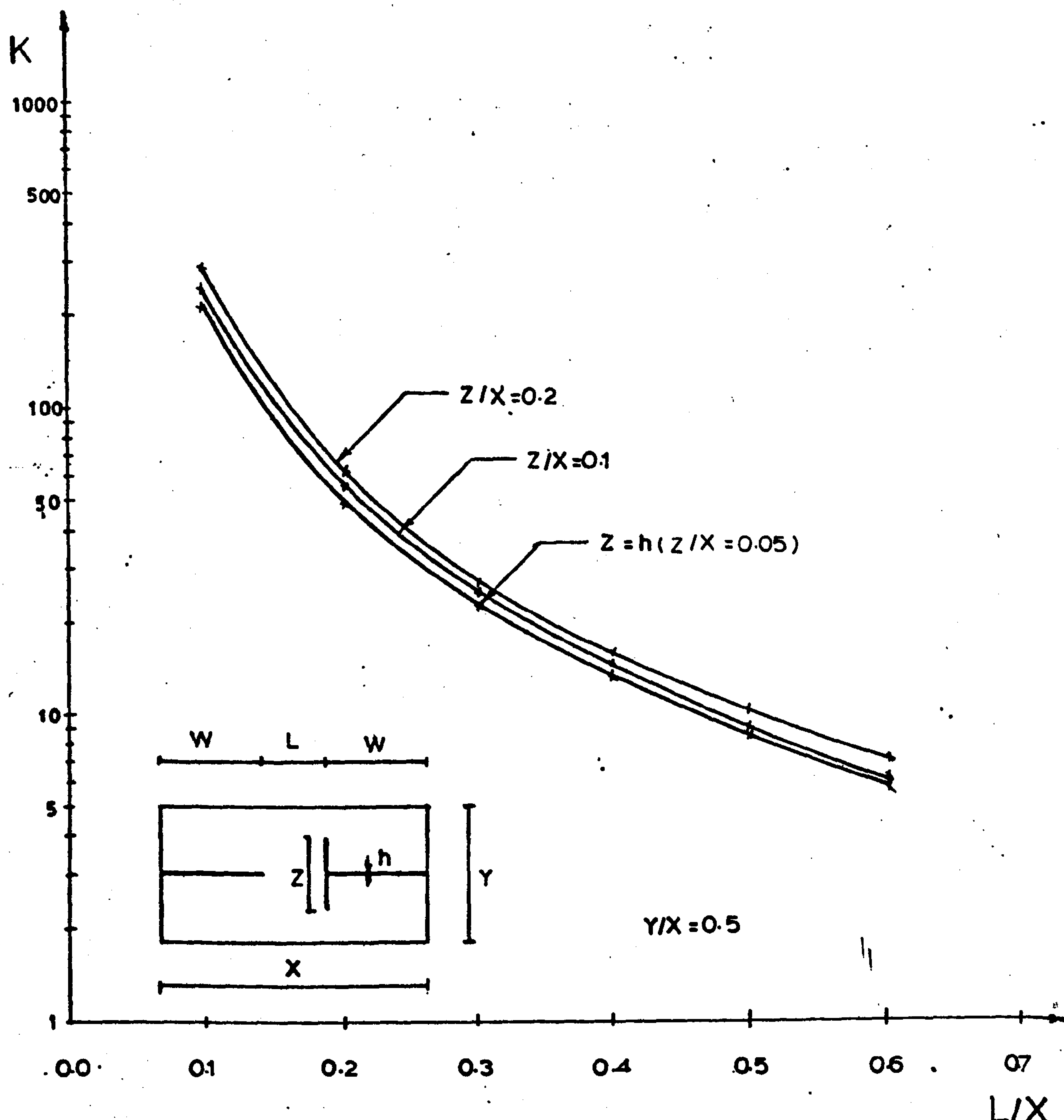


FIGURE 4.29 CHANGE IN THE STIFFNESS NUMBER K WITH THE CHANGE OF THE FLANGE WIDTH

CHAPTER FIVE

AN ELASTO-PLASTIC ANALYSIS OF SLABS COUPLING SHEAR WALLS

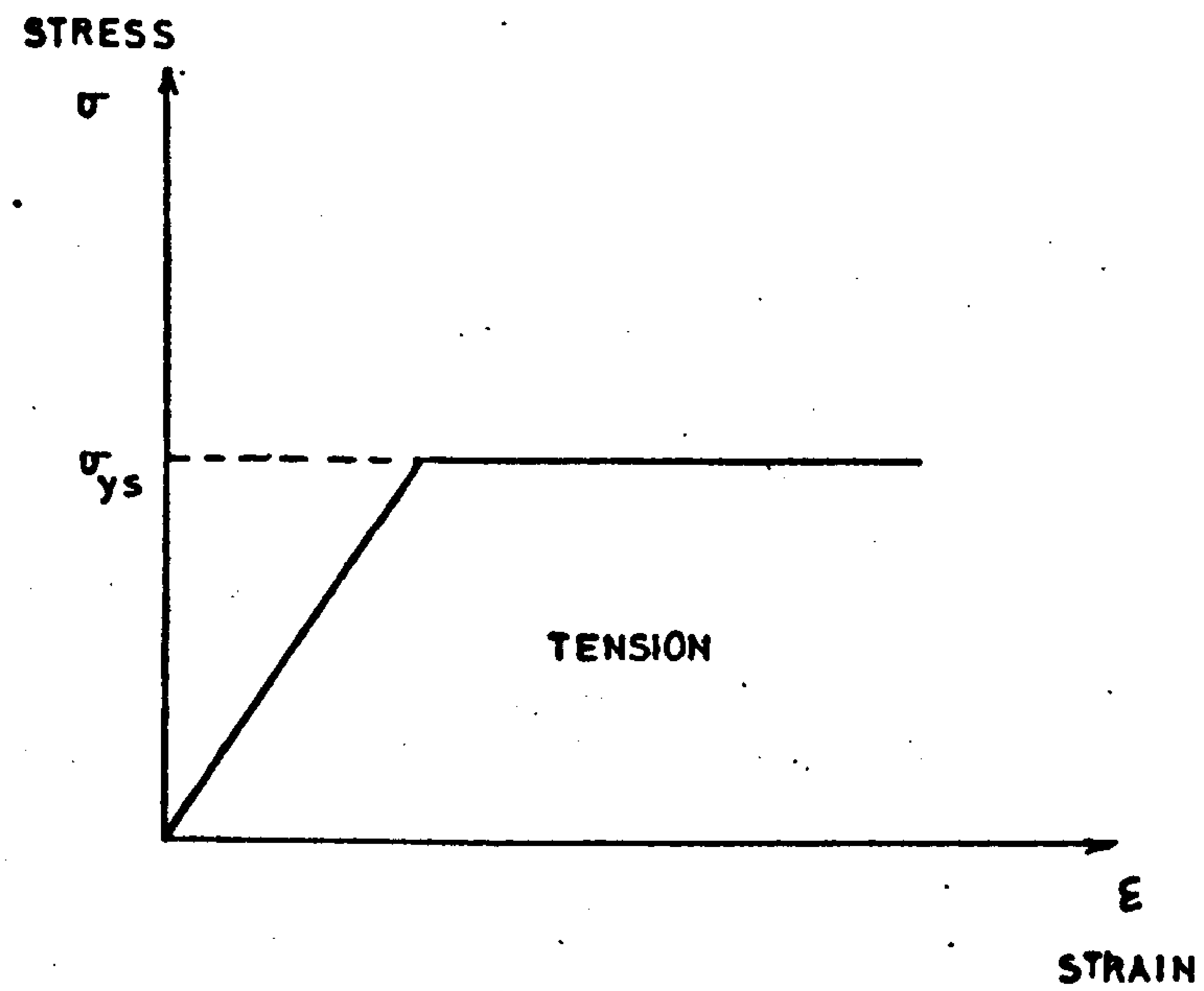
5.1 Introduction

Prior to this study estimations of elasto-plastic performance of coupled shear wall structures had not included investigation of complete wall slab structures. In this investigation the 'finite element' method is used together with the 'initial stress' technique to assess the performance of slabs coupling shear walls for the post-elastic situation.

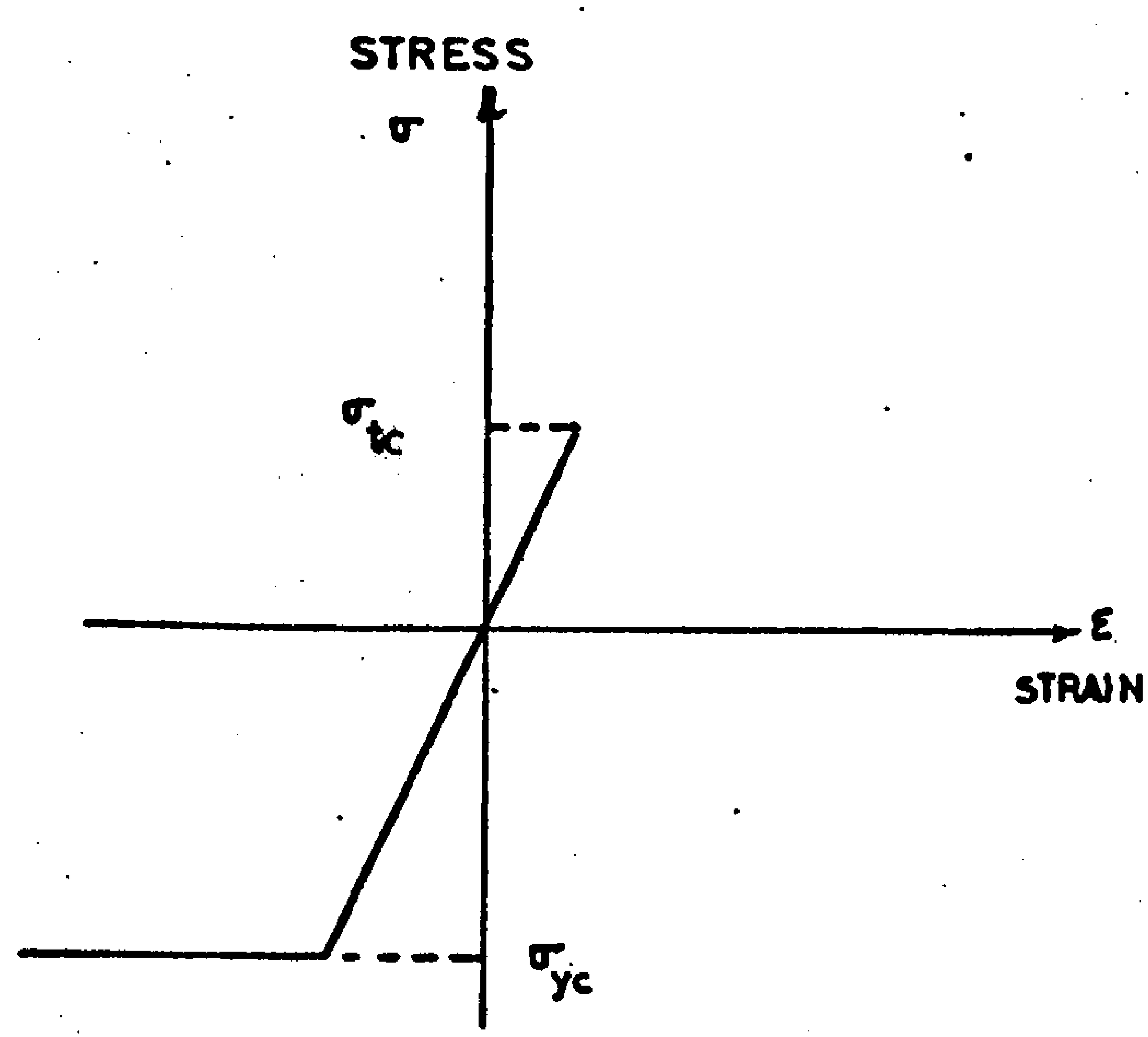
The rectangular plate in bending element described in Chapter Three is used to replace the slab by a finite number of inter-connected elements. For such an element it is difficult to represent both the steel and the concrete separately, the steel is assumed 'smeared' through the concrete. The calculations were carried out using the properties of the composite material.

5.2 Elasto-plastic stress analysis

In general the stress-strain relationship for the steel and concrete can be specified as shown in Figure 5.1. The concrete is assumed to behave in compression linearly up to the yield stress σ_{yc} and then as a perfectly plastic material. In tension the concrete is considered to possess only limited tensile strength σ_{tc}. The steel is assumed to behave linearly also until the yield stress σ_{ys}, and then as a perfect plastic material. In this analysis the reinforced concrete material is assumed to have a performance similar to that of the steel with a uniaxial stress at yield σ_y for the composite material.

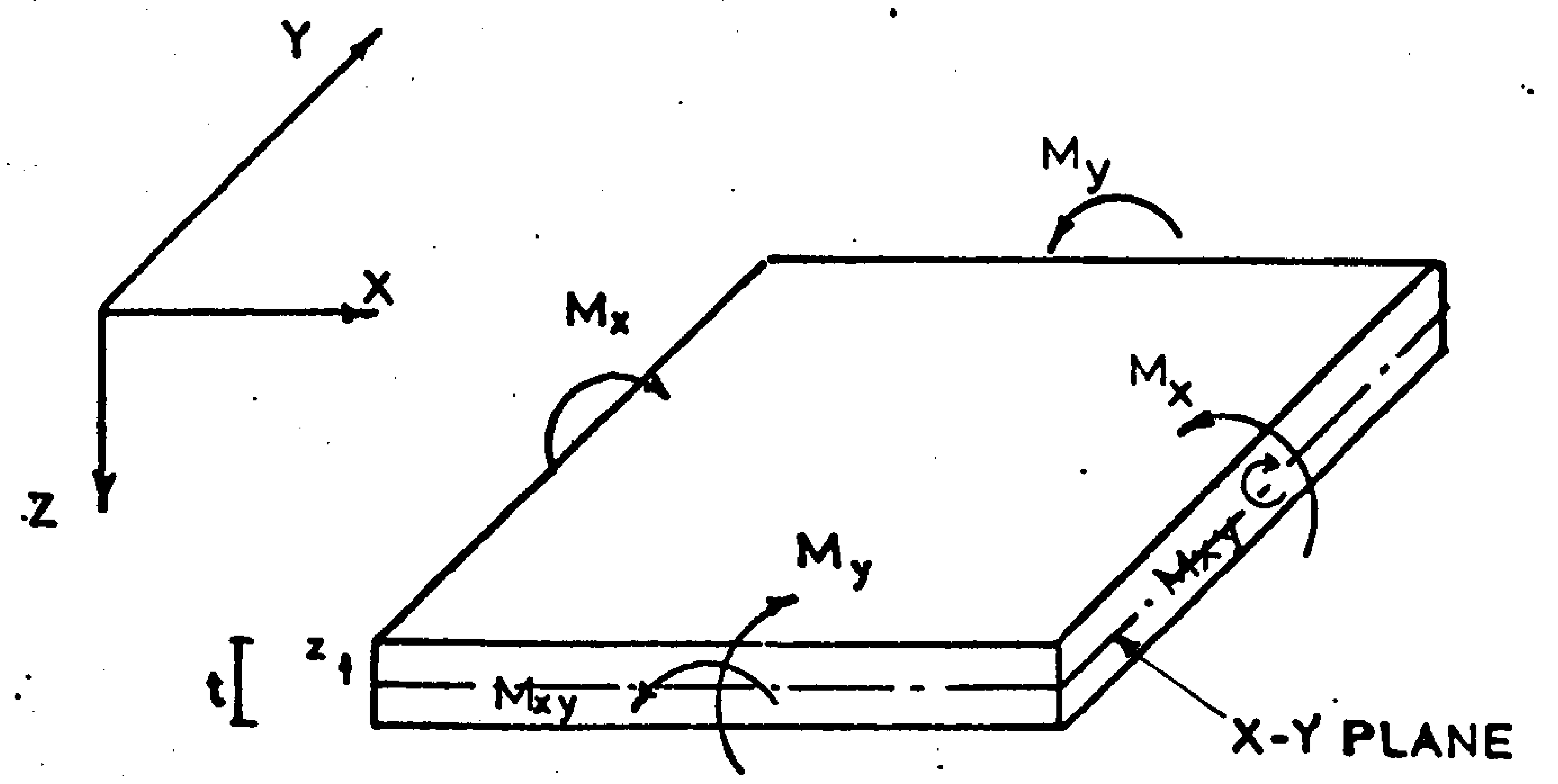


(a) STEEL



(b) CONCRETE

FIGURE 5.1 STRESS-STRAIN DIAGRAMS FOR STEEL AND CONCRETE



$$\sigma_x = \frac{12\ M_x}{t^3}\ Z$$

$$\sigma_y = \frac{12\ M_y}{t^3}\ Z$$

$$\sigma_{xy} = \frac{12\ M_{xy}}{t^3}\ Z$$

FIGURE 5.2 STRESSES

The method used in this study is one in which an elastic body is constrained to behave as an elastic-plastic one by the imposition of self equilibrating internal forces. The elastic-plastic stress-strain equations used are the Prandtl-Ressu relationship where the Von-Mises yield surface is used for the composite material. This is given by:

$$F(\sigma) = \left[\tfrac{1}{2}(\sigma_1-\sigma_2)^2 + \tfrac{1}{2}(\sigma_2-\sigma_3)^2 + \tfrac{1}{2}(\sigma_3-\sigma_1)^2 + 3\sigma_4{}^2 + 3\sigma_5{}^2 + 3\sigma_6{}^2\right]^{\frac{1}{2}} - \bar{\sigma} = 0 \tag{5.1}$$

This relation for the case of thin plate in bending reduces to:

$$F(\sigma) = \left[\tfrac{1}{2}(\sigma_x-\sigma_y)^2 + \tfrac{1}{2}\sigma_x{}^2 + \tfrac{1}{2}\sigma_y{}^2 + 3\sigma_{xy}{}^2\right]^{\frac{1}{2}} - \bar{\sigma}_{yield} = 0 \tag{5.2}$$

Where σ_x, σ_y, σ_{xy} are the stresses as shown in Figure 5.2.

$\bar{\sigma}_{yield}$ is the uniaxial stress at yield.

The particular method of analysis adopts the 'initial stress' concept. Figure 5.3 shows the process used applying this technique. The current applied force is P and the change in the next increment is DP. If DP is such as to reduce the absolute value of $\bar{\sigma}$ or if it makes the absolute value of $\bar{\sigma}_e$ less than the absolute value of $\bar{\sigma}_y$ then elastic behaviour results. On the other hand, if the current state is at A or E, as shown in Figure 5.3, and DP increases the absolute value of $\bar{\sigma}_e$ beyond $\bar{\sigma}_y$ then plastic yielding occurs. In the first case, the excess load to be redistributed is that corresponding to BC and in the second case is the load corresponding to HG. In the computations these excess loads are merely added in as internal self-equilibrating loads at the element nodes in the next cycle of iteration. The effect of these loads is to cause increased deformations. For a load increment the stress-strain behaviour during the elastic-plastic deformation can be expressed as:

$$\{d\sigma\} = \left[D_{ep}\right]^* \{d\varepsilon\} \tag{5.3}$$

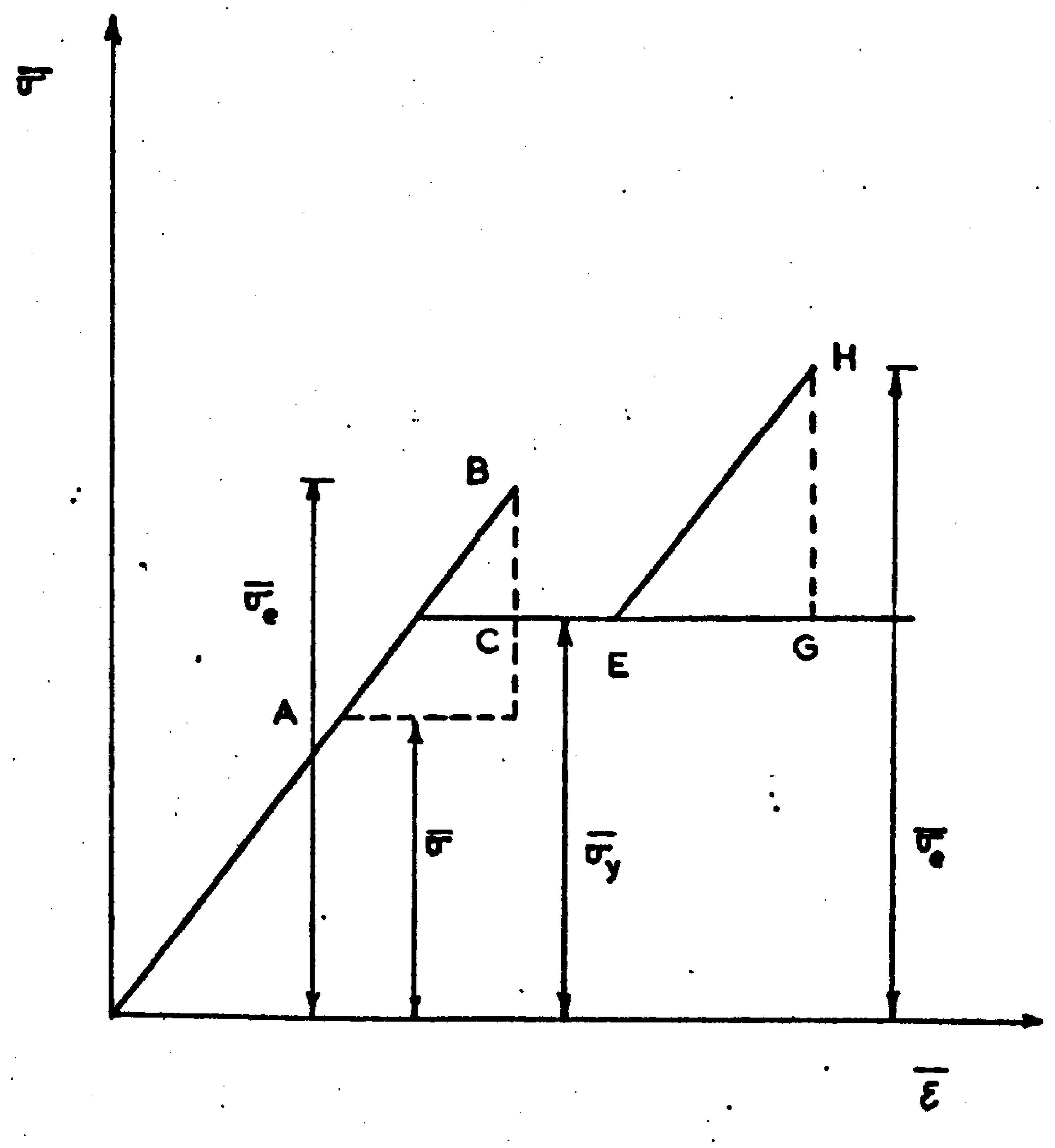

$\bar{\sigma}$
$\bar{\sigma}_e$
B
H
C
E
G
A
$\bar{\sigma}$
$\bar{\sigma}_y$
$\bar{\sigma}_e$
$\bar{\varepsilon}$

FIGURE 5.3

where $d\{\varepsilon\}$ is the total incremental strain composed of elastic and plastic strains and is given by:

$$d\{\varepsilon\} = \{d\varepsilon^e\} + \{\varepsilon^p\} \tag{5.4}$$

in which $\{d\varepsilon^e\}$ is the elastic strains vector as described in Chapter Three and

$$d\{\varepsilon^p\} = \lambda\{\frac{\partial F}{\partial \sigma}\} \tag{5.5}$$

λ is a proportionality constant for the material,

$[D_{ep}]^*$ is the elasto-plastic matrix relating the stresses and strains in an incremental loading and is given by:

$$[D_{ep}]^* = [D] - [D]\{\frac{\partial F}{\partial\{\sigma\}}\}\{\frac{\partial F}{\partial\{\sigma\}}\}^T[D]\left[A + \{\frac{\partial F}{\partial\{\sigma\}}\}^T[D]\{\frac{\partial F}{\partial\{\sigma\}}\}\right]^{-1} \tag{5.6}$$

From equation (5.3) it can be seen that when the material reaches the plastic stage an elasto-plastic matrix instead of the elastic matrix has to be used to relate stresses and strains (Zienkiewicz 1971). The elasticity properties are used in the initial stress method repeatedly and then making an adjustment in the stresses by successive corrections so as to reproduce the actual stresses which would have been obtained using an elasto-plastic matrix.

5.3 Outline of the computational process

For a typical load increment the main steps required can be described as follows:

1. Apply load increment and determine elastic increments of stress $\{\Delta\ \sigma_1'\}$ and the corresponding strains $\{\Delta\ \varepsilon_1'\}$.
2. Add $\{\Delta\ \sigma_1'\}$ to stresses at the start of increment $\{\sigma_0\}$ to obtain $\{\sigma'\}$.
3. The effective stress $F(\sigma')$ is calculated from these stresses and checked to see whether the element has yielded, that is,

whether $F(\sigma')$ has exceeded the yield stress of the material. If $F(\sigma')$ is less than the yield stress then only elastic strains occur.

4. If $F(\bar{\sigma}) \geq 0$, that is, if the element has yielded, then the actual stresses due to non-linear stress-strain relationship using the elasto-plastic matrix would be:

$$\{\Delta\ \sigma_1\} = \left[D_{ep}\right] \{\Delta\ \varepsilon_1'\} \tag{5.7}$$

with $\left[D_{ep}\right]^*$ computed from equation (5.6) with stresses $\{\sigma'\}$. The stresses which have to be supported by body forces (the initial stresses) are:

$$\{\Delta\ \sigma_1''\} = \{\Delta\ \sigma_1'\} - \{\Delta\ \sigma_1\} \tag{5.8}$$

The current stresses would be:

$$\{\sigma\} = \{\sigma'\} - \{\Delta\ \sigma_1''\} \tag{5.9}$$

and the current strains:

$$\{\varepsilon\} = \{\varepsilon'\} \quad \Delta\{\varepsilon_1'\} \tag{5.10}$$

5. If $F(\sigma) > 0$ but $F(\sigma_0) < 0$ find the intermediate stress value at which yield begins, and compute increment $\{\Delta\ \sigma_1\}$ by equation (5.6) starting from that point and then to to step 3.

6. The initial stresses are converted into the equivalent nodal forces by:

$$\{P_{ip}\}^e = \int \left[B\right]^T \{\Delta\ \sigma_1''\}\ d(vol)$$

where $\{P_{ip}\}^e$ is the initial load vector for element due to plasticity.

7. For the next iteration the structure is resolved using the original elastic properties and the load system $\{P\}$ to find

the stresses $\{\Delta \sigma_2'\}$ and the strains $\{\Delta \varepsilon_2'\}$. These stresses, strains and displacements are then added to their current values.

8. Steps 2 to 6 are then repeated until convergence is obtained, that is, within the same increment of load for every iteration and equilibrating body forces are compared with the respective forces obtained during the first iteration. If they are negligible this means that statical equilibrium has been achieved. If the convergence is not obtained in a reasonable number of cycles of iteration then a collapse condition is deemed to have been achieved and the process is stopped. Full details of this method are given by Nayak and Zienkiewicz (1972). Figure 5.4 shows the application of the method to the analysis of slabs coupling shear walls and theoretical values obtained using this method are compared with the results obtained experimentally for the 1:3 scale model described in Chapter Two.

In the theoretical analysis employing the rectangular plate elements, a value of 50 per cent of the uniaxial stress (δ_y) at yield of the composite material was used. This was calculated to be appropriate for the reinforced slab sections of the 1:3 scale model.

Two alternative approaches to the solution of the elasto-plastic problem were considered. In one case the depth of section on the tension side of the neutral axis was reduced once the limit tension stress of the concrete had been reached. In the second case once the slab structure reached yield then only the steel on the tension side and steel and concrete on the compression side were included in the post-elastic analysis. Unfortunately neither of these two methods gave a solution which converged satisfactorily in a reasonable number of iterations.

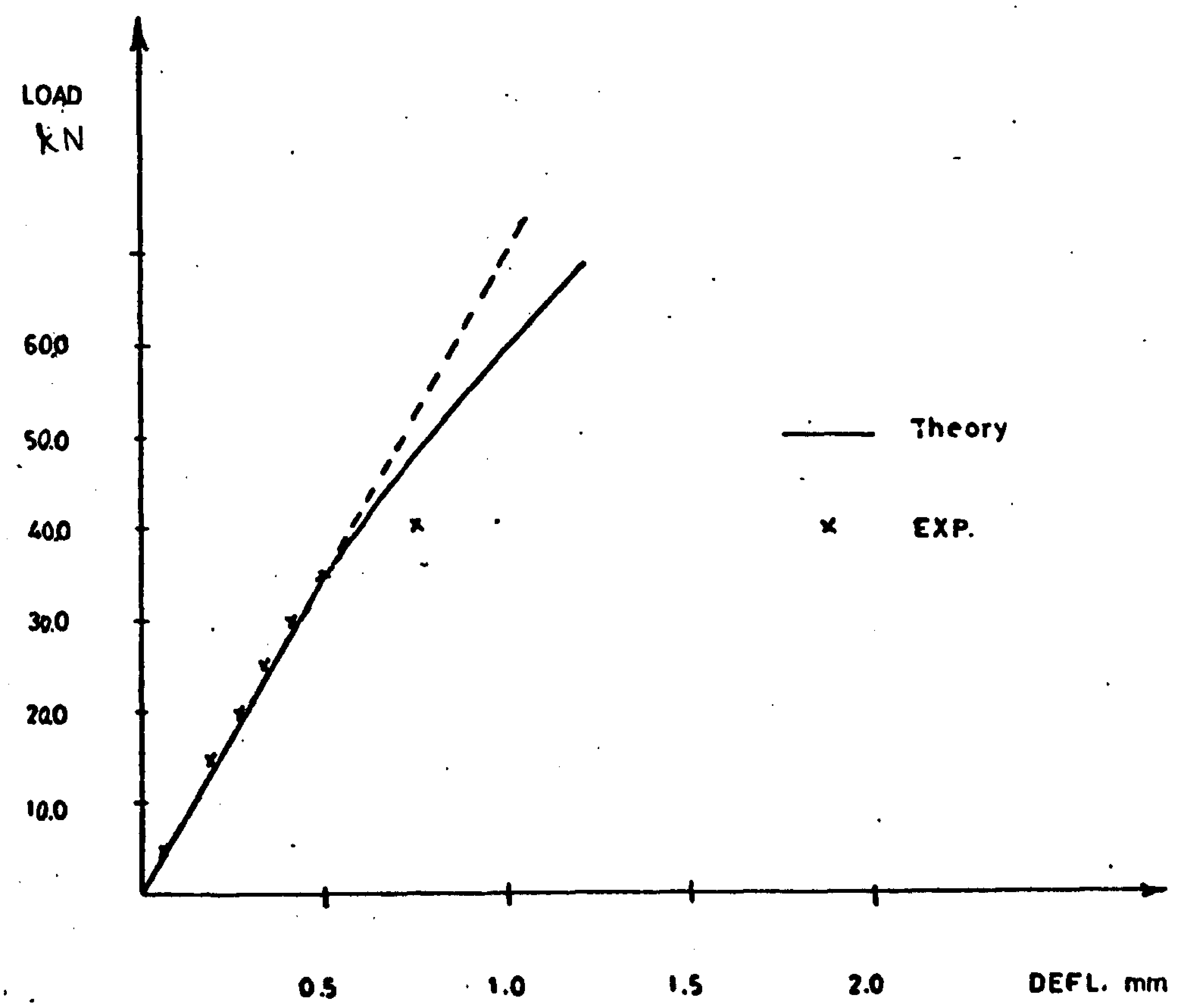


FIGURE 5.4 LOAD DEFLECTION CURVE FOR THE POINT OF MAXIMUM DEFLECTION AT THE SLAB (2-STOREY MODEL)

CHAPTER SIX

THEORETICAL AND EXPERIMENTAL RESULTS

6.1 Introduction

A proportion of the experimental and theoretical results obtained during this study are given in Chapters 2, 3, 4 and 5. Additional curves and results are given in Appendix I of this text and a comparison of some of the experimental and theoretical results is given in Chapter 7 along with the discussion of the results. For each model tested, the horizontal deflections of the walls and strain distribution in the floor slabs are given in graphical form for both the elastic and elasto-plastic behaviour. For the 1:3 scale model the deflections of the floor slabs are also given.

The theoretical and experimental deflection and strain distributions are given for a point lateral load for the three models tested. Limitations in power output of the available actuator prevented a rigorous dynamic test programme but the response curves for the 1:25 scale 13-floor model are given in Chapter 2. A comparison between the theoretical and the experimental results for the natural frequencies and estimation of damping is given in Chapter 7.

6.2 Experimental results

The experimental values of the deflections and strains for every model are given with the best straight line drawn through the plotted results. Due to the limited ability of concrete to sustain tension, the low strain static tests, especially for the

1 to 25 scale models, had to be conducted with limited deflections of the structures of the order of about 2mm. This resulted in relatively small experimental readings for deflection and strain increments. Measures were taken to correct the readings of the strain gauges due to change of temperature and humidity as described in Chapter 2.

The dynamic response curves for the 13-floor 1:25 scale model are plotted for the full range of frequencies considered and the peaks corresponding to the natural frequencies of the structure are clearly marked. For the cyclic load tests on the models the force-deflection cycles for a few significant points on the model are given. The force-strain cycles for the strain in the slabs are also given.

6.3 Theoretical results

Both the finite element method and the continuous connection technique described in Chapter 3 were used to compute the deflections and strains of the shear walls and the floor slabs of the three models described in Chapter 2. Although in both methods any type of load can be applied, the models were subjected only to point loads. The width of the floor slabs which was considered to act as deep beams spanning between the walls was calculated using the curves presented in Chapter 5. The theoretical results obtained are compared with the experimental results in Chapter 7. Curves for the effective width of slabs coupling different configurations of shear wall structures are presented in Chapter 5. Theoretical estimates of natural frequencies and percentage of critical damping for the 13-storey model were determined using a value of the dynamic modulus of elasticity obtained as described in Chapter 3.

CHAPTER SEVEN

DISCUSSION

7.1 Comparison of results

By the use of small scale models the overall failure mechanism of a laterally loaded shear wall building with vertical superimposed loading has been found experimentally to consist of a combination of shear and flexure cracks. The action of the floor slabs in the range of working stresses, both at large and small scale, have been found both theoretically and experimentally. In the elasto-plastic range the action of floor slabs connecting planar shear walls has been assessed both experimentally and theoretically and the failure pattern of such slabs has been studied. The photographic record in Appendix III shows cracking initiated at the points of the walls and later cracks radiated from this region.

It was found from the experimental investigations described in Chapter 2 and the theoretical investigations obtained by employing Fortran and Algol computer programs for the methods described in Chapter 3, that generally good agreement has been obtained between the theoretical and experimental results.

7.1.1 Deflections

Comparison of the theoretical and experimental values of deflection for the three models used in this analysis show that the methods of analysis presented in Chapter 3 yield results of sufficient accuracy for application of the method in practice. In fact the finite element analysis shows a more accurate result than those obtained using the continuous technique method. Figure 7.1

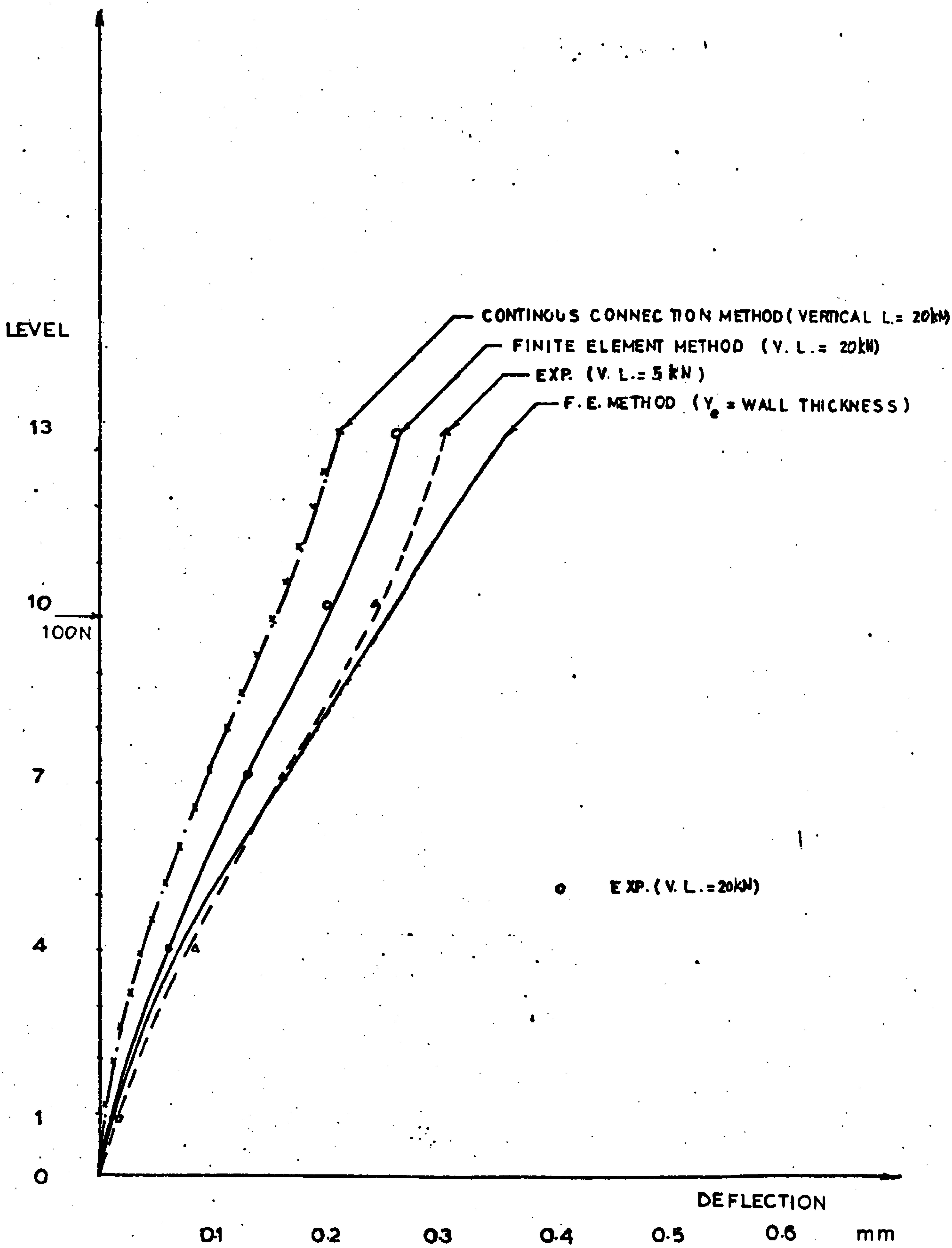


FIGURE 7.1 COMPARISON BETWEEN THE DIFFERENT THEORETICAL RESULTS AND THE EXPERIMENTAL RESULTS FOR 13-STOREY MODEL

shows the results of the theoretical analysis for lateral deflections using the two methods of analysis for the 13-floor 1:25 scale model. The experi-:mental readings obtained for the model are plotted on the same graph to enable the comparison. It is worth noting (Figure 2.10) the stiffening effect of increase in vertical loading on the 13-storey 1:25 scale model. It is thought that this is due to a combination of a slight increase in lateral restraint of the model by the vertical tie rods and increases in internal frictional forces between the micro-concrete particles tending to effectively increase the modulus of elasticity of the material.

In Figure 7.2a a comparison between the theoretical results of deflection of the slab-wall contact point using the finite element method and the experimental results obtained for the 1:3 scale model shows close agreement. Similar theoretical results were obtained for the 1:25 scale 9-storey model. Additional theoretical results are given in Appendix 1.

7.1.2 Strains and stresses

The experimental strain distribution in the slab connecting coupled planar shear walls is given in Chapter 2 and Appendix 1. The comparison between the theoretical results obtained from the finite element analysis and the experimental results of the 1:3 scale model is shown in Figure 7.3. From the Figure one can see that a reasonable accuracy can be achieved in determining the strains in the coupling slab using the program described in this report. The stresses in the shear walls were calculated theoret-:ically and the results are given in Appendix 1.

7.1.3 Dynamic analysis

The dynamic analysis of shear wall buildings allows assessment of the building natural frequencies, mode shapes and the probable response of the structures for both normal design wind speeds and severe earthquake loading. From these analyses maximum probable stresses in both working range and in ultimate conditions can be found and an assessment of the acceptability to the occupants of the building to their dynamic motion in wind storms can be made.

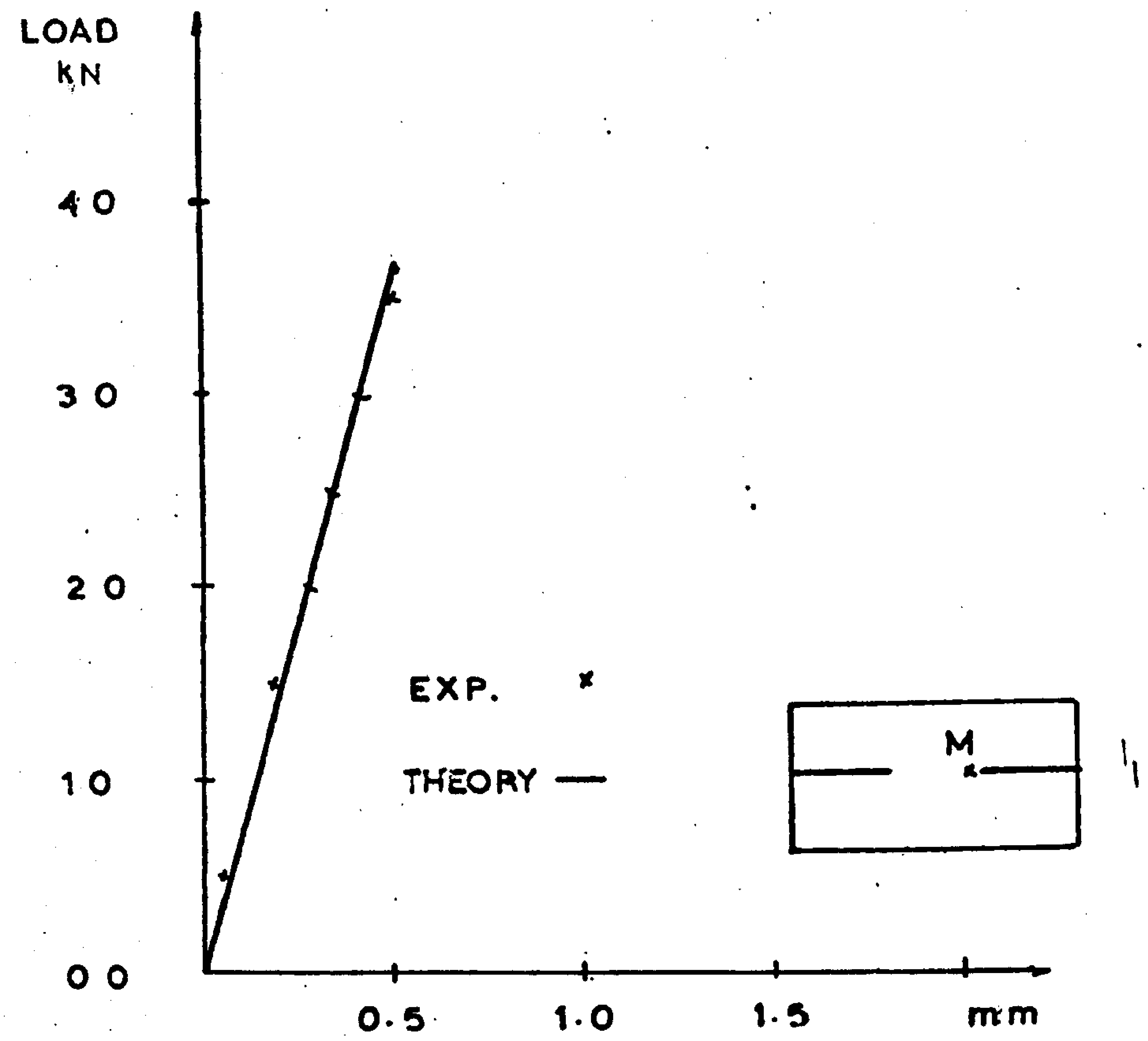


FIGURE 7.2 COMPARISON BETWEEN THE THEORY AND EXPERIMENTS FOR THE DEFLECTION AT POINT M (2-STOREY MODEL)

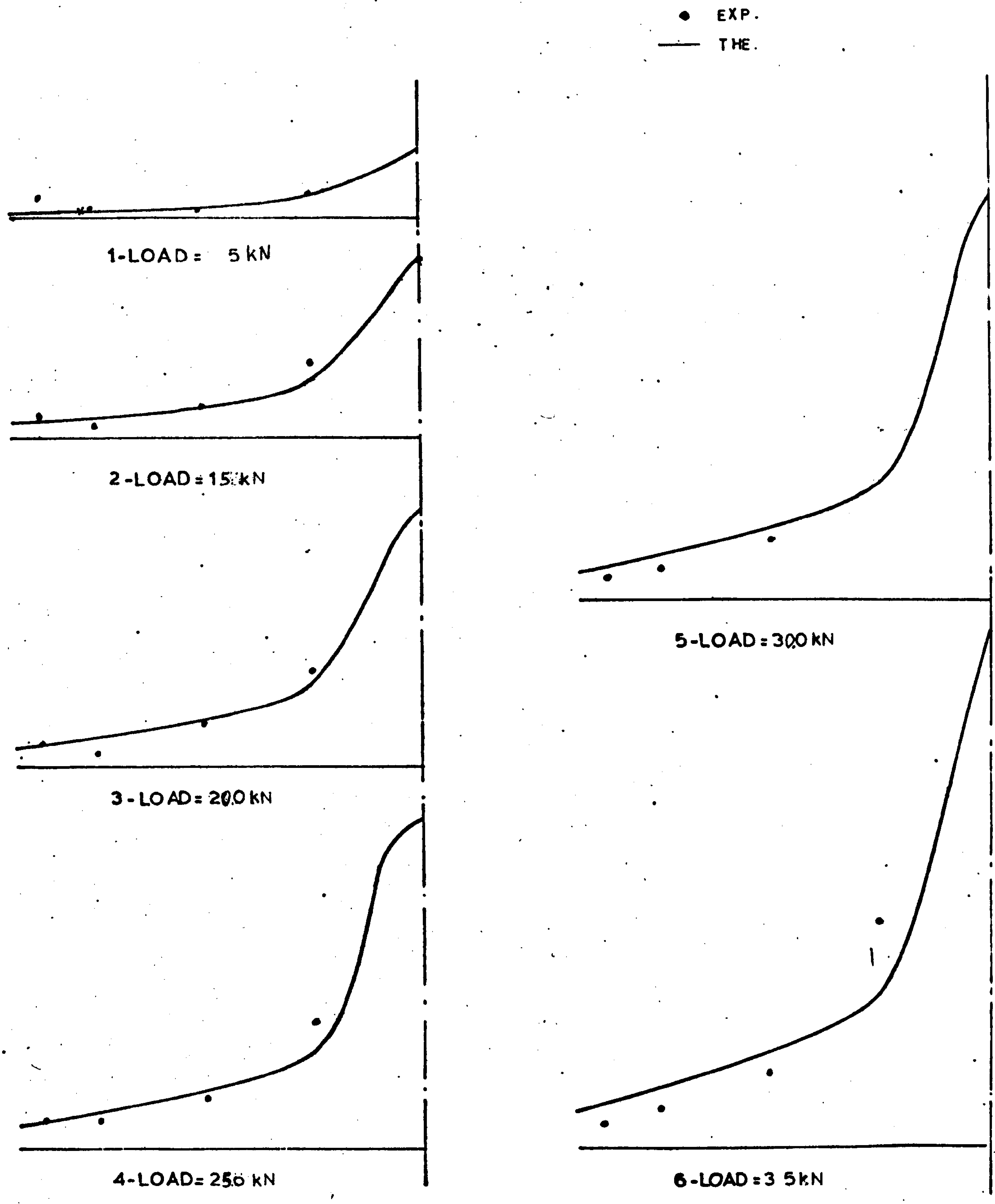


FIGURE 7.3 COMPARISON BETWEEN THE THEORETICAL AND EXPERIMENTAL RESULTS FOR THE STRESSES IN THE SLAB (2-STOREY MODEL)

The experimental procedures allow, in addition, values of structural damping to be found. It is interesting to examine the effect of vertical loading on the dynamic response of the structures since it is often difficult to estimate the super:imposed loadings likely to be present in a structure in advance of any event which may produce vibration of the structure. Damping tends to decrease with increasing vertical load for the same amplitude of vibration and the effects of added mass and vertical load tend to reduce natural frequency of oscillation in horizontal nodes. Table 7.1 shows a comparison of the results obtained for the natural frequencies and damping of the 13-floor model with no vertical loading applied, while Table 7.2 gives the same results for a vertical load of 5 kN applied to the model. Additional mass - of channels, load cells and the tie nodes - was added to the model for application of the vertical load as well as the prestressing which effectively increased the dynamic modulus of elasticity to 39.3 kN/mm^2.

	In the X direction		Torsion	
Natural Frequencies	Exp.	Theor.	Exp.	Theor.
	11.2	13.5	16.75	17.5
Damping	1.8%		2.3%	

TABLE 7.1 COMPARISON OF DYNAMIC RESULTS WITH NO VERTICAL LOAD

	In the X direction		Torsion	
Natural Frequencies	Exp.	Theor.	Exp.	Theor.
	10.6	10.63	13.8	12.4
Damping	1.4%		1.5%	

TABLE 7.2 COMPARISON OF DYNAMIC RESULTS WITH VERTICAL LOAD 5kN

7.2 Effective slab widths

In Chapter 5 the effective width of slabs coupling various wall shapes were presented and the effects of the different wall shapes and thicknesses on the effective slab widths and structural stiffness were given. In the analysis of the models mentioned above and described in Chapter 2, the effective slab widths derived theoretically were used to find the theoretical response of each model. It can be seen from Figure 6.1 that fairly accurate comparisons were obtained.

7.3 Elasto-plastic analysis

Prior to this study estimations of elasto-plastic performance of coupled shear wall structures had not included investigations of complete wall-slab structures by theoretical means, and experimental investigations had been limited to small scale structures. The theoretical results and experimental method presented in this study allows designers to assess with reasonable accuracy the likely performance of a proposed or existing structure if it is subjected to cycles of elasto-plastic behaviour.

The elasto-plastic tests results for the three models, given in detail in the Figures, are summarised in Table 7.3. From these results it is apparent that for the overall structures a ductility (i.e. ratio of maximum deflection to deflection at yield) in excess of 4 can be readily achieved, a value which is considered by many authorities to be a minimum ductility requirement for structures built in zones subject to strong ground motion. To achieve such an overall structural ductility much higher ductility is required of component parts of buildings, and here again it has been shown at large scale that for the critical regions where slabs couple walls that ductilities in excess of 12 can be obtained if care is taken in detailing the reinforcement. Other data which is obtained from the elasto-plastic tests concerns the stiffness degradation of the structures as cycling progresses, and the energy absorption capacit-:ies of the structures. This data can be used to assess both the strength potential of a structure subject to single large shock loadings

Model	Max. ductility factor - D	C_E kN/mm/kg	C_{LU} N/kg	K_1 kN/mm	K_2 kN/mm	K_3 kN/mm	K_4 kN/mm	K_6 kN/mm
1:25 scale 9-storey	6	0.44	72.6	1.25	1.18	1.14	1.0	0.84
1:3 scale 2-storey	12.8	0.78	11.4	76.9	26.1	15.6	19.9	-

C_E = Energy absorbed during cyclic load tests/mass of test unit

C_{LU} = Maximum load applied/mass of test unit (N/kg)

D = Maximum overall ductility of test unit = $\frac{\text{maximum deflection}}{\text{yield deflection}}$

K_i = Stiffness of test unit for loading i in kN/mm

TABLE 7.3 SUMMARY OF ELASTO-PLASTIC RESPONSE DATA FOR THE 9-STOREY AND 2-STOREY MODELS

and the likely response of a structure forced to cycle elasto-plastically for a number of oscillations. In general, the areas enclosed by the elasto-plastic load deflection curves indicate a considerable capacity to absorb energy with the structures retaining considerable stiffness for a number of loadings. This is borne out by the high percentage of the initial stiffness retained by the 9-storey model as shown in Table 7.3. It should be noted that part of the energy input to the 2-storey, 1:3 scale model is used to overcome frictional forces at the support rollers.

7.4 General discussion

Generally in any zone requiring earthquake or hurricane resist:ance a minimum overall structural ductility of 4 is usually essential. To achieve this the ductility requirement of the connecting beams or slabs is greatly in excess of 4. For both working stresses and ultimate load conditions the action of the floor slabs or coupling beams is such that bending stresses at the junction with the walls may be positive or negative, thus requiring both faces to be reinforced to resist lateral deflections of the structure in addition to normal positive bending moment from gravity floor loading.

In practice bending of the floor slabs may be caused by gravity loads, differential settlements, lateral deflections and torsional movements of buildings. The combined effects of these actions must be considered in assessing the requirements of floor slabs or coupling beams. Torsional movement can be caused by eccentricity of structure stiffness, mass or loading and can result in wall and slab stresses of equal magnitude to the usual bending and vertical stresses, especially in severe dynamic conditions.

The overall structural analyses used in this study are suitable for buildings where the walls are continuous throughout their height and care must be taken if construction joints are present. These construction joints may affect the stress distribution especially in elasto-plastic conditions. Such construction joints are often a haxard in extreme conditions of lateral load since relative horizontal movement can occur at these joints, resulting in reduced building stiffness and degredation of a structure.

During this study, models were tested using various vertical loadings varying from zero to that corresponding to maximum probable vertical building loading. The effect of the vertical load in the range of working stresses is to reduce the overall deflections of the structure and reduce the stresses in the floor slabs. In the post-elastic range the absence of imposed vertical loading tends to allow rapid flexural failure, especially if there is no torsion of the structure. If a substantial vertical loading is applied more cracking is encouraged and hence greater ductility is achieved before failure of the structure as is generally the case in practice.

CHAPTER EIGHT

CONCLUSIONS AND SUGGESTIONS FOR FUTURE WORK

8.1 Conclusions

1. Theoretical solutions of complete multi-storey shear wall buildings and theoretical solutions for coupled shear walls have been presented. The theoretical analyses are based on both the continuous connection method and the finite element techniques. The effects of the vertical uniformly distributed load and the effective width of the slabs connecting the shear walls, as given in Chapter 5, have been considered.

   The continuous connection method can be applied to both two-dimensional and complex three-dimensional systems. If the buidling is uniformly loaded and consists of several identical coupled shear wall load bearing units only, the analysis using the finite element method is performed directly. In this case the structure can be treated as a two-dimensional coupled shear wall.

   The finite element analysis as described in Chapter 3 can be applied to symmetrical and non-symmetrical coupled shear walls with any number of openings.

2. Comparison of the results from theoretical analysis and the results obtained from elastic tests on the models described in Chapter 2 show that the theoretical methods presented can be used to produce accurate values of stresses and deflections for two-dimensional coupled shear walls and for three-dimensional

multi-storey shear wall buildings subject to any system of lateral loading.

3. Comparison between the dynamic test results obtained for the 1:25 scale 13-storey model and the theoretical dynamic analysis shows that the method can be used for estimating, with sufficient accuracy for practical purposes, the natural frequencies of regular symmetric shear wall structures with and without superimposed vertical load.

4. Agreement between the experimental and theoretical values for the effective widths of the slabs coupling shear walls is, in general, good when the wall openings are not small. The effective width of the floor slabs derived from the experimental results, in the case of small wall openings, is less than that found theoretically. The difference is partially due to the fact that local wall deformations are not allowed for in the theory and partly to difficulties in obtaining exact values experimentally. As indicated in Figure 4.13, as the wall opening to the slab length ratio decreases the system becomes stiffer and the deflections are reduced. It was found, in general, that the stiffness of the slab is significant for small values of wall openings and large values of the slab width/slab length ratio.

5. From Figures 4.13, 4.25 one can see that for the elastic range the large values of stresses are mainly concentrated in the parts of the slab close to the wall for the planar configuration. The use of T-section or box-type wall configurations assist in distributing the stresses over larger areas of the slab.

6. The high strain cycle performance obtained from the reinforced concrete models supports the view that shear wall structures can be suitably designed to withstand severe earthquakes. This also shows that if care is taken in the design of the connecting slabs coupling shear walls, a stiffer, more ductile and economical structure can be achieved.

7. Although the elasto-plastic theoretical analysis presented for the slabs coupling shear walls is a first step towards a more accurate analysis, one can use the theoretical results in conjunction with the experimental values to obtain a reasonably good assessment of the ductility of these slabs.

8. In Chapter 1 it is shown that it is not only structural design considerations which control the construction of tall buildings, but a number of additional concepts such as the sociological aspects and the human reactions to such buildings have to be catered for.

8.2 Suggestions for future work

Although many papers and much work has been published concern:ing the theoretical and experimental analysis of shear wall buildings, there still remain a large number of problems which need to be thoroughly studied, and these include:

1. Theoretical elasto-plastic cyclic analysis for complete buildings subject to bending and torsional force, and also for the slabs and beams coupling the shear walls. This can be done, for example, by one of the step-by-step integration methods which can readily be applied in the finite element programme.

2. The effect of the type of the foundations - such as individual wall footings, rafts, cassions or pile foundations - and the effect of the strength of the foundation on the performance of shear wall buildings in resisting normal working forces, dynamic forces and severe forces which cause elasto-plastic deformations of the structure.

3. The effect of very stiff services floors.

4. The use of discrete steel and concrete elements in the finite element analysis to take into account the effect of the bond and bond length between the steel and concrete and to study the effect of the cracking and the crack development in the post-elastic situations.

5. Although the effect of the vertical uniformly distributed load on the building was studied in this work, still more is required to find the effect of applied superimposed loads, differential settlement and differential temperature movements which may occur prior to active loadings.

6. The effect of large vertical loads over only part of the height of the building, on the performance of the building for the dynamic, elastic and elasto-plastic conditions.

7. The effect of steel to concrete ratios and the steel arrangements in walls and beams on the overall structural performance.

8. The effect of non-structural units and materials on the static, dynamic and elasto-plastic behaviour.

9. Possible use of units combining fibres and normal reinforcement in the concrete to assist in crack prevention and to improve the elasto-plastic structural performance.

10. Performance of structures which have been damaged and subsequently repaired using epoxy resin pressure grouting, sprayed on glass fibre reinforced cement or steel fibre reinforced cement. The performance of structures repaired by any of these means would have to be studied at a range of damage conditions and account taken of the number of cycles undergone in elasto-plastic range when initial occurred. Aging effects on epoxy repairs, steel carbon and glass fibre reinforced units must also be studied to assess their durability and long term properties.

11. Study of the amplitude dependance of damping and frequency of oscillation and the effect of Tuned Mass Dampers to minimise oscillations of tall structures in wind storms.

12. The isolation of tall buildings from ground borne vibrations using, for example, rubber pads or steel rollers.

13. Experimental work to determine the effect of local bending deformations at wall/slab intersections on the performance of coupled shear wall systems. Such experiments are especially needed for the cases where the walls are T-section of box type.

14. The effect of construction and expansion joints on overall and local structure performance.

15. Examination of local slab stresses and the effects of slab/

wall thickness ratios.

16. Demolition of tall buildings.

BIBLIOGRAPHY

1. ALEXANDER C.M. et al — 'Cyclic load tests on shear-wall panels'
   Proc. 5th World Conf. on Earthquake Eng. Rome, 1973.

2. ARVIDSSON K. — 'Shear-walls with door openings near the edge of the wall'
   A.C.I. Journal, July 1974, Title No 71-72 pp. 353-357.

3. AUTENBURG A.V. — 'Wall-beam frames under static lateral load'
   Discussion, Proc. A.S.C.E. Struct. Div., Aug. 1975, ST8, pp. 1727-1729.

4. BARNARD P.R. — 'The shear-wall-slab building'
   The Canadian Architect, Mar. 1966, pp. 66-68.

5. BHATT P. — 'Effect of beam-shear-wall junction deformation on the flexibility of the connecting beams'
   Building Science, 1973, Vol.8, pp. 149-151.

6. BECK H. — 'Contribution to the analysis of coupled shear walls'
   J. A.C.I., Aug. 1962, Title No 59, 39 pp. 1055-1070.

7. BISWAS J.K., TSO W.K. — 'Three dimensional analysis of shear-wall buildings to lateral load'
   Proc. A.S.C.E. Struct. Div., May 1974, ST5, pp. 1019-1036.

8. CHANG F.K. — 'Wind and movement in tall buildings'
   Civ. Eng. A.S.C.E. Vol 37, 1967, No 8, pp. 70-72.

9. CHEN P.W., ROBERTSON L.E. — 'Human perception thresholds of horizontal motion'
   Proc. A.S.C.E. Struct. Div., Aug. 1972, ST8 pp. 1681-1695.

10. CLOUGH R.W., PENZIEN J. — 'Dynamics of structures'
    McGraw-Hill, 1975.

11. COULL A. — 'Free vibrations of regular symmetrical shear-wall buildings'
    Building Science, 1975, Vol.10, pp.127-133.

12. COULL A., CHOUDHURY J.R. — 'Stresses and deflections in coupled shear-walls'
    A.C.I. Journal, Feb. 1967, Title No 64-6 pp. 65-72.

13. COULL A., CHOUDHURY J.R. 'Analysis of coupled shear-walls'
A.C.I. Journal, Sept 1967, Title No 64-51, pp. 587-593.

14. COULL A., EL HAG A.A. 'Effective coupling of shear-walls by floor slabs'
A.C.I. Journal, Aug 1975, Title No 72-31, pp. 429-431.

15. COULL A., IRWIN A.W. 'Model investigation of shear wall structures'
Proc. A.S.C.E. Struct. Div., June 1972, ST6, pp. 1223-1237.

16. COULL A., IRWIN A.W. 'Analysis of load distribution in multi-storey shear-wall structures'
The Structural Engineer, Aug 1970, No 8, V.48, pp. 301-306.

17. COULL A., IRWIN A.W. 'Design of connecting beams in coupled shear-wall structures'
A.C.I. Digest Paper, Mar 1969, Title No 66-20.

18. COULL A., IRWIN A.W. 'Torsional analysis of multi-storey shear-wall structures'
A.C.I. Paper SP 35-6, pp. 211-238.

19. COULL A., PURI R.D. 'Analysis of coupled shear-walls of variable thickness'
Building Science, 1967, Vol 2, pp. 181-188.

20. COULL A., PURI R.D., TOTTENHAM H. 'Numerical elastic analysis of coupled shear-walls'
Proc. Inst. Civ. Eng., Paper 2, pp.109-129.

21. COULL A., MUKHERJEE P.R. 'Natural vibrations of shear-wall buildings on flexible supports'
Earthquake Engineering and Structural Dynamics, Vol 6, pp. 295-315, 1978.

22. COULL A., STAFFORD-SMITH B. 'Torsional analysis of symmetric building structures'
Proc. A.S.C.E. Struct. Div., Jan 1973, ST1, pp. 229-233.

23. COULL A., STAFFORD-SMITH B. 'Structural analysis of tall concrete buildings'
Proc. Inst. Civ. Eng., Part 2, Mar 1973, Vol 55, pp. 151-167.

24. CURRIE A.O., SMITH R.G. 'Wall-beam frames under static lateral load'
Discussion, Proc. A.S.C.E. Struct Div., Dec 1975, ST12, pp. 2679-2681.

25. DE LISLE D.J. 'Effect of repeated cyclic lateral loading on load bearing shear-wall panels'
M.Eng. thesis, McMaster Univ., Canada, 1971.

26. ELKHOLY I.A.S., ROBINSON H. 'An inelastic analysis of coupled-shear-walls'
Building Science, 1974, Vol 9, pp. 1-8.

27. ELKHOLY I.A.S., ROBINSON H. 'Analysis of multi-bay coupled shear-walls'
Building Science, 1973, Vol 8, pp. 153-157.

28. FINTEL M. 'Ductile shear walls in earthquake resistant multi-storey buildings'
A.C.I. Journal, June 1974, pp. 296-305.

29. GLUCK J. 'Elasto-plastic analysis of coupled shear-walls'
Proc. A.S.C.E. Struct. Div., Aug 1973, ST8, pp. 1743-1760.

30. HANSON, REED and VANMARCKE 'Human response to wind-induced motion of buildings'
Proc. A.S.C.E. Struct. Div., July 1973, ST7, pp. 1589-1605.

31. HARRISON T. 'The effect of transverse straps on the bending and torsional behaviour of a thin-walled prismatic beam of open cross-sectional profile'
Int. J. Mech. Sci., Pergman Press, 1969, Vol 11, pp. 225-239.

32. HARRISON T., SIDDALL J.M., YEADON R.E. 'A modified beam stiffness matrix for inter-connected shear walls'
Building Science, 1975, Vol 10, pp. 89-94.

33. HART G.C., DIJULIO R.H., LEW M. 'Torsional response of high-rise buildings'
Proc. A.S.C.E. Struct. Div., Feb 1975, ST2, pp. 397-417.

34. HOLMES M., ASTILL A.W., MARTIN L.H. 'Experimental stresses and deflections of a model shear-wall structure'
A.C.I. Jnl., Vol 66, 1969, pp. 667-677.

35. HUSSEIN W.A. 'Analysis of multi-storey shear-wall structures by the shear connection method'
Building Science, 1972, Vol 7, pp. 69-73.

36. IABSE, ISE 'Tall buildings and people?'
Residential Conference, St. Catherine's College, Oxford, 17-19 Sept 1974.

37. IRWIN A.W. 'Static, dynamic and elasto-plastic performance of reinforced concrete tall building core structures'
Int. Conf. on the Behaviour of Slender Structures, City Univ. London, Sept 1977.

38. IRWIN A.W. 'Human reactions to oscillations of building-acceptable limits'
Build International, 1975, Vol 8, pp. 43-55.

39. IRWIN A.W. 'Analysis of shear-wall structures'
Ph.D. thesis, Univ. of Strathclyde, Dept. of Civil Eng., 1970.

40. IRWIN A.W. 'Vibrations of asymmetrical multi-storey shear-wall buildings'
I.C.E. paper No 7397, Sept 1971, Vol 50, pp. 77.

41. IRWIN A.W. 'Dynamic tests on small scale structures, applications and analysis of data'
Build International, 1975, Vol 8, pp. 495-509.

42. IRWIN A.W. 'Human response to dynamic motion of structures'
The Structural Engineer, Sept 1978, No 9, Vol 56A, pp. 237-244.

43. IRWIN A.W. 'Analysis of tall shear-wall buildings including in-plane floor deformations'
Build International (8), 1975, pp. 43-55.

44. IRWIN A.W. 'Static and dynamic tests on model shear-wall structures'
Proc. Inst. Civ. Eng., 1972, 51, April, pp. 701-710.

45. IRWIN A.W., ANDREW N. 'Torsional performance of coupled channels in tall buildings'
Proc. Inst. Civ. Eng., Part 2, 1976, 61, Dec, pp. 773-783.

46. IRWIN A.W., BAIN W.R.L. 'Planned obsolescence and demolition of tall buildings'
Build International (7) (1974), pp. 549-561.

47. IRWIN A.W., ORD A.E.C. 'Cyclic load tests on shear-wall coupling beams'
Proc. Inst. Civ. Eng., Part 2, 1976, 61, June, pp. 331-342.

48. IRWIN A.W., SALAMA A.E. 'Structural and human response criteria in the design of tall buildings'
1st Conf. for Postgraduate Egyptian Students, London, March 1979.

49. IRWIN A.W., YOUNG R.W. 'Tests on a reinforced concrete model shear-wall building'
Proc. Inst. Civ. Eng., Part 2, Mar 1976, pp. 163-177.

50. JAEGER L.G., MUFTI A.A., MAMET J.C. 'The structural analysis of tall buildings having irregularly positioned shear walls'
Building Science, 1973, Vol 8, pp. 11-22.

51. KALER I., GULK J. 'Elasto-plastic finite element analysis'
Int. Jour. for Num. Methods in Engineering, Vol II, 1977, pp. 875-881.

52. KAZIMI S.M.A., AGARWAL P.D. 'Stresses and deflections of intermediate height shear-walls with rectangular openings'
Building Science, 1974, Vol 9, pp. 109-114.

53. KHAN A.H., STAFFORD-SMITH B. 'A simple method of analysis for deflections and stresses in wall-frame structures'
Building & Environment, 1976, Vol 11, pp. 69-78.

54. KRISHNAMOORTHY, C.S., PANNEERSELVAM A. 'A finite element model for non-linear analysis of reinforced concrete framed structures'
The Structural Engineer, Aug 1977, No 8, Vol 55, pp. 331-338.

55. LIAUW T.C., LEUNG K.W. 'Torsional analysis of core wall structures by transfer matrix methods'
The Structural Engineer, April 1975, No 4, Vol 53, pp. 187-194.

56. MACLEOD I.A. 'New rectangular finite element for shear-wall analysis'
Proc. A.S.C.E. Struct. Div., March 1969, ST3, pp. 399-409.

57. MACLEOD I.A. 'Lateral stiffness of shear-walls with openings'
"Tall Buildings", Pergamon Press Ltd. Oxford, 1967, pp. 223-250.

58. MACLEOD I.A. 'Analysis of shear-wall buildings by the frame method'
Proc. Inst. Civ. Eng., Part 2, pp. 593-603.

59. MACLEOD I.A., GREEN D.A. 'Frame idealization for shear-wall support systems'
The Structural Engineer, Feb 1973, No 2, Vol 51, pp. 71-74.

60. MACLEOD I.A., HOSNY H. 'The distribution of vertical load in shear-wall buildings'
The Structural Engineer, Feb 1976, No 2, Vol 54, pp. 67-71.

61. McKENZIE W.M.C. 'A study of staggered and shear-wall tall building systems'
Thesis for degree of Ph.D., Heriot-Watt Univ., Nov 1976.

62. MEE A.L., JORDAAN I.J., WARD M.A. 'Wall-beam frames under static lateral load'
Proc. A.S.C.E. Struct. Div., Feb 1975, ST2, pp. 377-395.

63. MENDELSON E., BARUCH M. 'Damped earthquake response of non-symmetric multi-storey structures'
The Structural Engineer, April 1975, No 4, Vol 53, pp. 165-171.

64. MICHAEL D. 'The effect of local wall deformations on the elastic interaction of cross-walls coupled by beams'.
"Tall Buildings", Pergamon Press Ltd., Oxford, 1967, pp. 253-270.

65. MICHAEL D. 'Torsional coupling of core walls in tall buildings'
The Structural Engineer, Feb 1969, No 2, Vol 47, pp. 67-71.

66. NAYAR K.K., COULL A. 'Elasto-plastic analysis of coupled shear walls'
A.S.C.E., Struct. Div., ST9, Sept 1976, pp. 1845-1860.

67. NAYAK G.C., ZIENKIEWICZ O.C. 'Elasto-plastic stress analysis. A generalization for various constitutive relations including strain softening'
Int. Jnl. for Numerical Methods in Engineering, Vol 5, 1972, pp. 113-135.

68. NEVILLE A.M. 'Properties of concrete'
Pitman Publishing, 1972.

69. NEWMARK N.M., ROSENBLUETH E. 'Fundamentals of earthquake engineering'
Civil Eng. and Eng. Mechanic Series, Prentice-Hall Inc., Englewood Cliffs, N.J., 1971.

70. NEWMARK N.M. 'A method of computation for structural dynamics'
Jnl. of Eng. Mechanics Div., Proc. A.S.C.E., July 1959, EM3, pp. 67-94.

71. PAULAY T. 'Design aspects of shear-walls for seismic areas'
Can. J. Civ. Eng., 2, 1975, pp. 321-344.

72. PAULAY T. 'The coupling of reinforced concrete shear-walls'
Proc. 4th World Conf. on Earthquake Eng., Chile - B - 2.

73. PAULAY T. 'An elasto-plastic analysis of coupled shear-walls'
A.C.I. Jnl., Nov 1970, Title No 67-60, pp. 915-922.

74. PAULAY T., SANTHAKUMAR A.R. 'Ductile behaviour of coupled shear-walls'
A.S.C.E. Struct. Div., ST1, Jan 1976, pp. 93-108.

75. PAULAY T. 'Coupling beams of reinforced concrete shear-walls'
Proc. A.S.C.E. Struct. Div., Mar 1971, ST3, pp. 843-861.

76. PAULAY T. 'Simulated seismic loading of spandrel beams'
Proc. A.S.C.E. Struct Div., Sept 1971, ST9, pp. 2407-2419.

77. PAULAY T. 'Some seismic aspects of coupled shear-walls'
Proc. 5th World Conf. on Earthquake Eng., Rome, 1973.

78. PECKNOLD D.A. 'Effective width of orthotropic plate'
Jnl. A.S.C.E., Struct. Div., May 1978, ST5, pp. 867-872.

79. PECKNOLD D.A. 'Slab effective width for equivalent frame analysis'
A.C.I. Jnl., April 1975, Title No 72-13, pp. 135-137.

80. QUADEER A., STAFFORD-SMITH B. 'The bending stiffness of slabs connecting shear-walls'
A.C.I. Jnl., June 1969, Title No 66-37, pp. 464-473.

81. QUADEER A., STAFFORD-SMITH B. 'Actions in slabs connecting shear-walls'
Proc. Symposium on Tall Buildings, Planning, Design and Construction, Vanderbilt Univ., Nashville, Tenn. Nov. 1974, pp. 315-338.

82. ROBINSON H., ELKHOLY I.A.S. 'Finite difference methods for analysing plane coupled shear-walls'
Report No 72-10, Oct 1972, Faculty of Engineering, McMaster Univ., Canada.

83. ROLL F. 'Materials for structural models'
Proc. A.S.C.E. Struct Div., June 1968, ST6, pp. 1353-1381.

84. ROSMAN R. 'Stability and dynamics of shear-wall frame structures'
Building Science, 1974, Vol 9, pp. 55-63.

85. SALAMA A.E. 'Numerical solutions for pile-driving'
A dissertation submitted to Manchester Univ. in partial fulfilment of require-:ments for the Degree of MSc., Dec 1976.

86. SALSE E.A.B., FINTEL M. 'Strength stiffness and ductility properties of slender shear-walls'
Proc. 5th World Conf. on Earthquake Eng. Rome, 1973.

87. SCHWAIGHOFER J., MICROYS H.F. 'Analysis of shear-walls using standard computer programs'
A.C.I. Digest Paper, Title No 66-89, Dec 1969.

88. SCHWAIGHOFER J., WING NING HO 'An elasto-plastic analysis of a core structure'
Building and Environment Vol 12, pp. 199-204, Pergamon Press, 1977.

89. STAFFORD-SMITH B. 'Modified beam method for analysing symmetrical inter-connected shear-walls'
A.C.I. Jnl., Dec 1970, Title No 67-68, pp. 977-980.

90. STAMATO M.C. 'Three-dimensional analysis of tall buildings'
Planning and design of tall buildings, A.S.C.E. New York, Vol III, 1973, pp. 683-699.

91. STAMATO M.C., STAFFORD-SMITH B. 'Approximate method for the three-dimensional analysis of tall buildings'
Proc. Inst. of Civ. Eng., July 1969, 43, pp. 361-379.

92. TSO W.K., BISWAS J.K. 'Analysis of core wall structures subjected to applied torque' Building Science, 1973, Vol 8, pp. 251-257.

93. TSO W.K., MAHMOUD A.A. 'Effective width of coupling slabs in shear-wall buildings' A.S.C.E. Struct Div., Mar 1977, ST3, pp. 573-586.

94. TSO W.K., RUTENBERG A., HEIDEBRECHT A.C. 'Cyclic loading in externally reinforced masonry walls confined by frames' Can. J. Civ. Eng., Vol 2, 1975, pp. 489-493.

95. VALLIAPPAN S., ASCE M., DOULAN T.F. 'Non-linear stress analysis of reinforced concrete' Proc. A.S.C.E. Struct. Div., April 1972, ST4, pp. 885-898.

96. WARD H.S. 'Dynamic characteristics of multi-storey concrete building' Proc. Inst. of Civ. Eng., Aug 1969, pp. 553-572.

97. WINKOUR A., GLUCK J. 'Ultimate strength analysis of coupled shear-walls' A.C.I. Jnl., Dec 1968, Title No 65-81, pp. 1029-1036.

98. YUZUGULLU O., SCHNOBRICH W.C. 'A numerical procedure for the determination of the behaviour of a shear wall frame system' A.C.I. Jnl., July 1973, Title No 70-74, pp. 474-479.

99. ZELMAN M.I., HEIDEBRECHT A.C., TSO W.K., JOHNSTON W.A. 'Practical problems and costs of fabricating multi-storey models' Models for Concrete Structures - A.C.I. publication No 24, Paper P.24-7, pp. 159-185.

100. ZIENKIEWICZ O.C. 'The finite element method in engineering science' McGraw-Hill, London, 1978.

101. ZIENKIEWICZ O.C., CHEUNG Y.K. 'The finite element method for analysis of elastic isotropic and orthotropic slabs' Proc. Inst. Civ. Eng., 28, 1964, pp. 471-488.

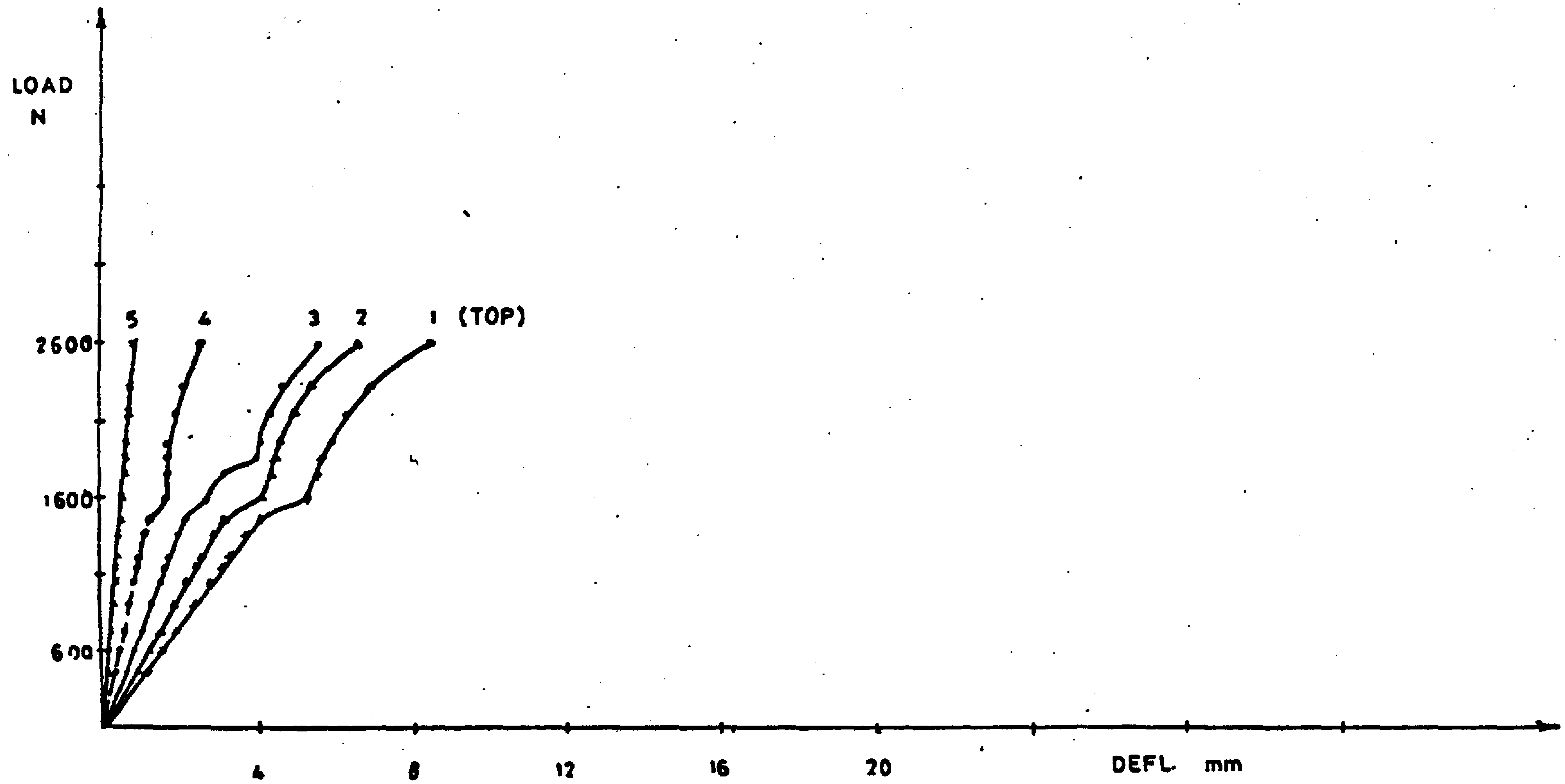
102. ZIENKIEWICZ O.C., VALLIAPPAN S., KING I.P. 'Elasto-plastic solutions of engineering problems "Initial Stress", finite element approach' Int. Jnl. for Numerical Methods in Eng., Vol 1, 1969, pp. 75-100.

103. 'Planning and Design of Tall Buildings'
Proc. A.S.C.E. and I.A.B.S.E. Conference, Aug 1972, Lehigh Univ., Bethlehem, Pennsylvania, Vol Ia, Ib and II.

APPENDIX I

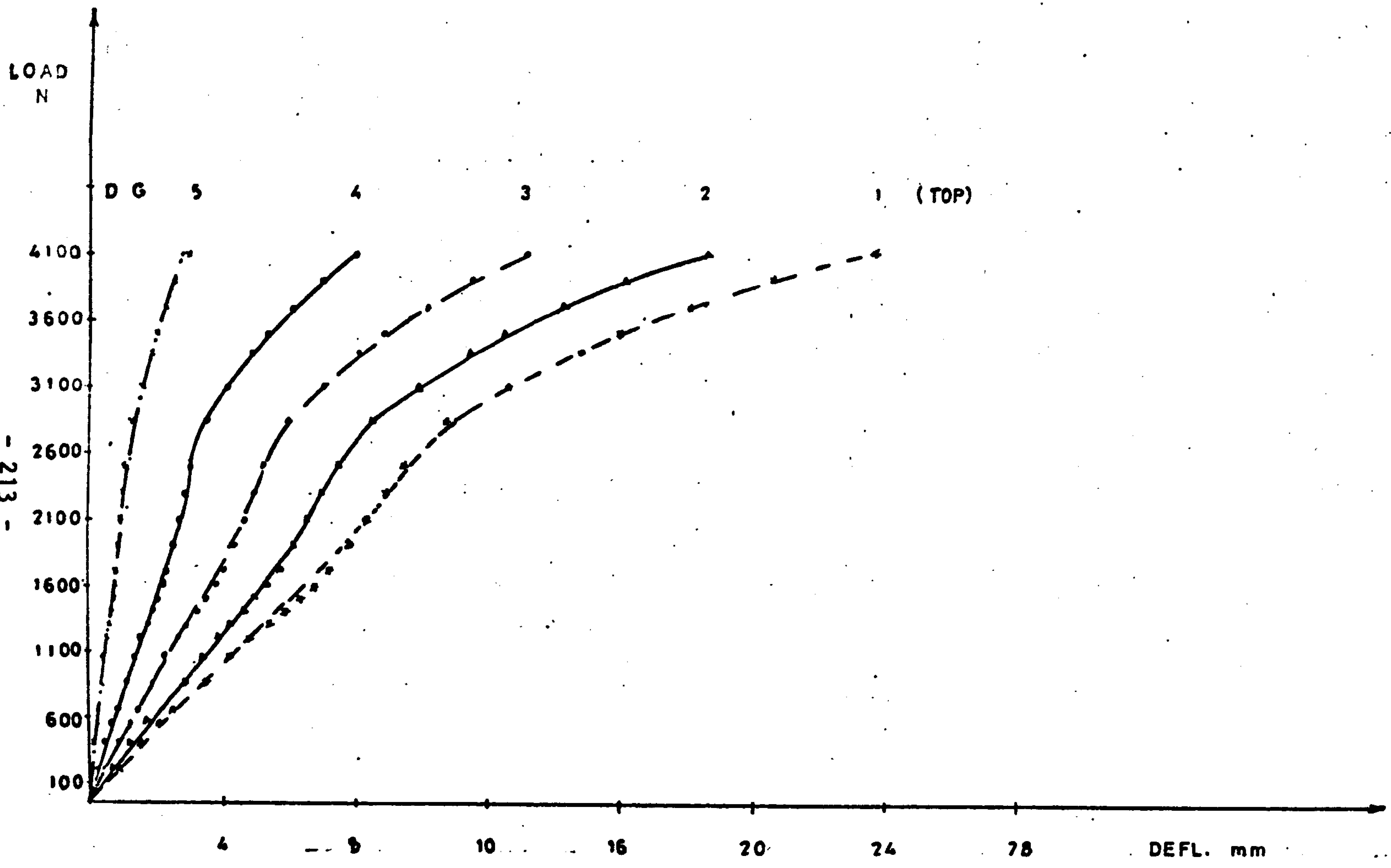
EXPERIMENTAL AND THEORETICAL RESULTS

EXPERIMENTAL RESULTS



1.1 LOAD DEFLECTION CURVES FOR FIRST LOADING (13-STOREY MODEL)

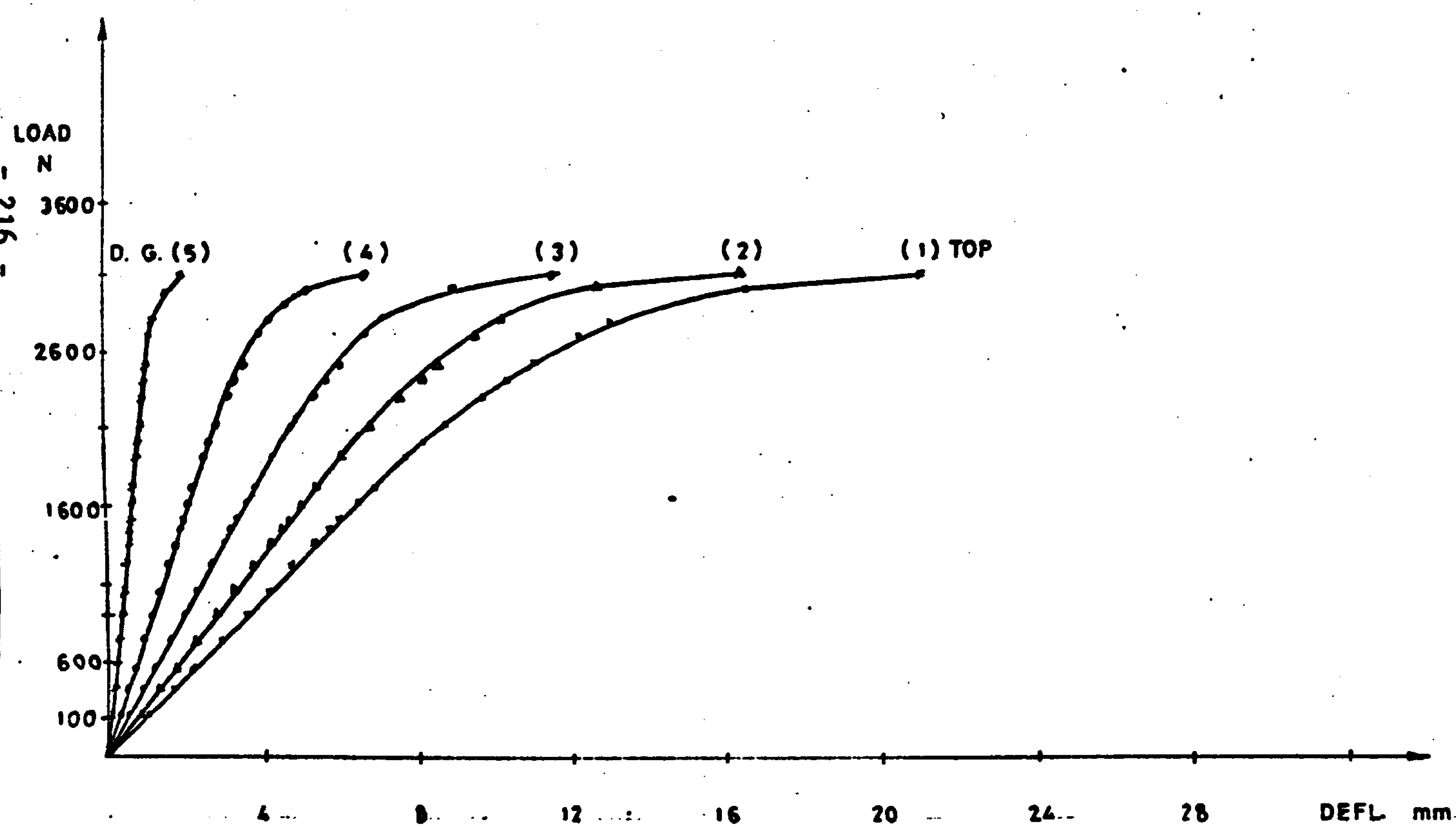
1.2 LOAD-STRAIN CURVES FOR THE FIRST LOADING (13-STOREY MODEL)



1.3 LOAD DEFLECTION CURVES FOR THE SECOND LOADING (13-STOREY MODEL)

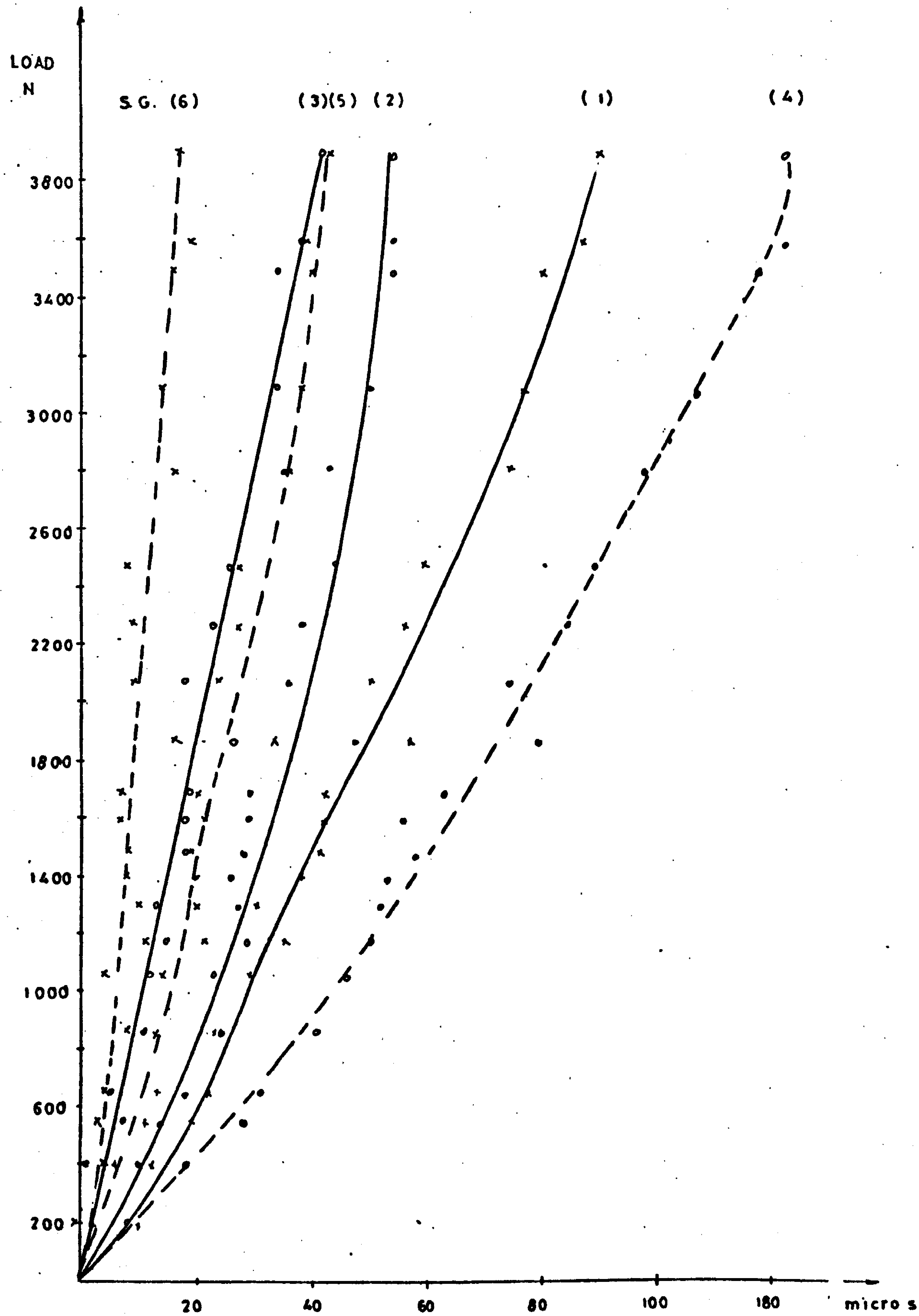
1.4 STRAIN DISTRIBUTION IN THE SLAB DURING SECOND LOADING (13-STOREY MODEL)

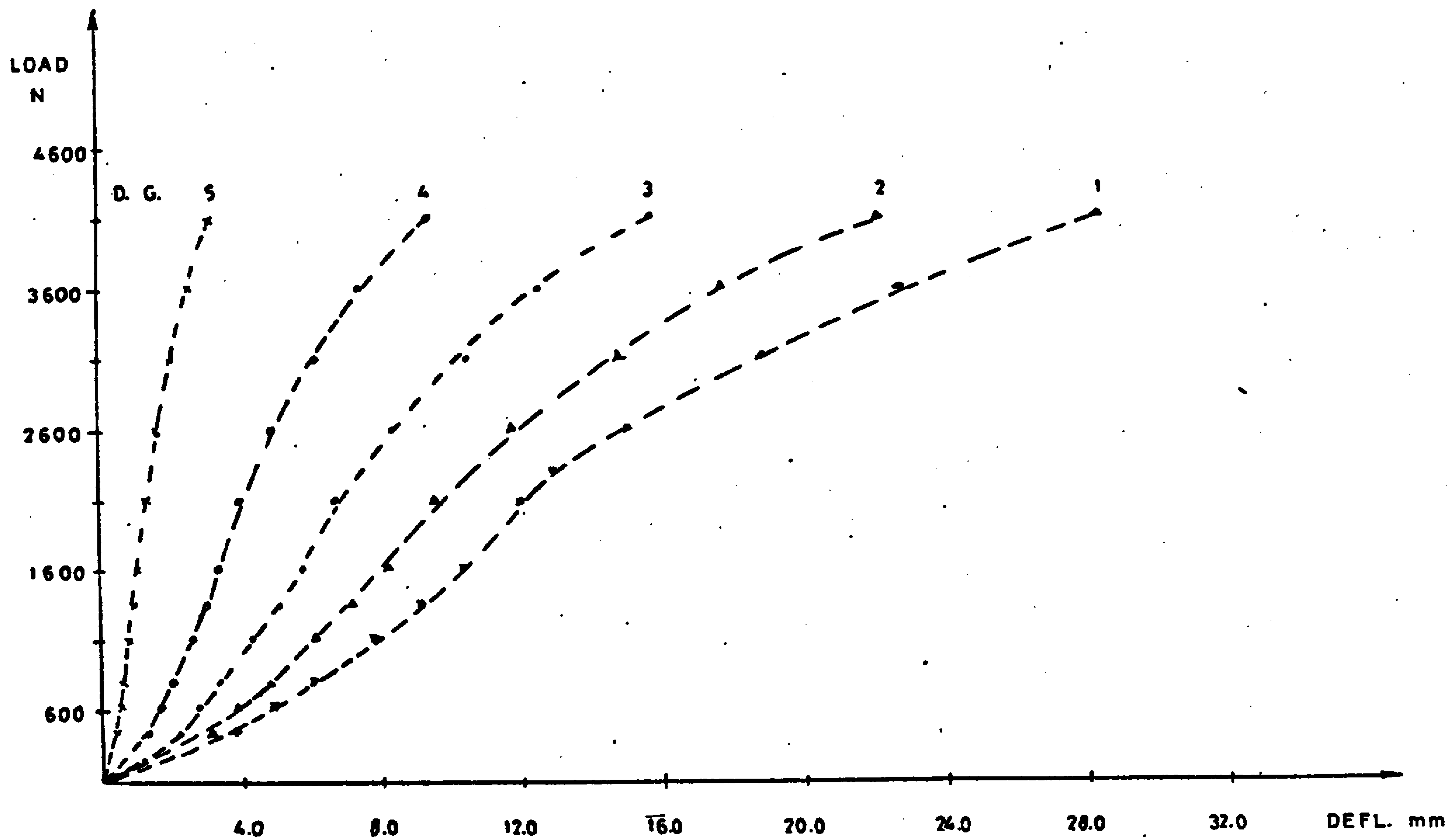
I.5 STRAIN DISTRIBUTION IN THE SLAB DURING SECOND LOADING (CONT.)



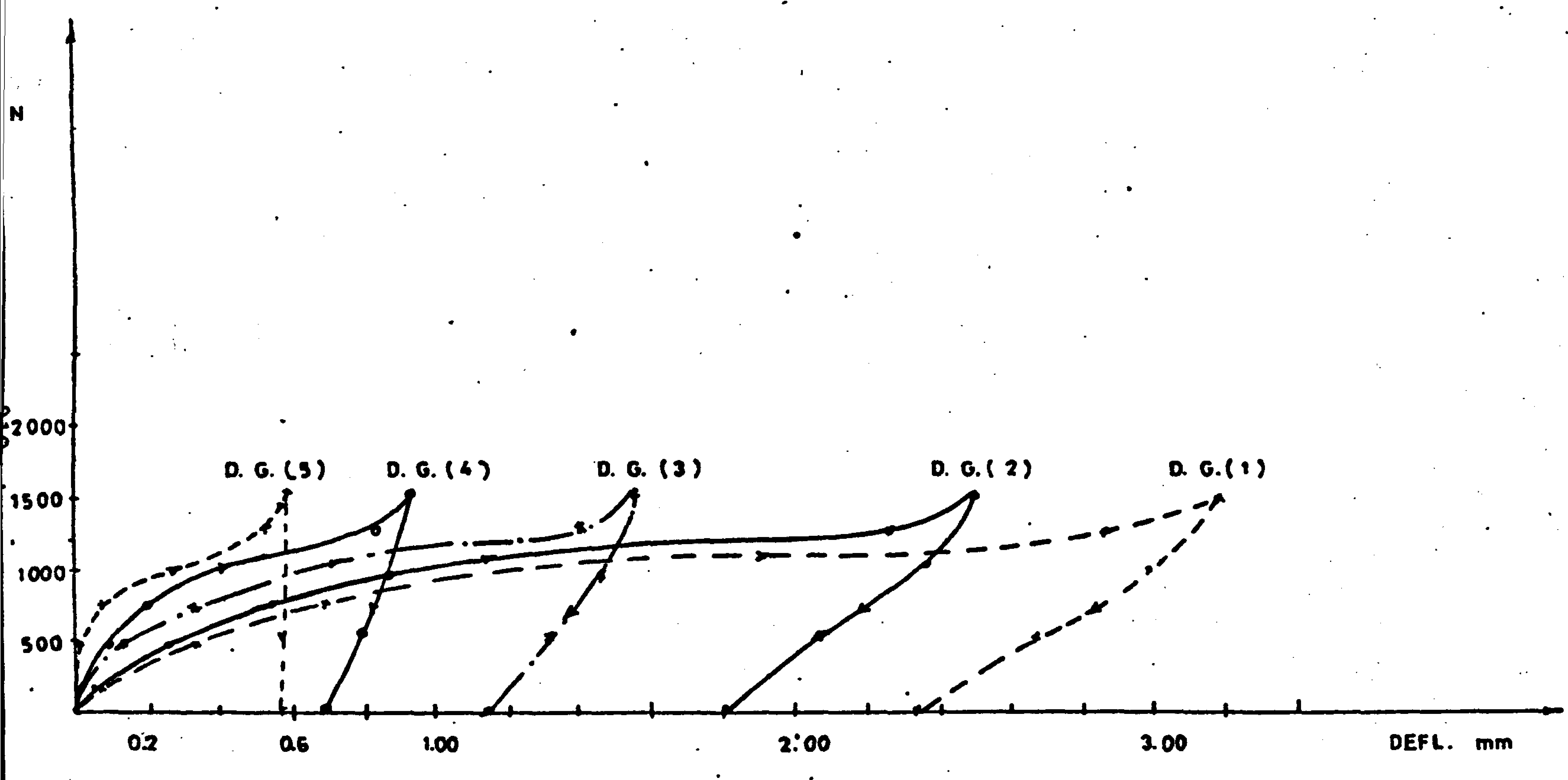
1.6 LOAD DEFLECTION CURVES FOR THE THIRD LOADING (13-STOREY MODEL)

1.7 LOAD STRAIN RELATIONSHIP FOR THE THIRD LOADING (13-STOREY MODEL)

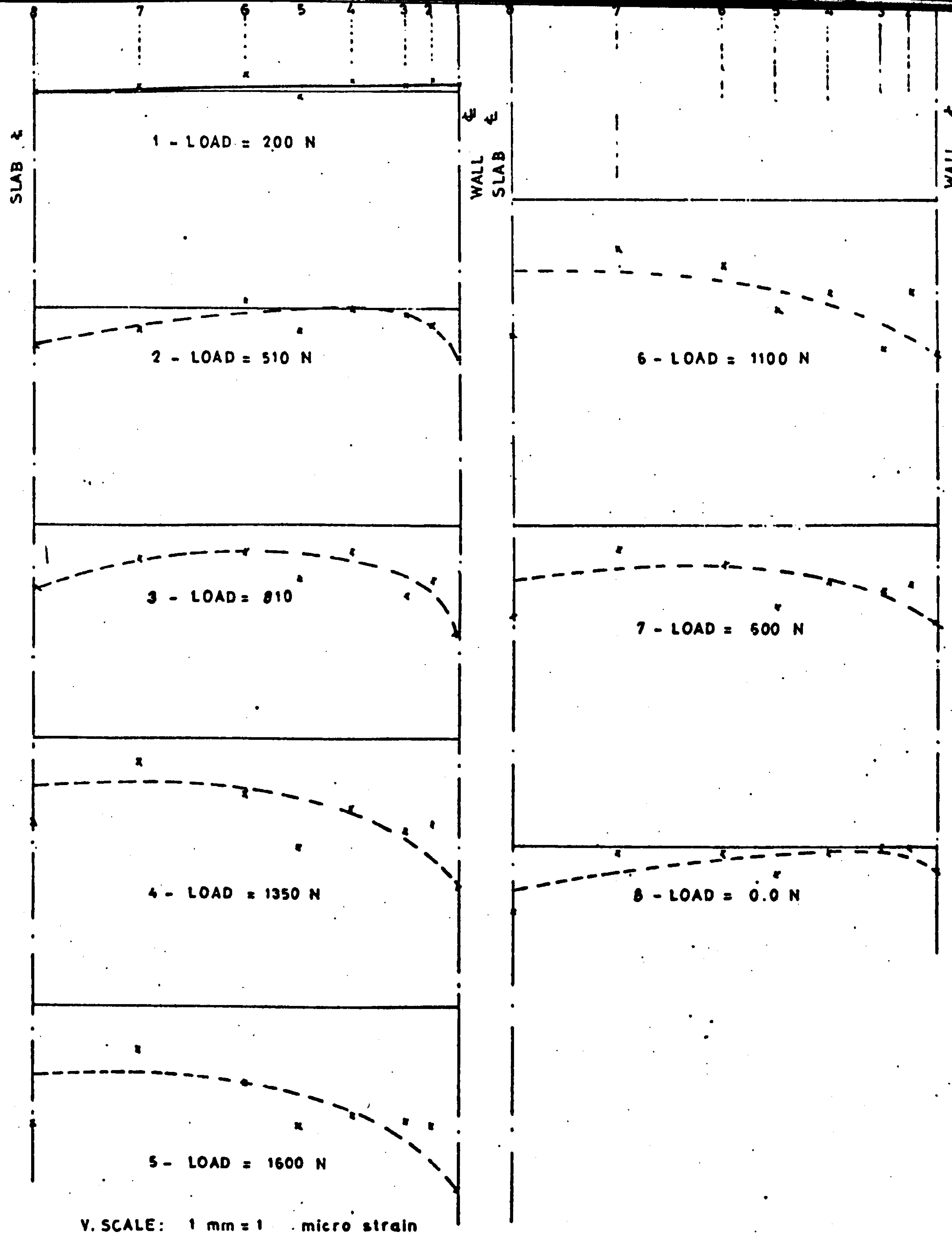




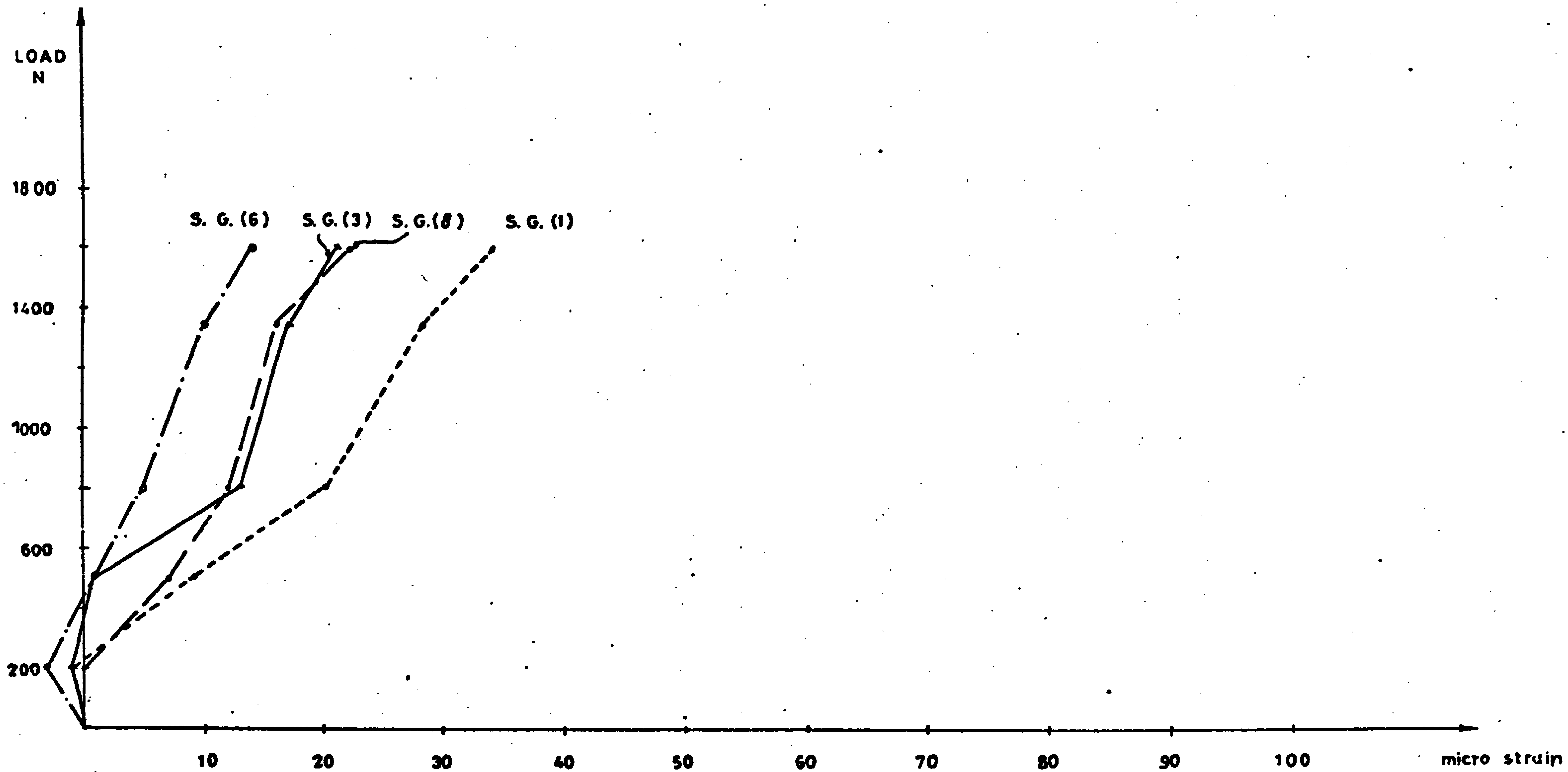
1.8 LOAD DEFLECTION RELATIONSHIP FOR THE FOURTH LOADING (13-STOREY MODEL)



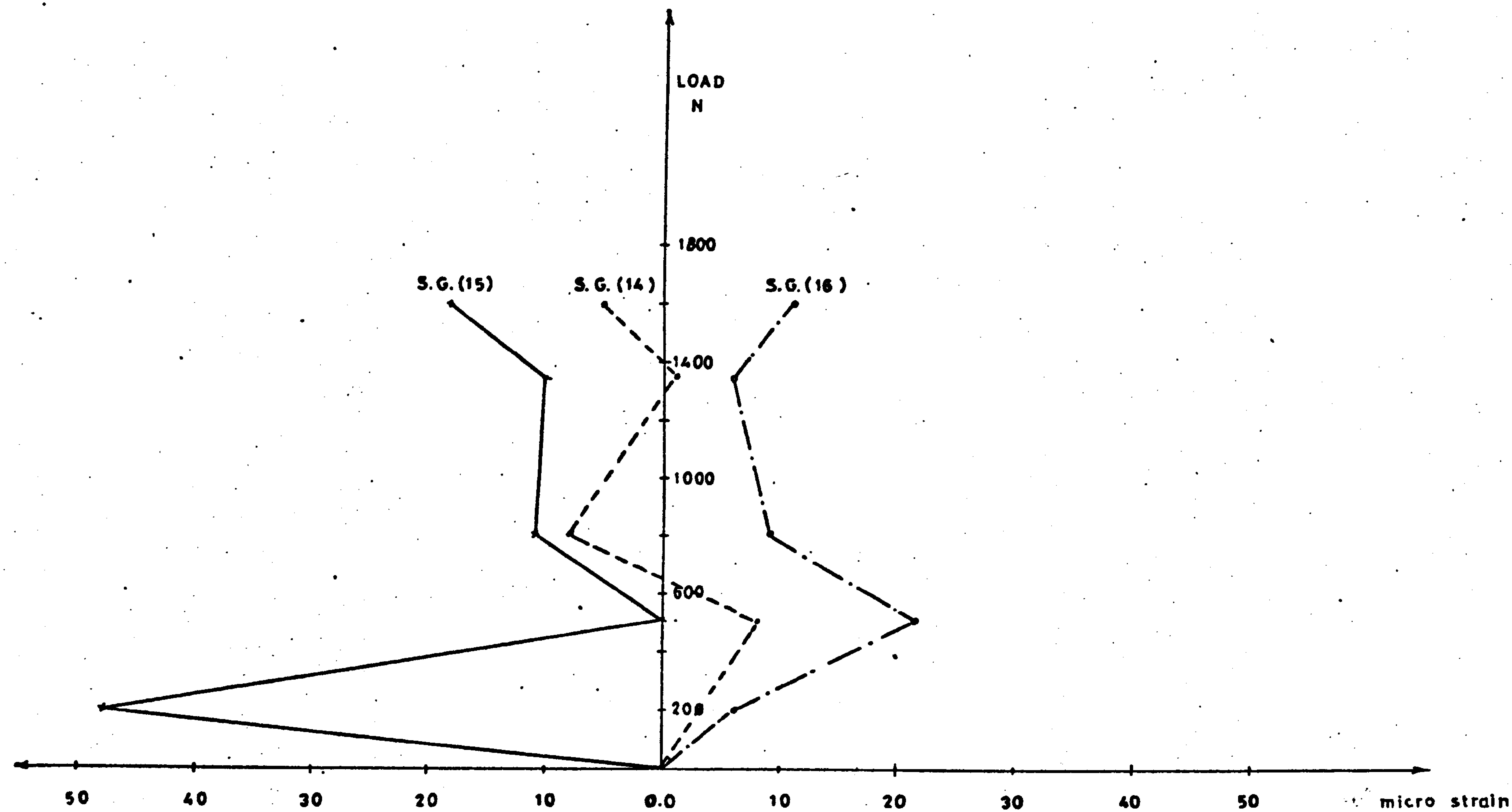
,1.9 LOAD DEFLECTION RELATIONSHIP FOR SECOND LOADING (9-STOREY MODEL)



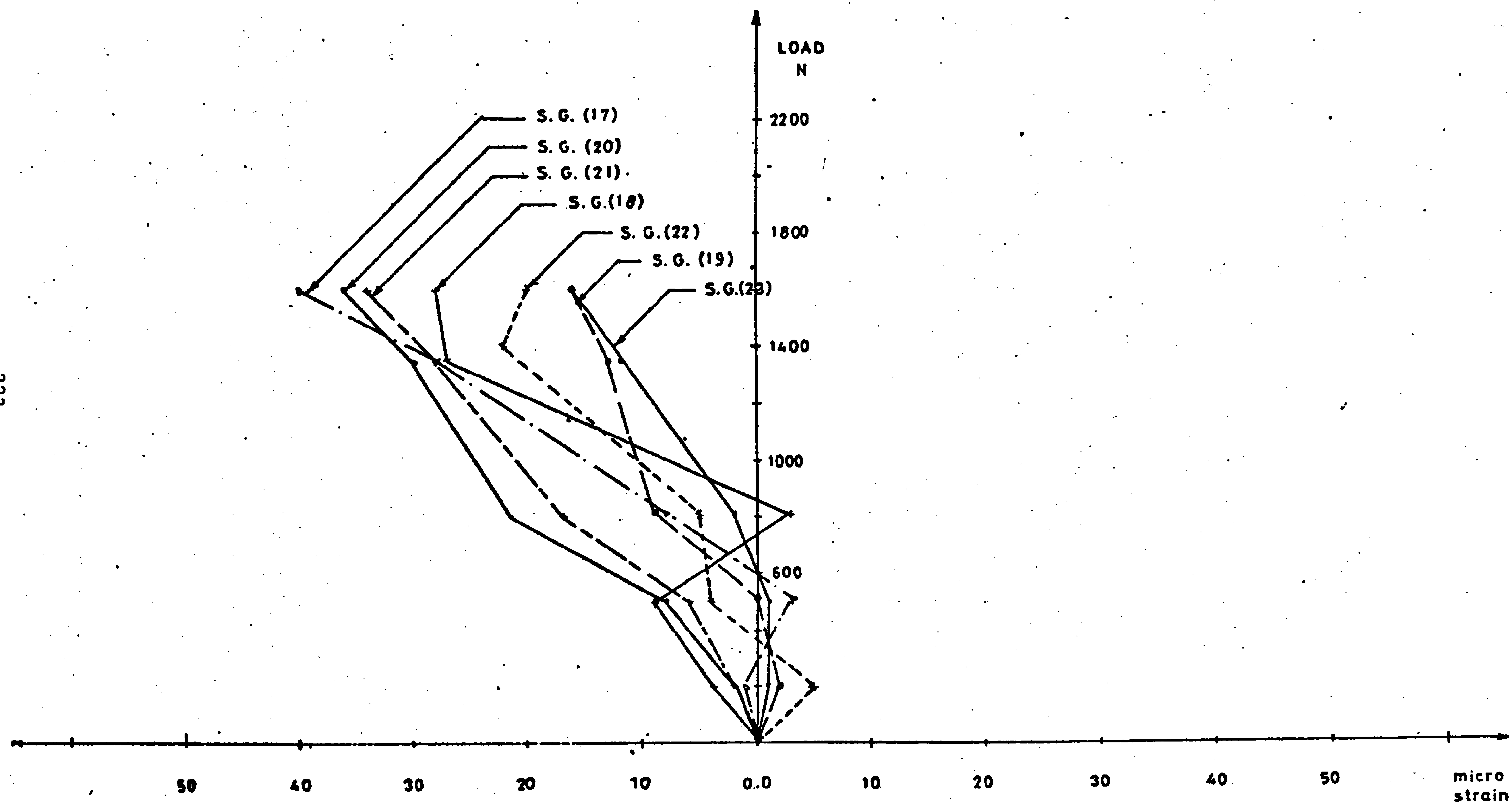
1.10 STRAIN DISTRIBUTION IN THE SLAB DURING SECOND LOADING (9-STOREY MODEL)



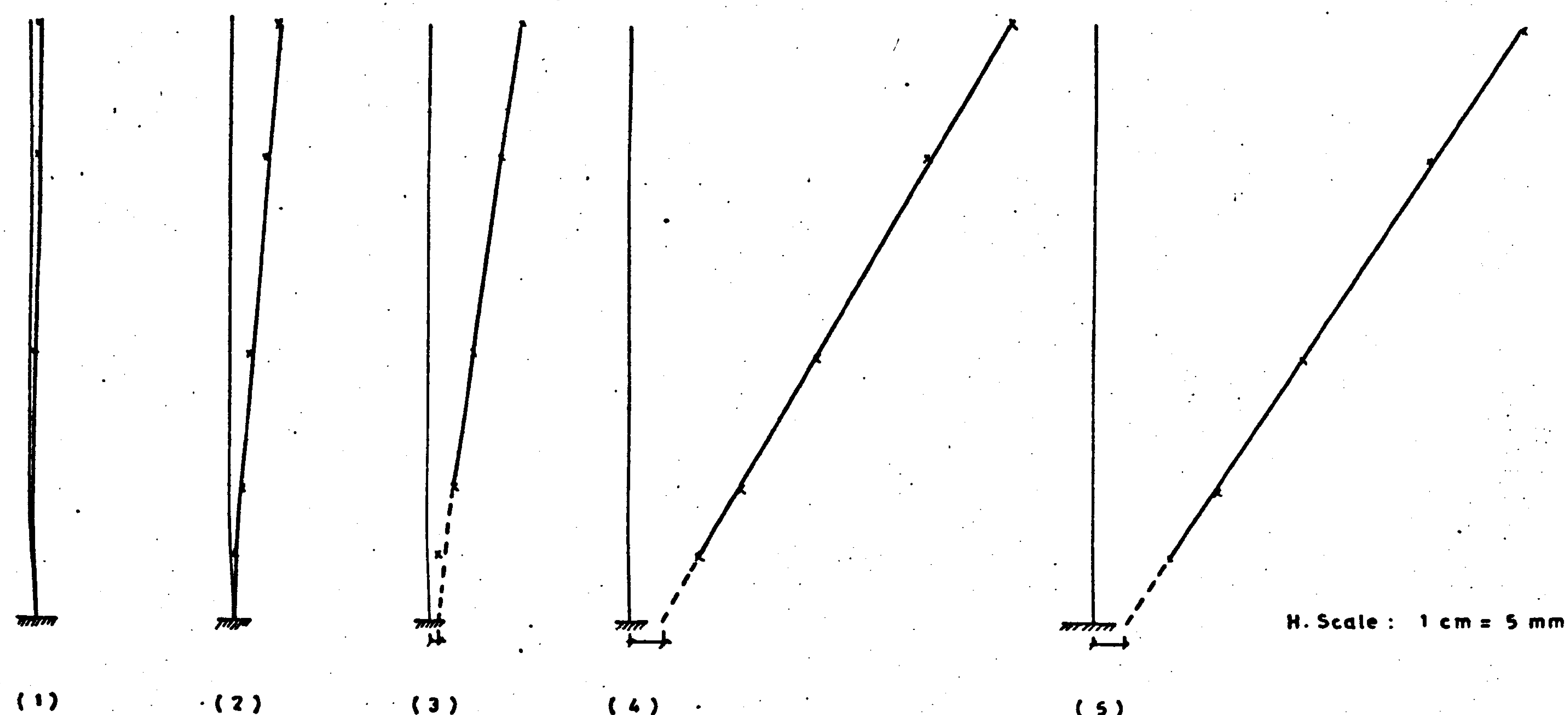
1.11 VARIATION OF STRAINS AT VARIOUS POINTS DURING SECOND LOADING (9-STOREY MODEL).



1.12 VARIATION OF STRAINS AT VARIOUS POINTS DURING SECOND LOADING (9-STOREY MODEL)

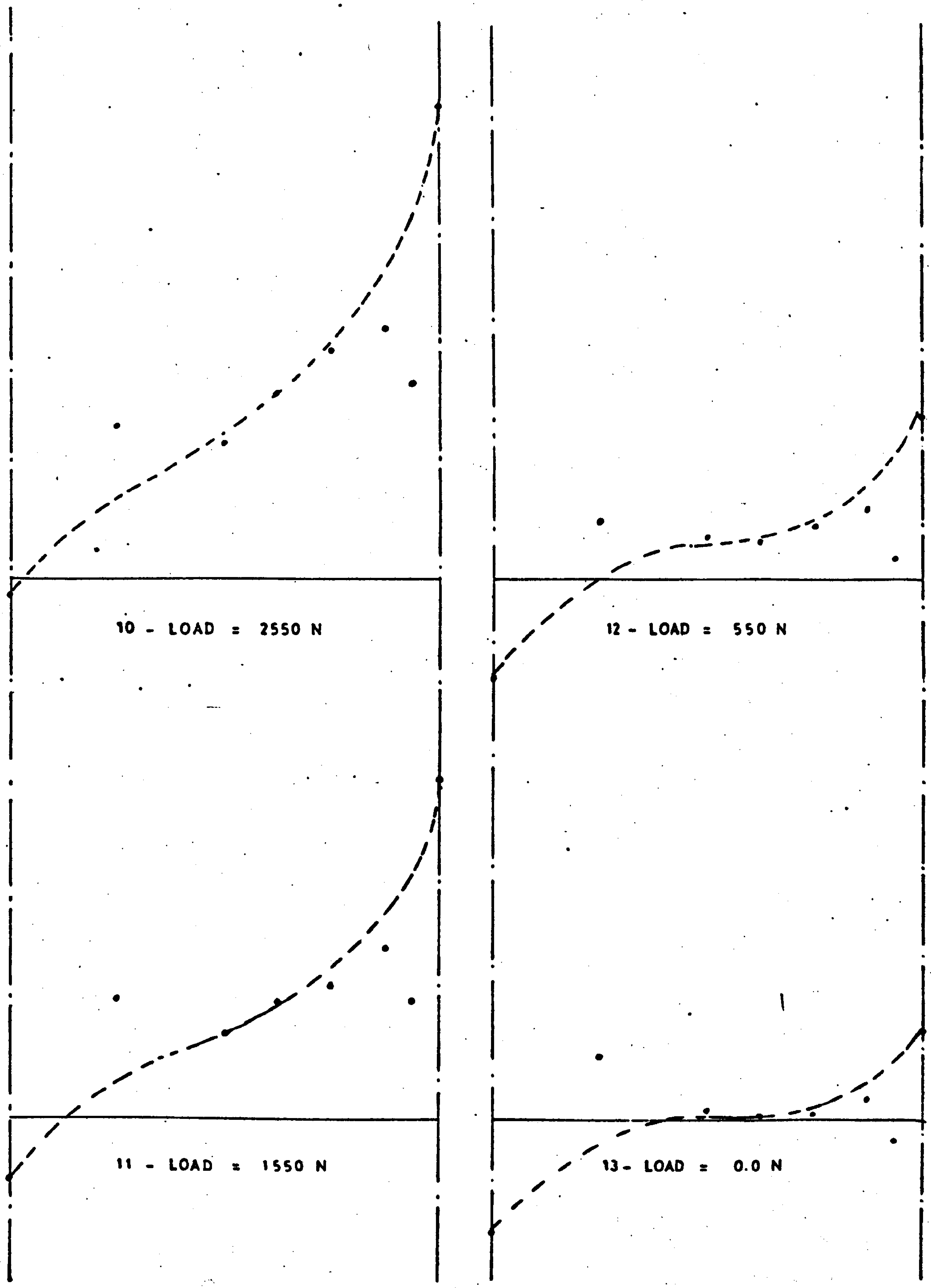


1.13 VARIATION OF STRAINS AT VARIOUS POINTS DURING SECOND LOADING (9-STOREY MODEL)



1.14 DEFLECTION OF MODEL AT VARIOUS STAGES DURING SECOND LOADING (9-STOREY MODEL)

1.15 STRAIN DISTRIBUTION IN THE SLAB FOR THIRD LOADING (9-STOREY MODEL)



1.15b STRAIN DISTRIBUTION IN THE SLAB (CONT.)

1.16 VARIATION OF STRAINS AT VARIOUS POINTS DURING THIRD LOADING (9-STOREY MODEL)

1.17 VARIATION OF STRAINS AT VARIOUS POINTS DURING THIRD LOADING (9-STOREY MODEL)

1.18 VARIATION OF STRAINS AT VARIOUS POINTS DURING THIRD LOADING (9-STOREY MODEL)

1.19 LOAD DEFLECTION RELATIONSHIP FOR THIRD LOADING (9-STOREY MODEL)

1.20 STRAIN DISTRIBUTION DURING THE FIFTH LOADING (9-STOREY MODEL)

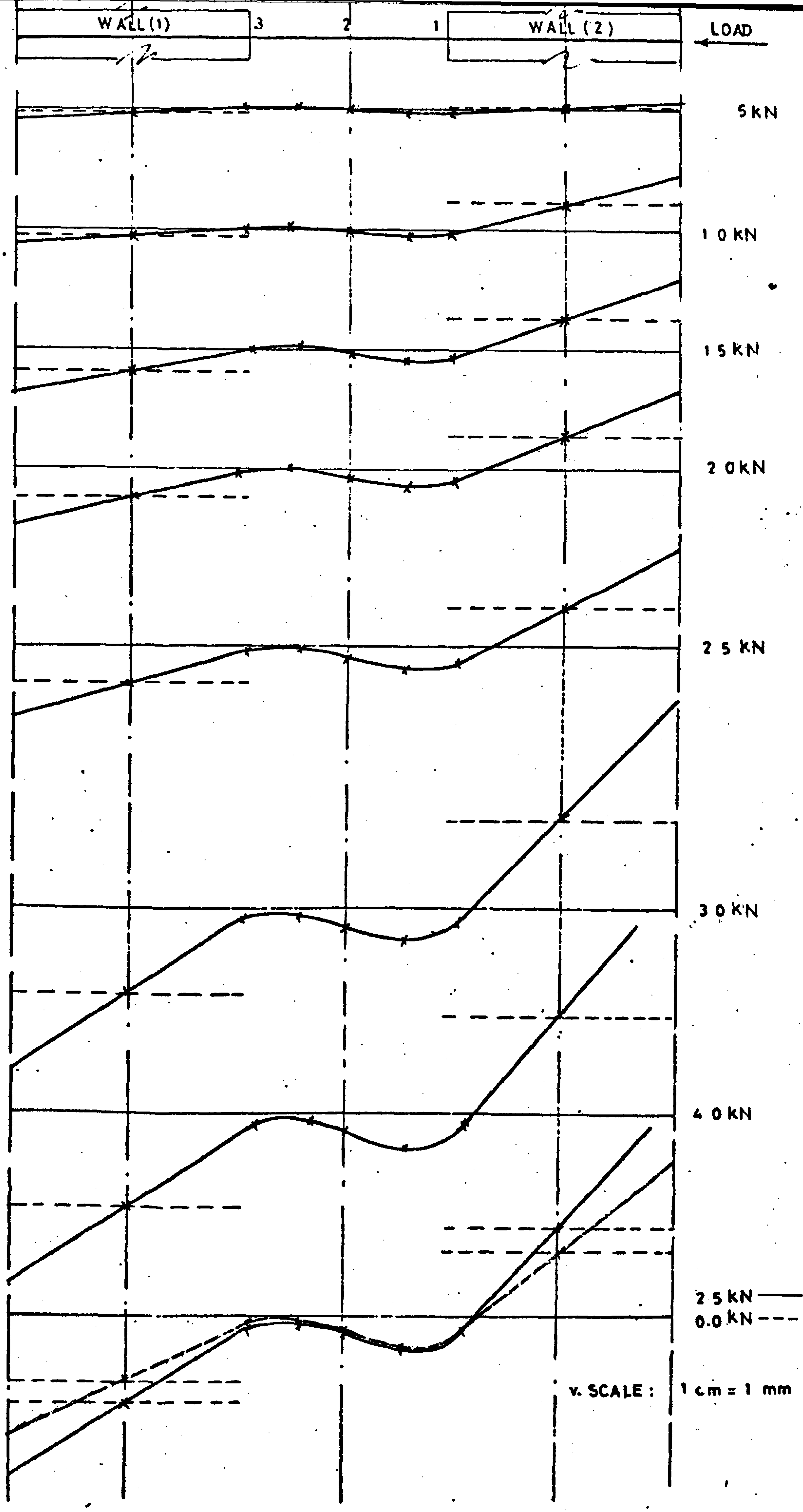
1.20b STRAIN DISTRIBUTION (CONT.)

1.21 LOAD DEFLECTION RELATIONSHIP FOR THE FIFTH LOADING (9-STOREY MODEL)

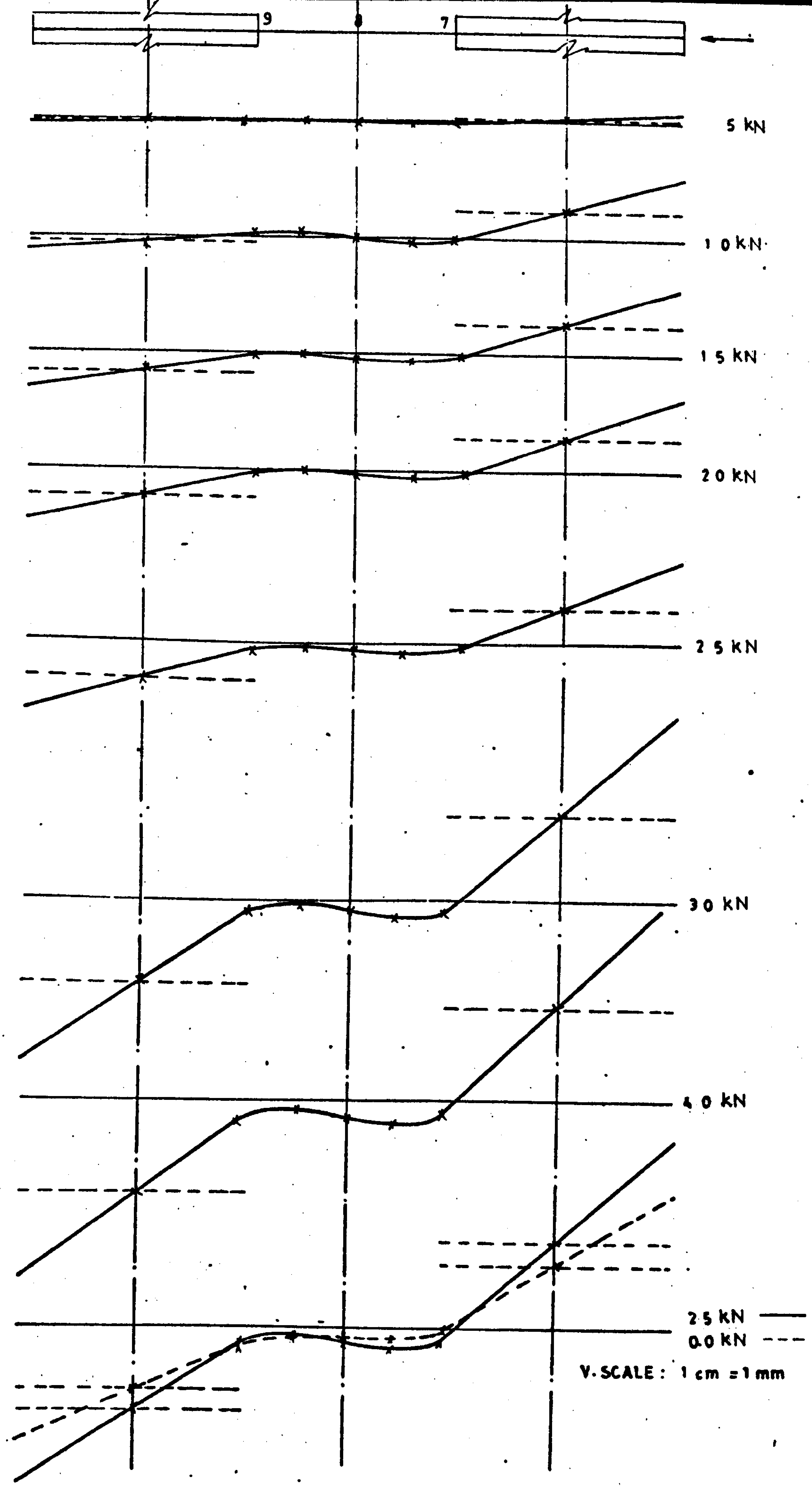
1.22 STRAIN DISTRIBUTION DURING SIXTH LOADING (9-STOREY MODEL)

V. SCALE 1 cm = 10 micro st.

1.23 STRAIN DISTRIBUTION DURING SEVENTH LOADING (9-STOREY MODEL)

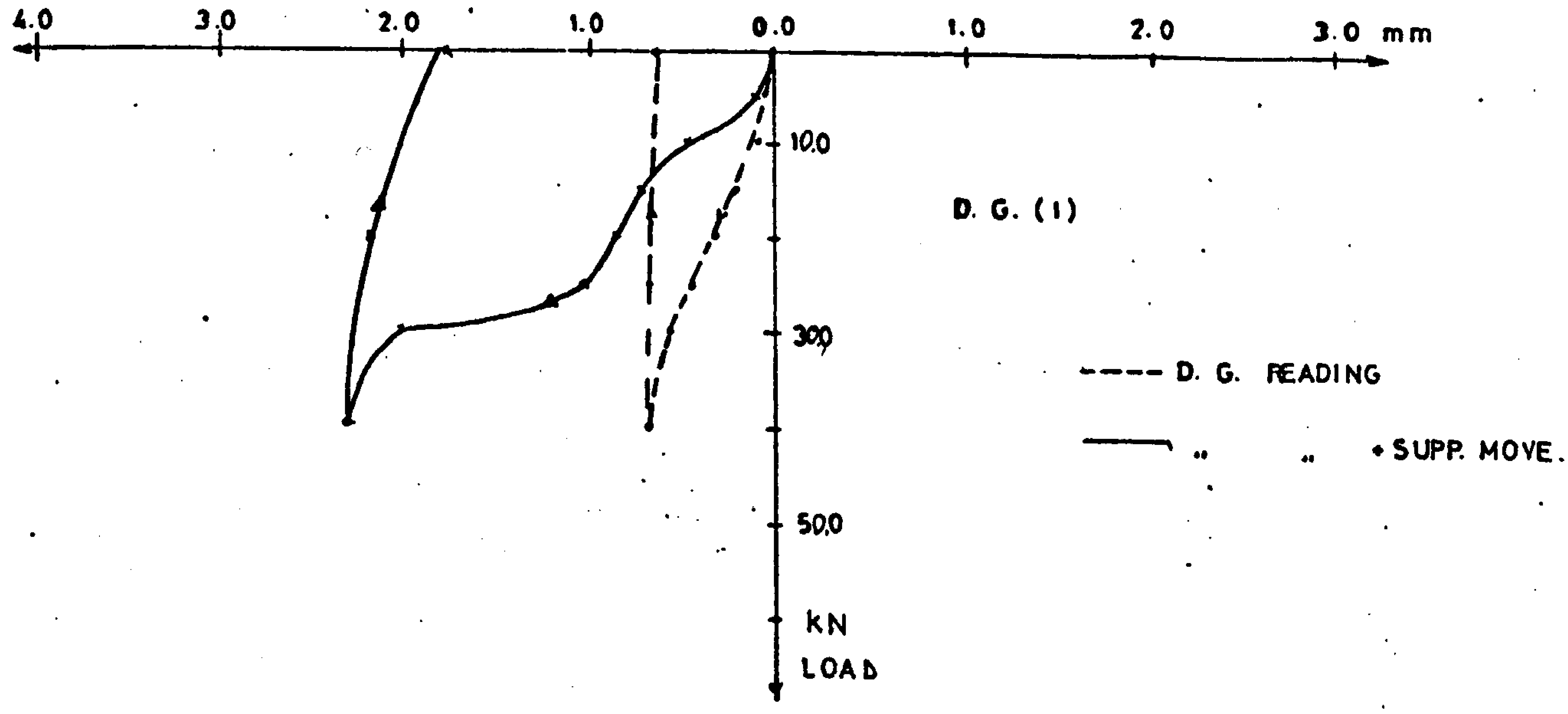


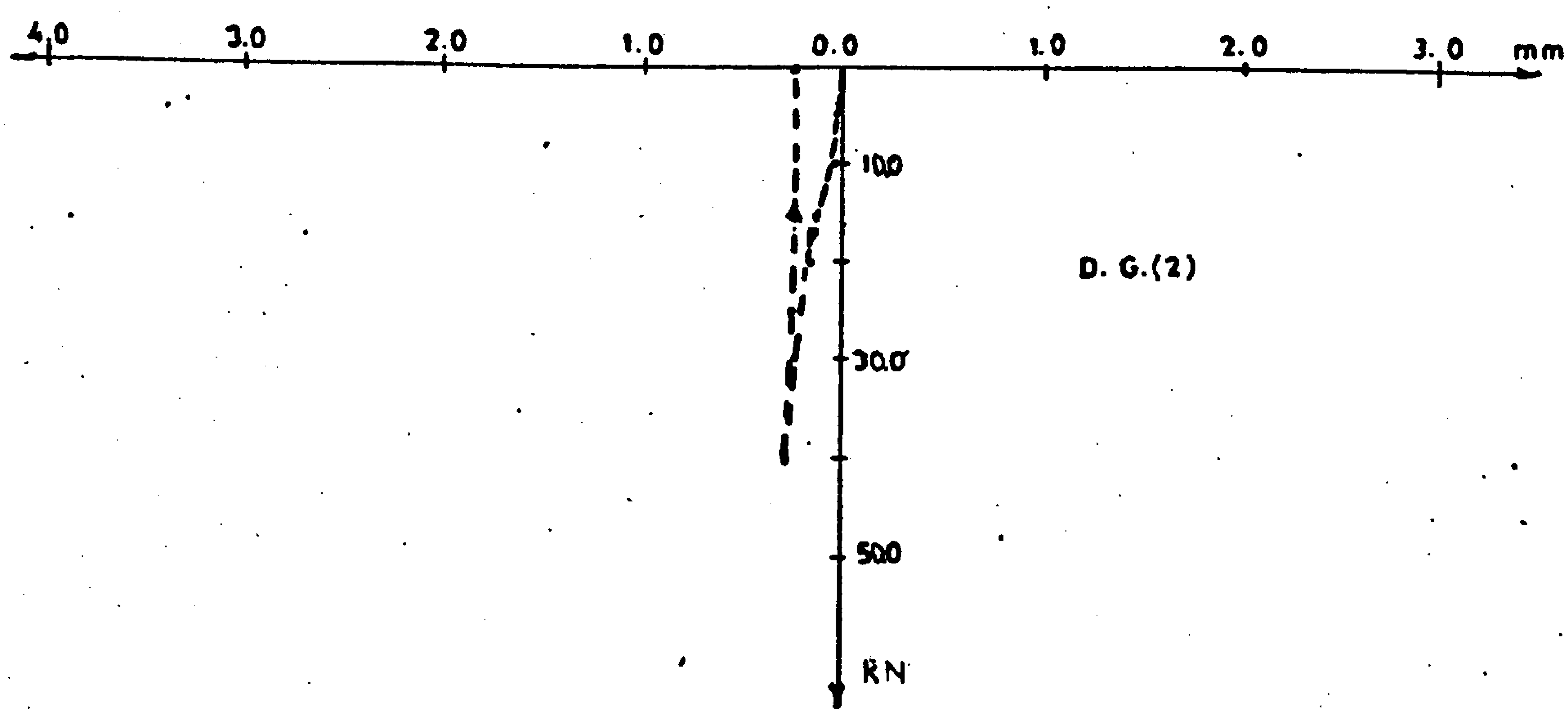
1.24 SLAB 1 DEFORMATION DURING SECOND LOADING (2-STOREY MODEL)

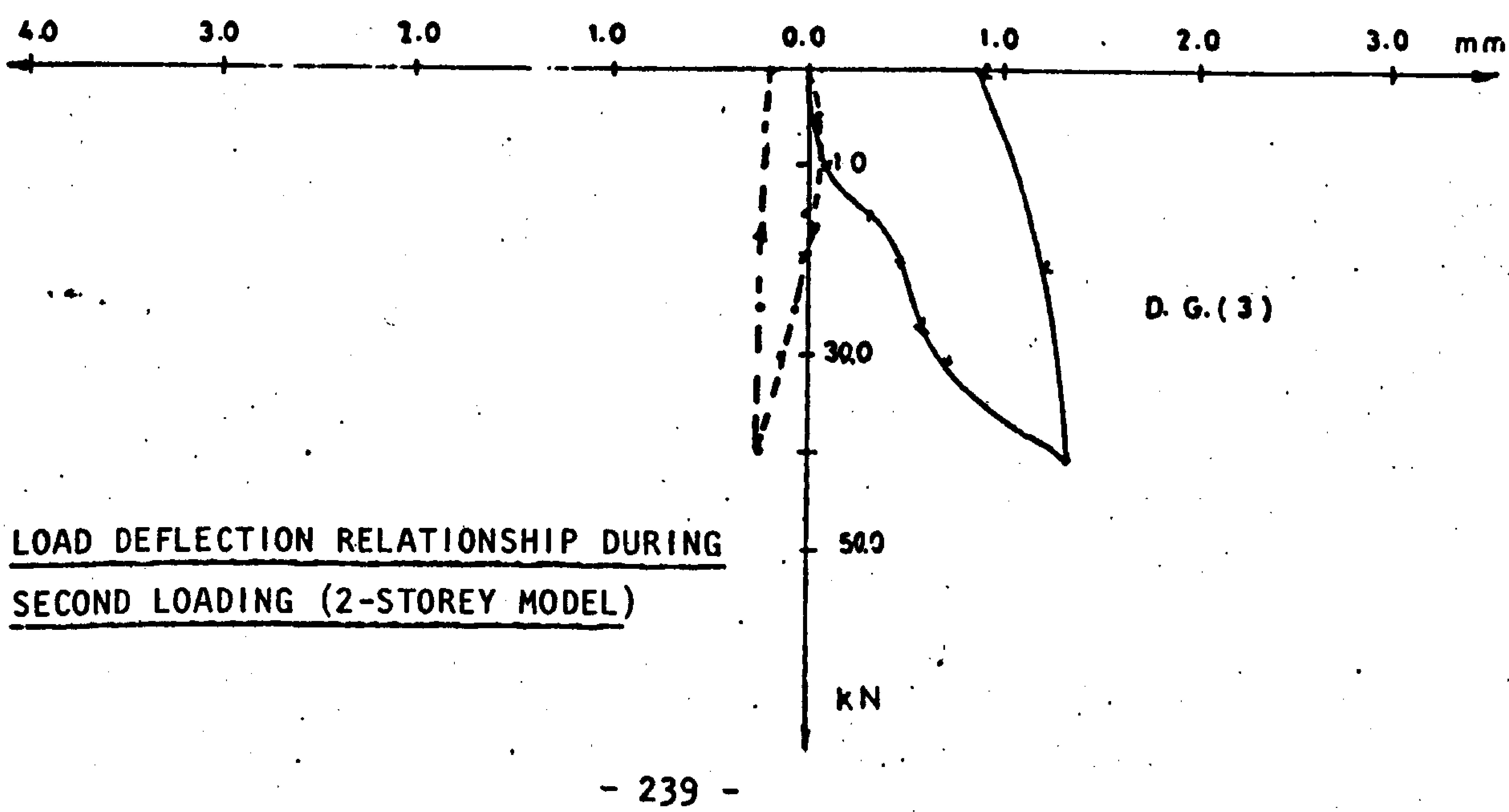


I.25 SLAB 1 DEFORMATION DURING SECOND LOADING (2-STOREY MODEL)

1.26 SLAB 1 DEFORMATION DURING SECOND LOADING (2-STOREY MODEL)



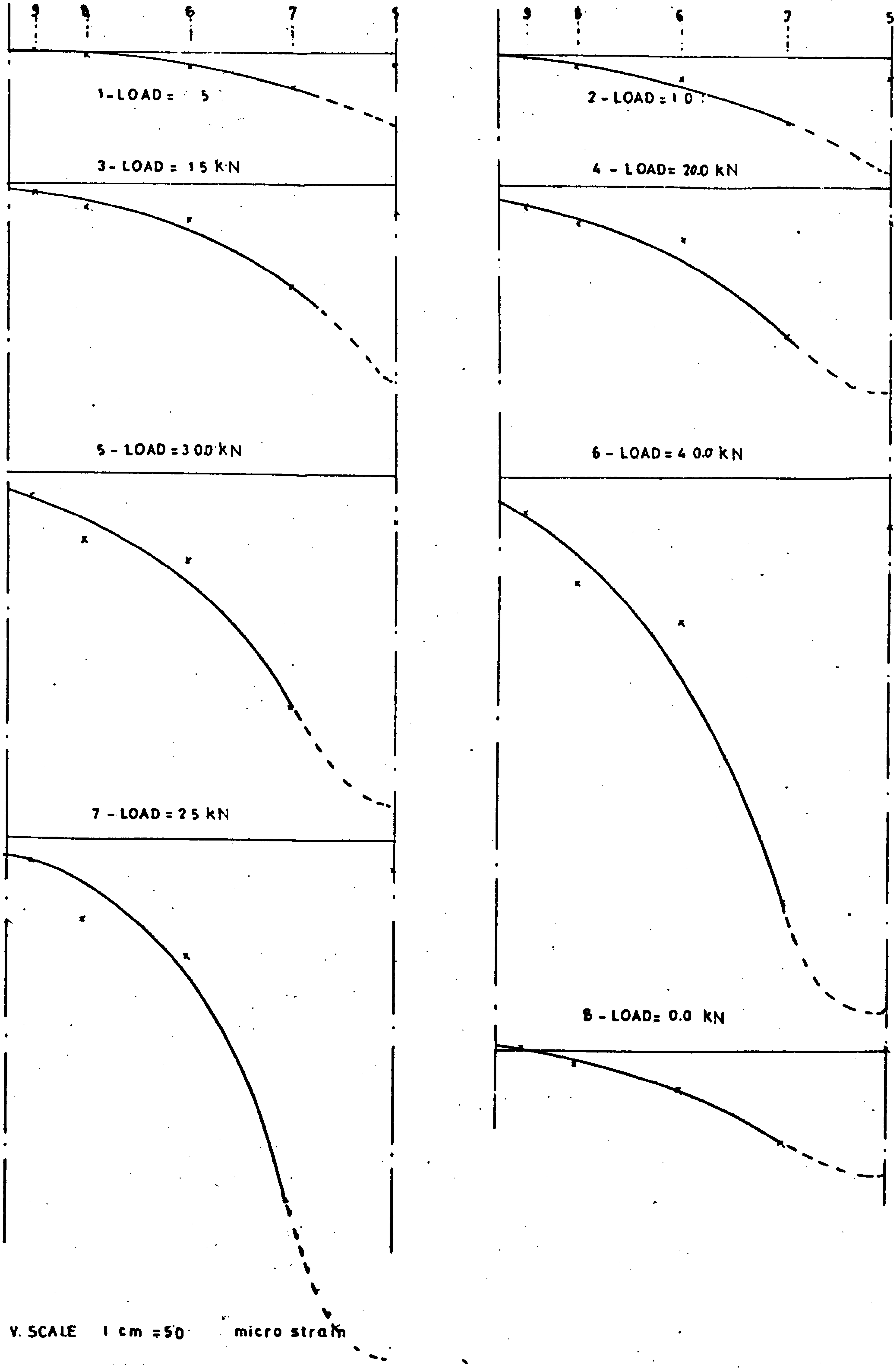




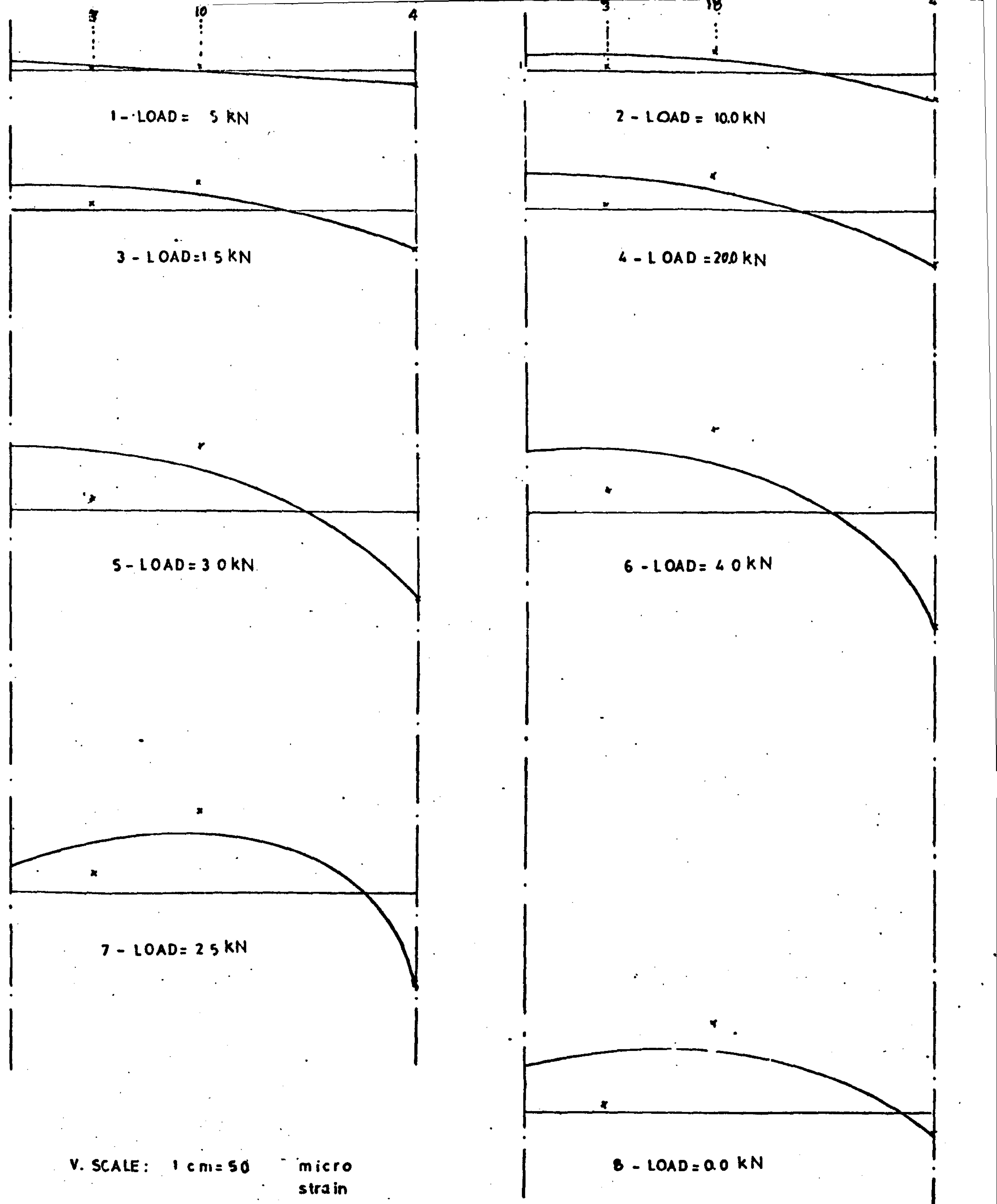
1.27 LOAD DEFLECTION RELATIONSHIP DURING SECOND LOADING (2-STOREY MODEL)

1.28 RELATIVE DISPLACEMENT BETWEEN THE WALLS

1.29 LOAD-STRAIN RELATIONSHIP DURING SECOND LOADING (2-STOREY MODEL)

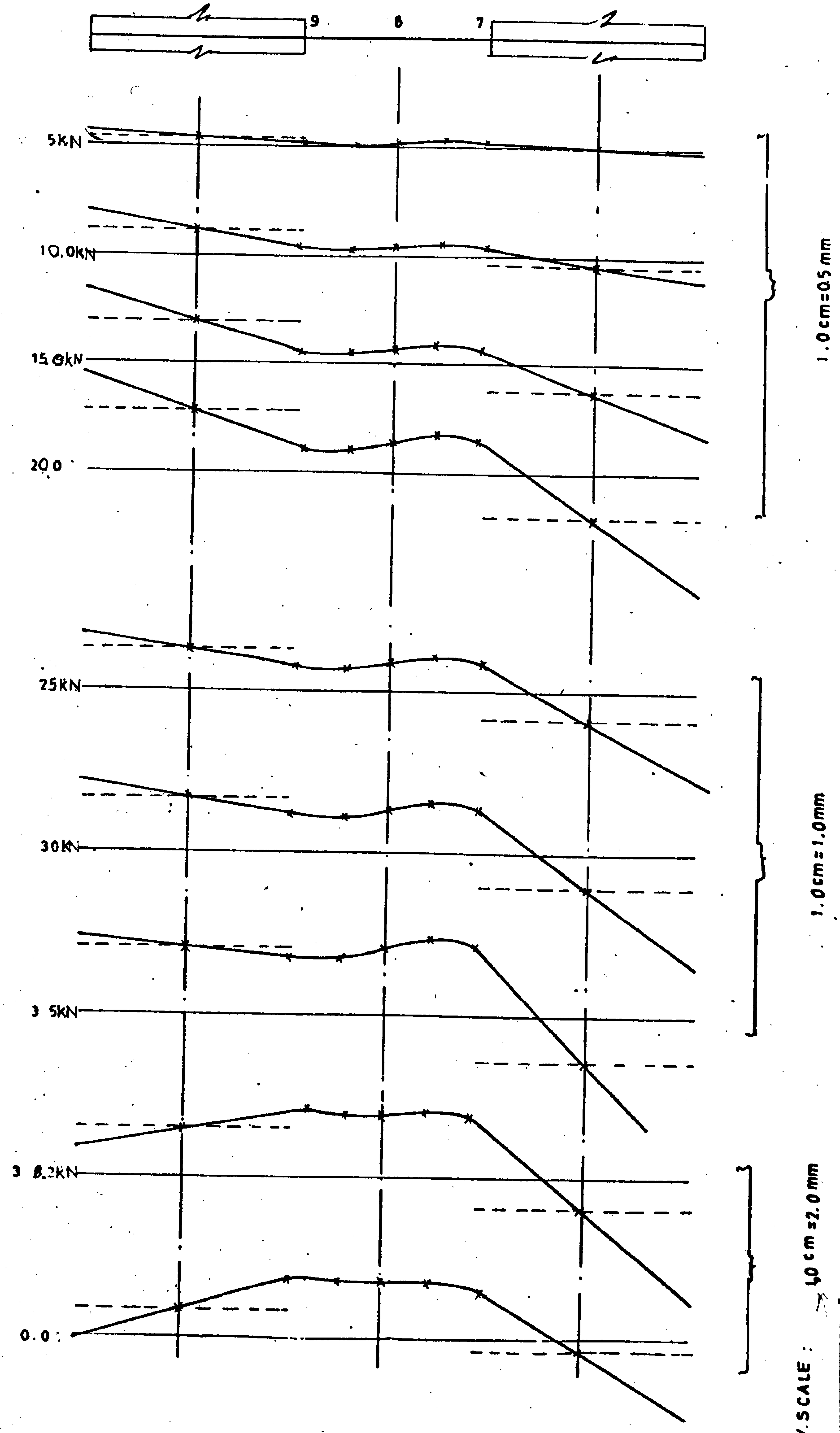


1.30 ACCUMULATED STRAIN DISTRIBUTION IN SLAB 1 (2-STOREY MODEL)



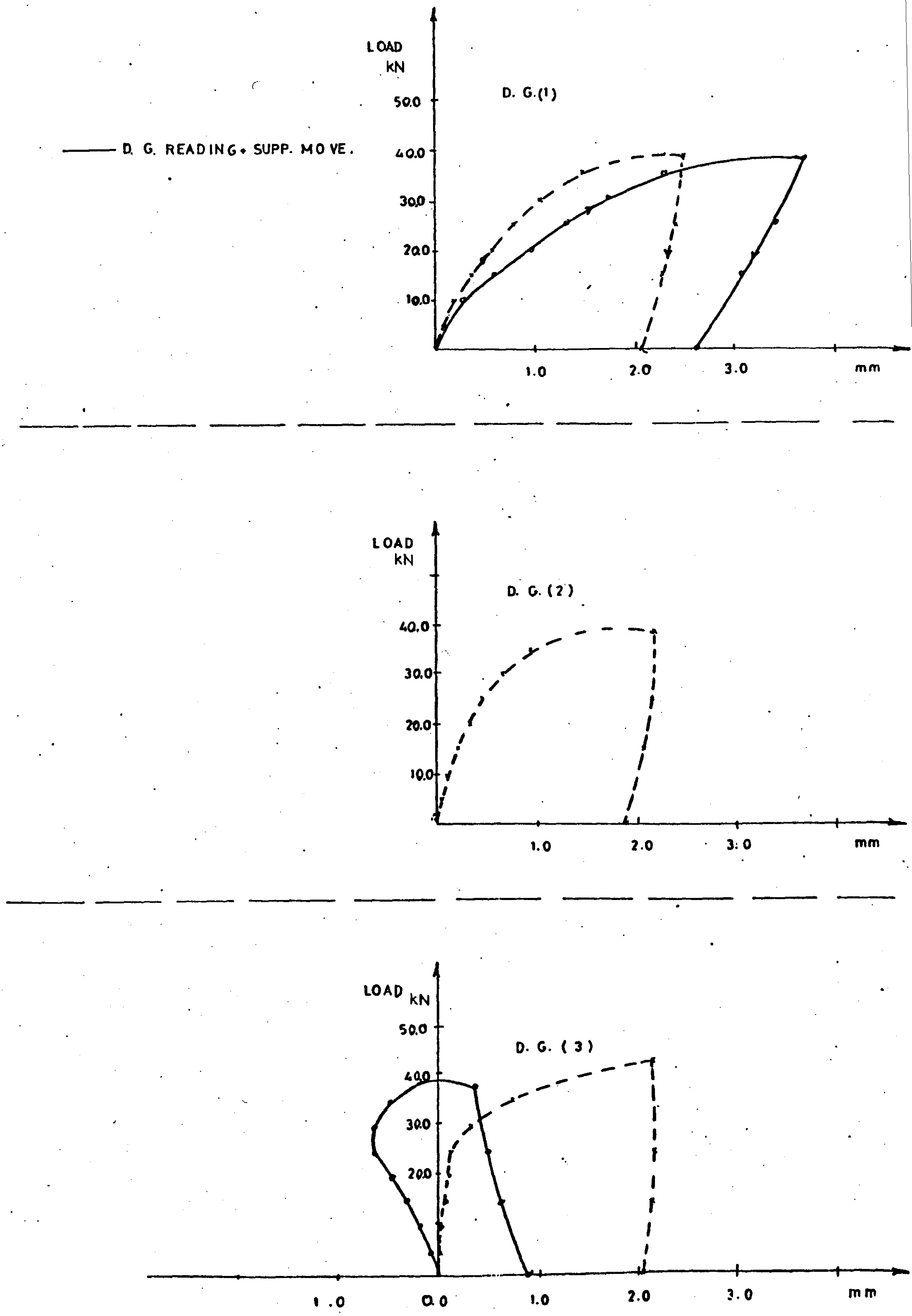
1.31 ACCUMULATED STRAIN DISTRIBUTION IN SLAB 1 (2-STOREY MODEL)

1.32 SLAB 1 DEFORMATION DURING THE THIRD LOADING (2-STOREY MODEL)

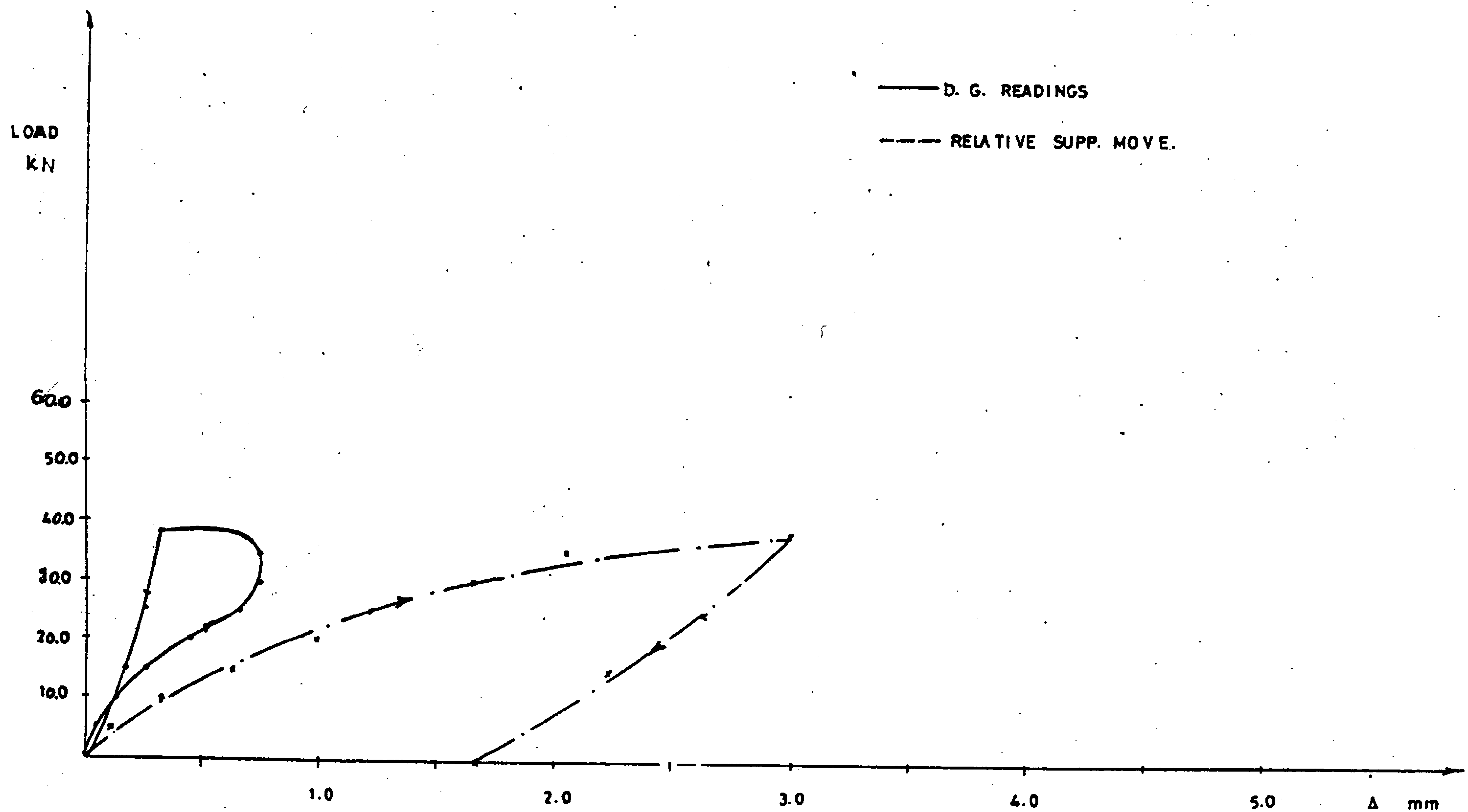


1.33 SLAB 1 DEFORMATION DURING THE THIRD LOADING (2-STOREY MODEL)

1.34 SLAB 2 DEFORMATION DURING THE THIRD LOADING (2-STOREY MODEL)



1.35 LOAD DEFLECTION RELATIONSHIP DURING THE THIRD LOADING (2-STOREY MODEL)



1.36 RELATIVE WALLS DISPLACEMENT DURING THIRD LOADING (2-STOREY MODEL)

V. SCALE: 1 cm=50 micro strain

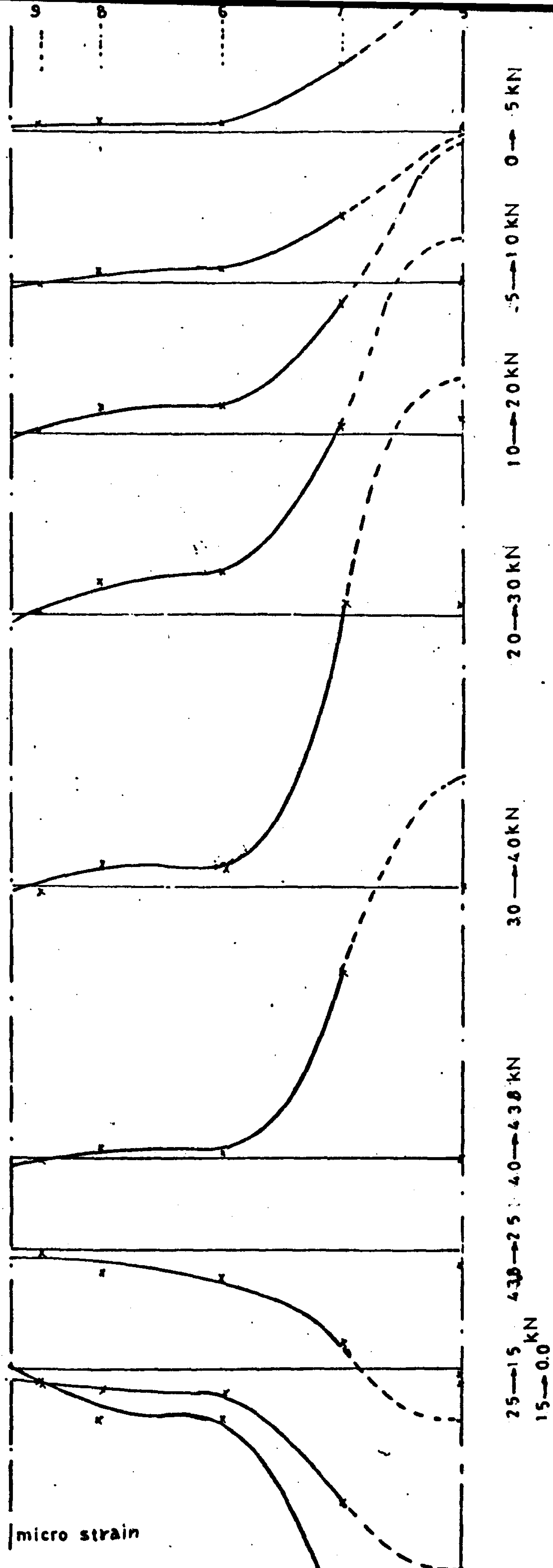
1.37 CHANGE OF STRAINS DURING EACH LOAD INCREMENT DURING THIRD LOADING (2-STOREY MODEL)

V. SCALE : 1 cm=50 micro strain

1.38 CHANGE OF STRAINS DURING EACH LOAD INCREMENT DURING THIRD LOADING (2-STOREY MODEL)

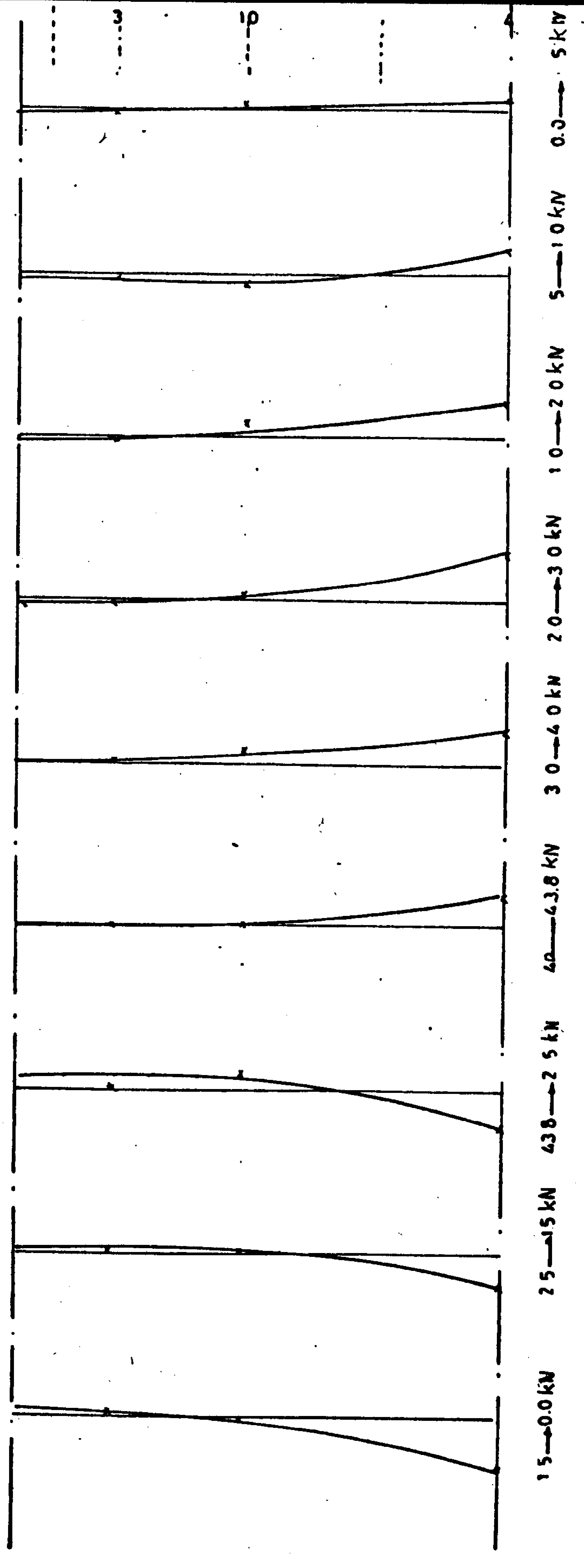
1.39 LOAD-STRAIN RELATIONSHIP FOR THE THIRD LOADING (2-STOREY MODEL)

1.40 SLAB 1 DEFORMATIONS DURING THE FIFTH LOADING (2-STOREY MODEL)



V. SCALE: 1 cm = 50 micro strain

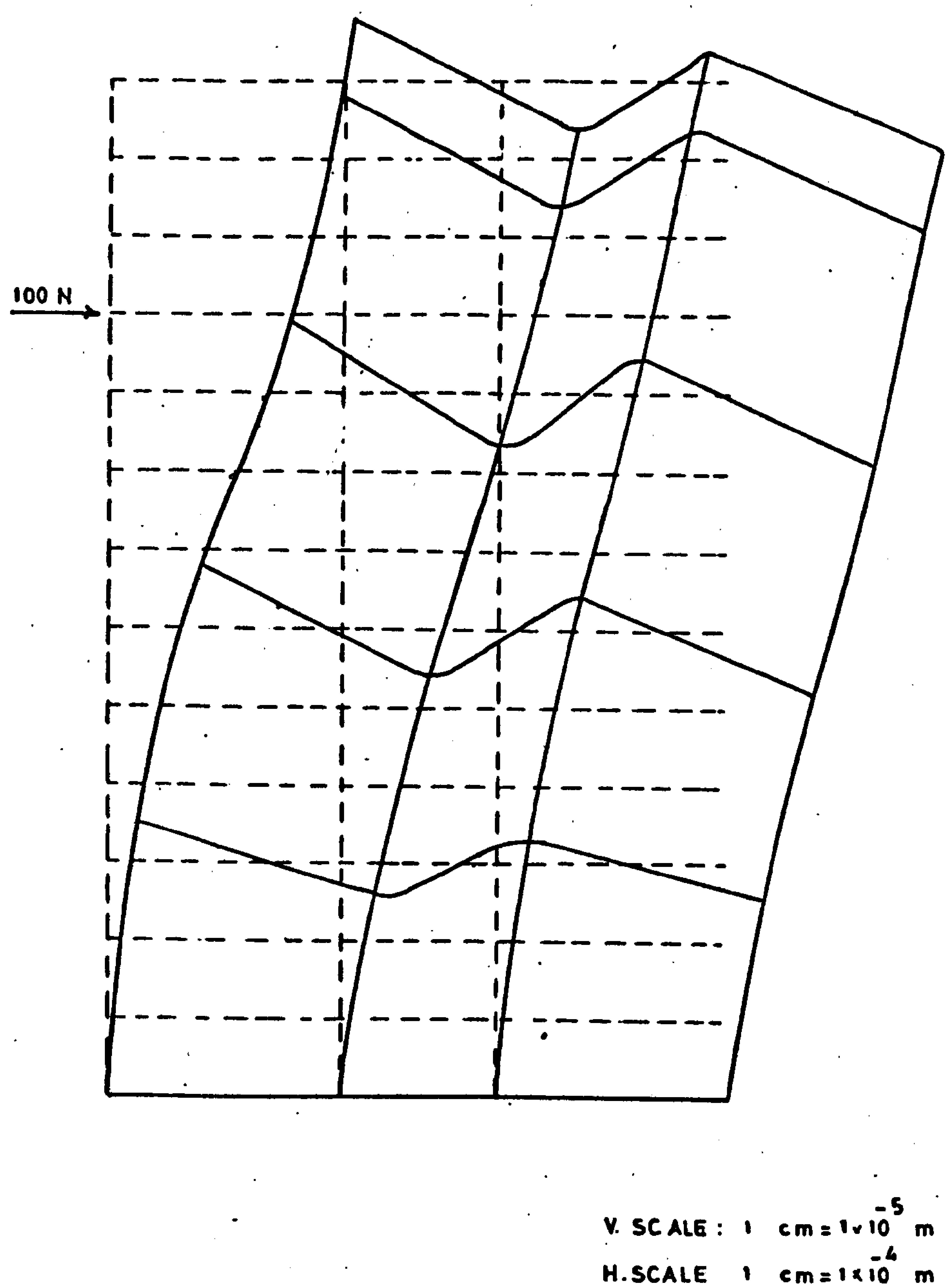
1.41 CHANGES IN THE STRAINS DURING THE FIFTH LOADING (2-STOREY MODEL)



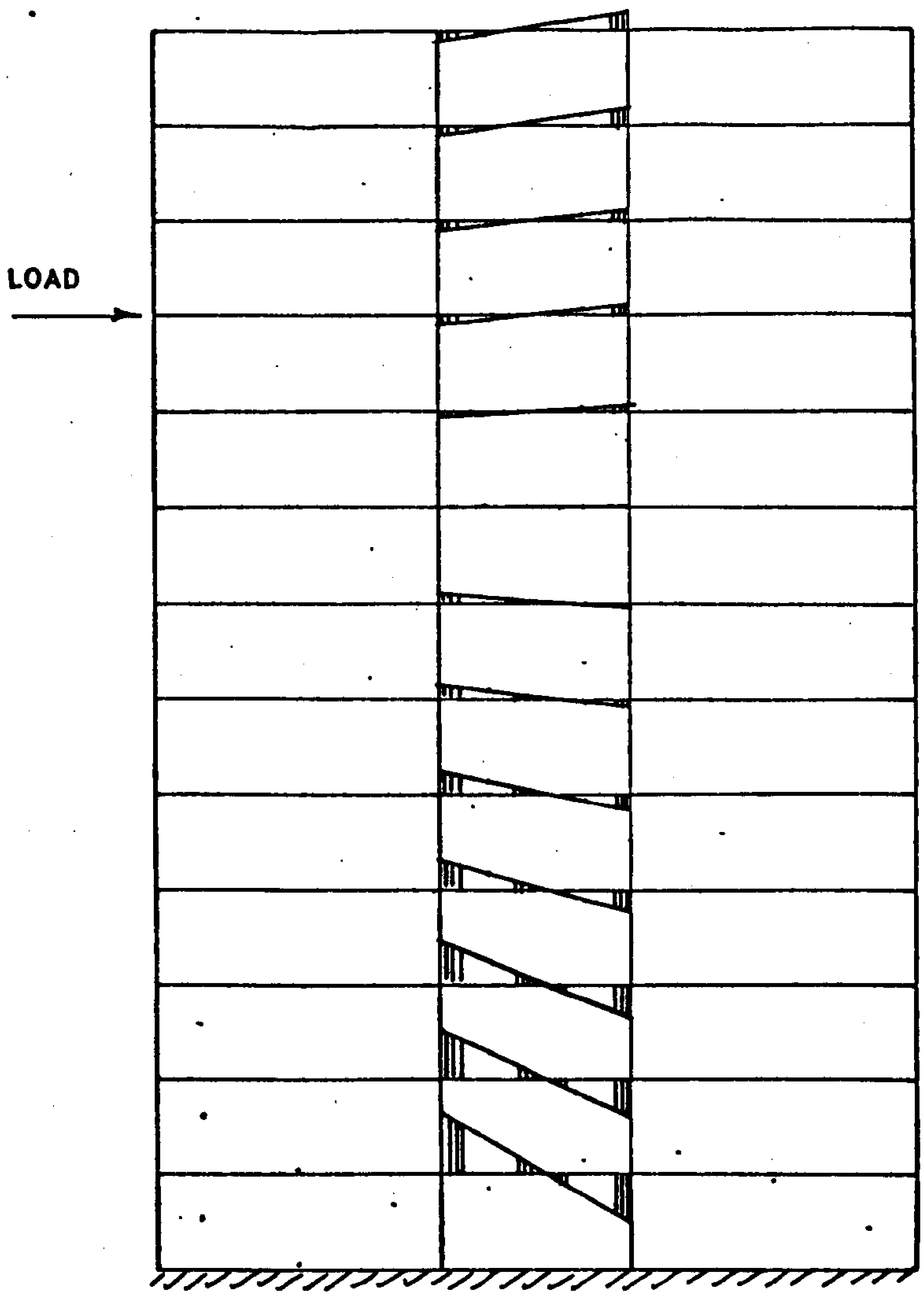
V. SCALE: 1 cm = 50 micro strain.

1.42 CHANGES IN STRAINS DURING FIFTH LOADING (2-STOREY MODEL)

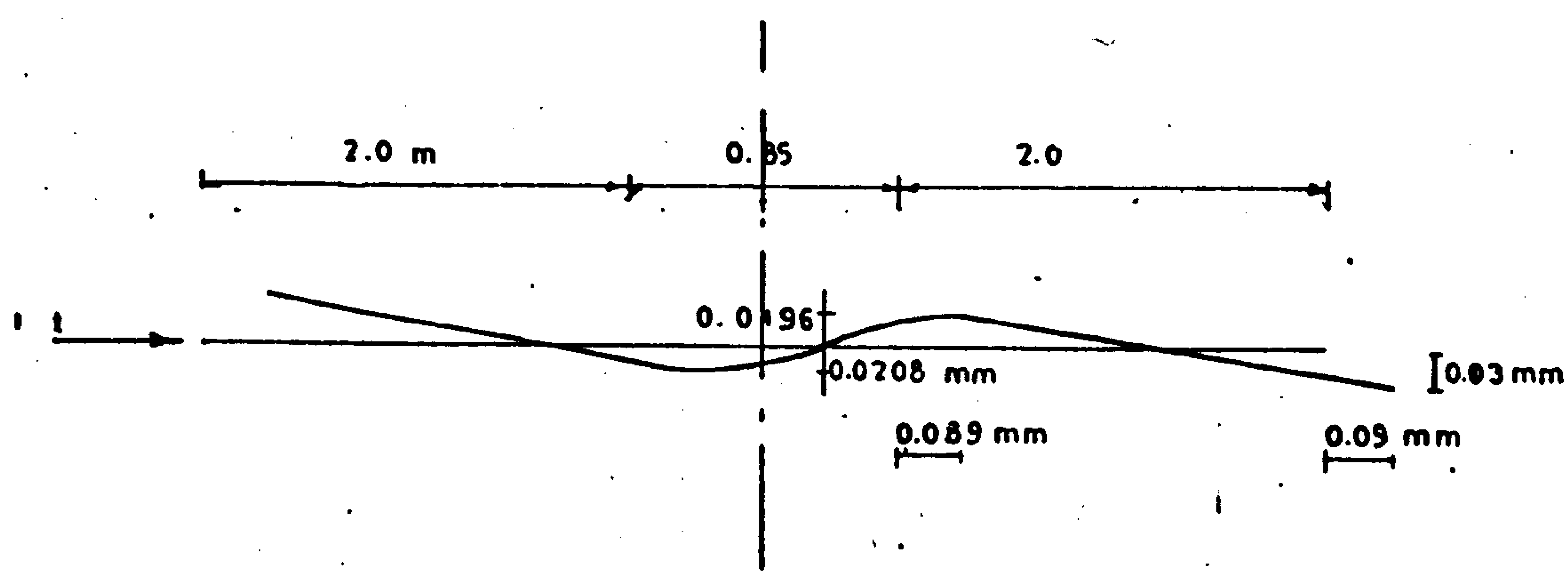
THEORETICAL RESULTS



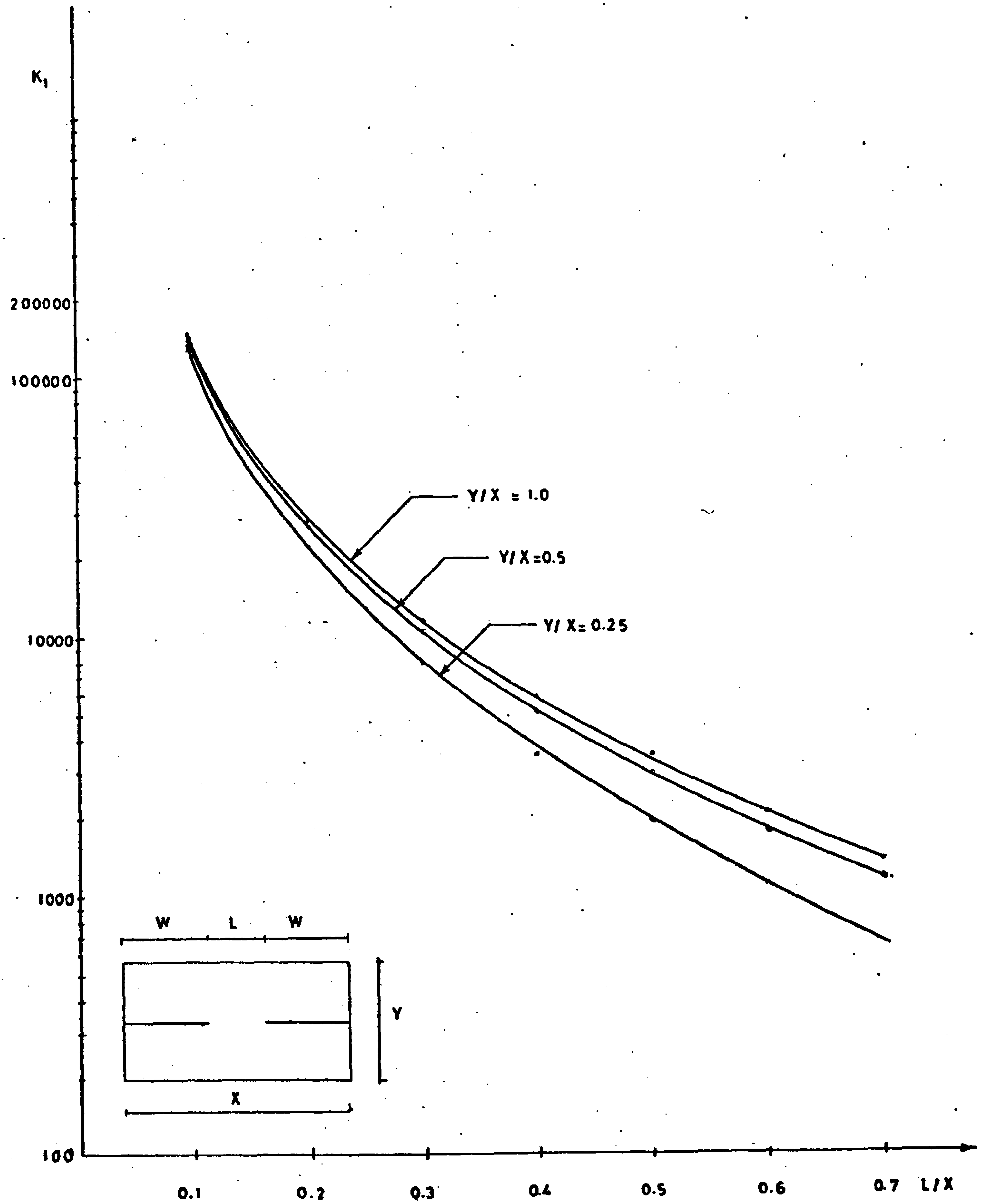
1.43 DEFLECTION OF THE 13-STOREY MODEL USING THE FINITE ELEMENT METHOD



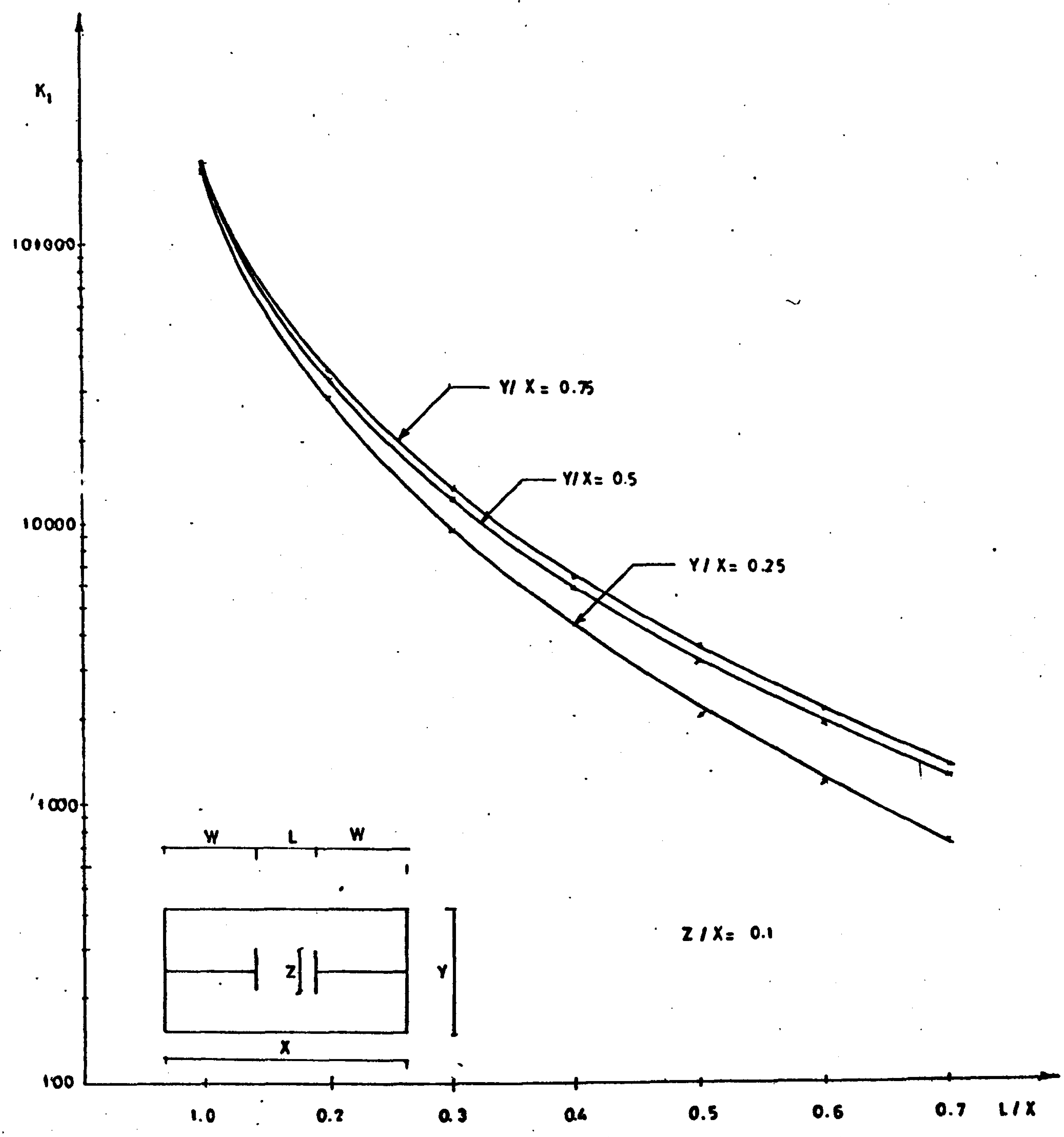
1.44 STRESS DISTRIBUTION IN THE SLAB CONNECTING THE SHEAR WALLS



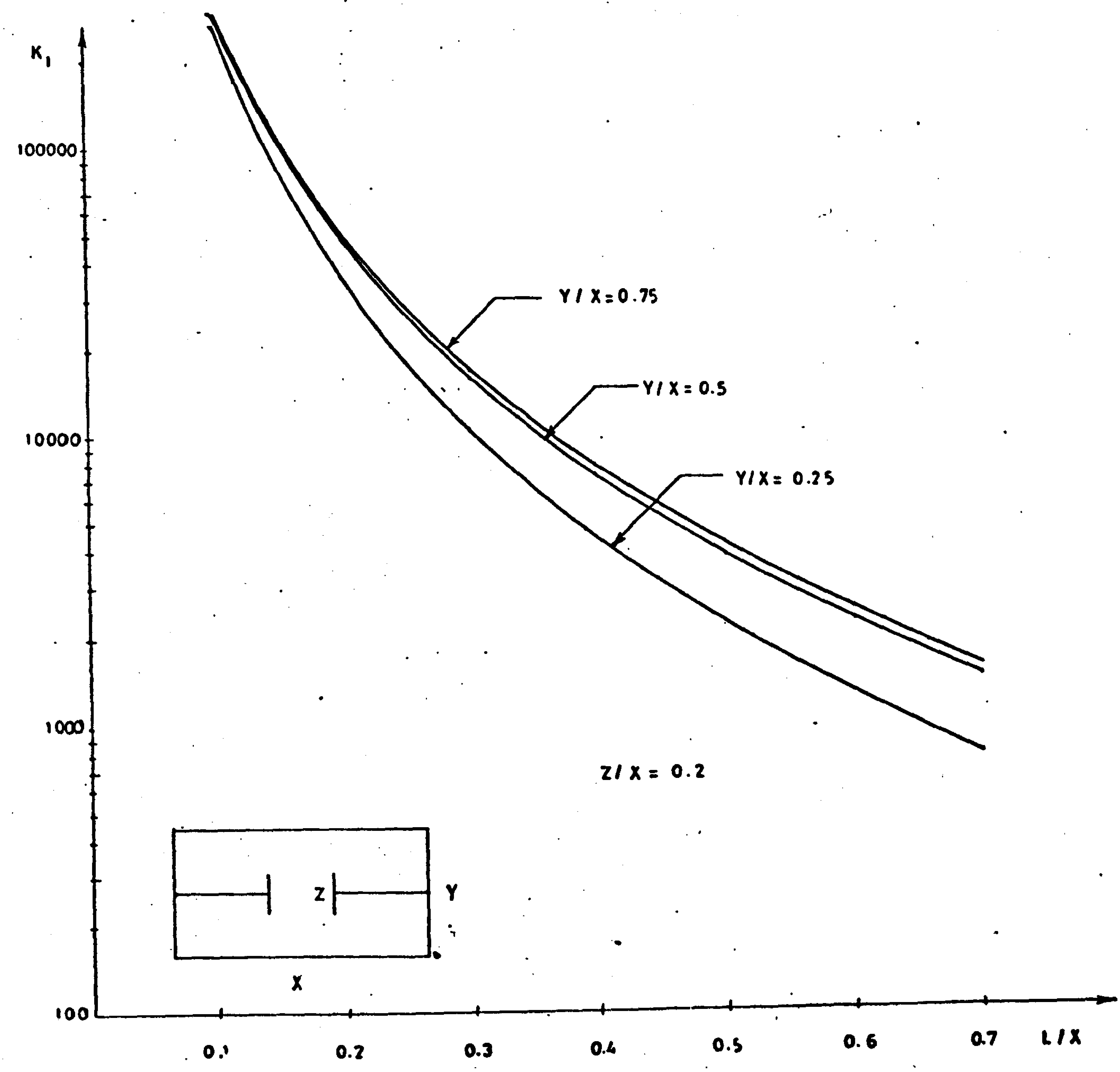
1.45 DEFORMATION OF SLAB 1 USING THE F.E. METHOD (2-STOREY MODEL)



1.46 VALUES OF THE STIFFNESS NUMBER K_1



1.47 VALUES OF THE STIFFNESS NUMBER K_t



1.48 VALUES OF THE STIFFNESS NUMBER K_1

APPENDIX II

GENERAL DESCRIPTION OF THE PROGRAMS

AND SAMPLE PROGRAMS

1. INPUT OF DATA

In a finite element program four fundamental sets of data are required. These sets consist of data specifying the geometry of the idealised structure, its material properties, boundary conditions and loading details.

1.1 Details of the geometry

The nodes of the idealised continuum are assigned numbers as economically as possible i.e. always numbering in the shorter direction as shown in Figure 1. This system of numbering ensures minimum band width for the matrices. The data required for the computer are:

NXE Number of elements along the x direction.

NYE Number of elements along the y direction.

W Half band width (equal to the maximum difference between non-zero degrees of freedom in any element of the continuum.

NN Number of nodes (total).

NOD Number of element nodes.

AA,BB Element dimensions in the x, y directions.

1.2 Details of boundary conditions

Boundary conditions can be applied either during element generation or by modification of the simultaneous equations immed-:iately prior to solution. In this analysis the first method is used as follows:

Associated with each node are degrees of freedom which are numbered in the same order as the nodes. In the example shown in Figure 1 each degree of freedom with zero denoting a free degree of freedom and a 1 denoting a fixed degree of freedom. This information about the degrees of freedom present in specific problems is stored in array [NF] which has NN rows and NODOF columns. The data input required are:

RN	Number of restrained nodes.
NL	Number of loaded nodes.
NODOF	Number of degrees of freedom per node.
DOF	Number of degrees of freedom per element.

1.3 Material properties

The material properties may vary from element to element so that it is convenient to give each element a number and to read in separately the information that describes element properties.

The material properties required are:

V	The Poisson's ratio of the material of the structure.
E	Young's modulus of the material.
RHO	Density of the material.
HI	Thickness of the element.

1.4 Loading details

This includes both the external forces and body forces. The external loads may be read in as a single vector while body forces are included using a special routine to generate the load vector.

Data read			Resulting NF array	
			1	2
			3	4
3	1	1	0	0
6	1	1	5	6
9	1	1	7	8
10	1	1	0	0
11	1	1	9	10
12	1	1	11	12
			0	0
			0	13
			0	14
			0	0

TABLE 1

2. BUILD UP OF THE NODE FREEDOM ARRAY

The node freedom matrix is introduced to reduce the storage space required for the overall stiffness and mass matrices. The terms corresponding to the restrained degrees of freedom are omitted from the matrices using the node freedom array. The nodal freedom array [NF] is formed by specifying the number at any node which is restrained in some way, followed by the digit 1 if the node is restrained in that sense and by the digit 0 if it is not. The array is built up in this program using a procedure FORMNF. For example the data read and resulting [NF] array for the problem shown in Figure 1 are as shown in Table 1.

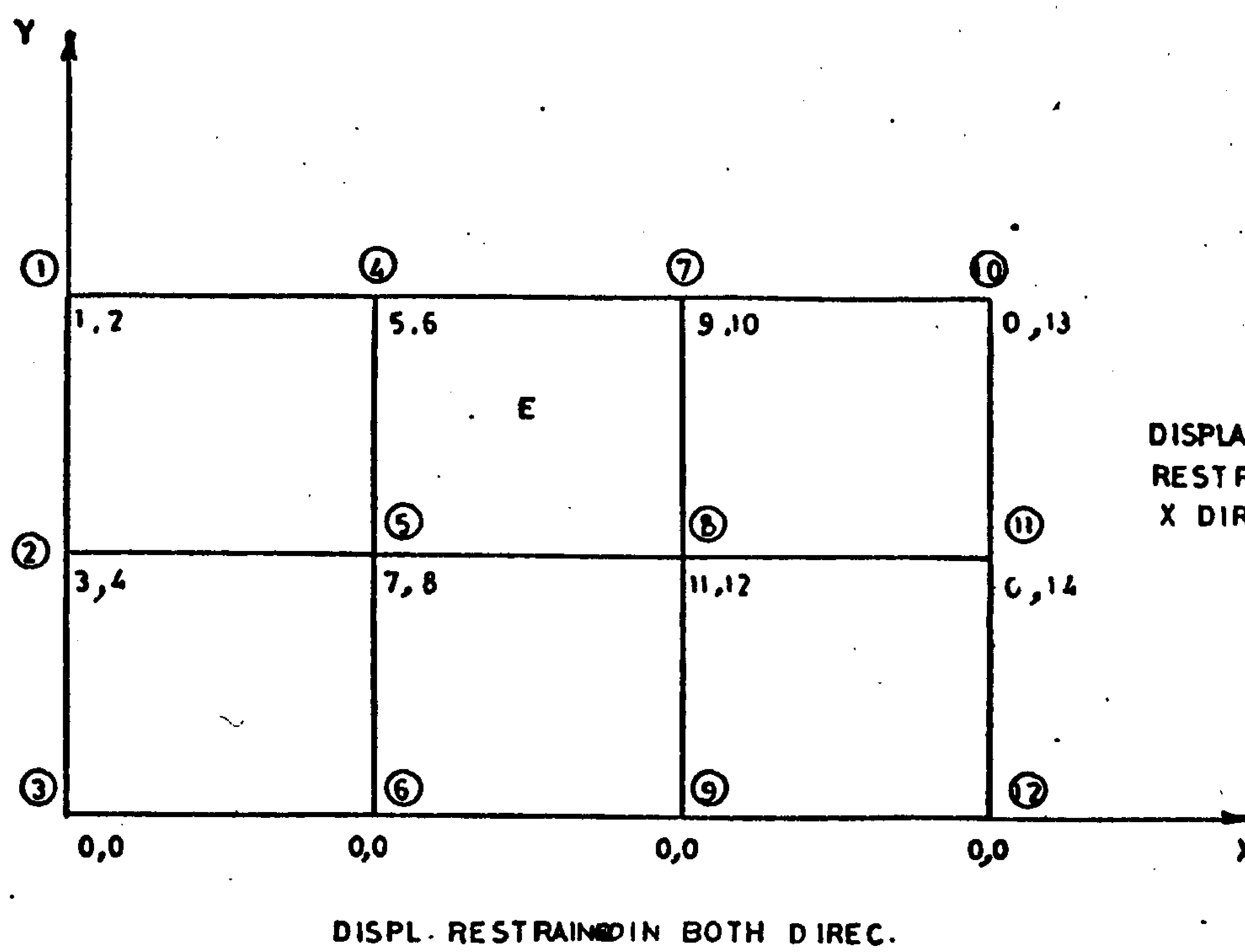


FIG.(1) TWO DEGREES OF FREEDOM NODE PLANE STRESS ELEMENTS

No Dof = 2 NXE = 3 NYE = 2 NN = 12 N = 14 Dof = 8

RN = 6 W = 7 NOD = 4

3. BUILD UP OF THE ELEMENT STIFFNESS MATRIX

After normalizing the local coordinates for the element as indicated in the last chapter, the local coordinates (ξ,η) are then related linearly to the global (x,y) coordinates as follows:

$$x = N_i\, x_i + N_j\, x_j + N_k\, x_k + N_\ell\, x_\ell$$

$$y = N_i\, y_i + N_j\, y_j + N_k\, 1/k + N_\ell\, y_\ell$$

y_K

In a matrix form this becomes:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_i & x_j & x_k & x_\ell \\ y_i & y_j & y_k & y_\ell \end{bmatrix} \begin{bmatrix} N_i \\ N_j \\ N_k \\ N_\ell \end{bmatrix}$$

or

$$\begin{bmatrix} x \\ y \end{bmatrix} = [COORD]^T \, [FUN]$$

where the $[FUN]^T$ matrix for the quadrilaterial is given by:

$$[FUN]^T = \tfrac{1}{4}\, [(1-\xi)(1-\eta)(1-\xi)(1+\eta)\ (1+\xi)(1+\eta)\ (1+\eta)(1-\xi)]$$

?

The matrix $[DER]$ in the program represents the gradients of shape function quantities:

$$[DER] = \begin{bmatrix} \dfrac{\partial FUN}{\partial \xi} \\[2ex] \dfrac{\partial FUN}{\partial \eta} \end{bmatrix} = \tfrac{1}{4} \begin{bmatrix} -(1-\eta) & -(1+\eta) & (1+\eta) & (1-\eta) \\ -(1-\xi) & (1-\xi) & (1+\xi) & -(1+\xi) \end{bmatrix}$$

As the properties of elements involve integration of [FUN] and [DER] over the element area the present work uses Gauss's rule to carry out all the integration required where the functions being evaluated and summed at nine points within the element area as shown in Figure 2. In applying the numerical integration for the rectangular element the integral:

$$I = \int_{-1}^{1} \int_{-1}^{1} f(\xi,\eta)\, d\xi\, d\eta \tag{1}$$

becomes:

$$I = \sum_{i=1}^{n} \sum_{j=1}^{n} H_i\, H_j\, f(\xi,\eta) \tag{2}$$

where H_i, H_j are the Gaussian multipliers.

The double summation can be readily interpreted as a single one over (nxn) points i.e. over (3x3) points in our case. The coordinates of these points and the relevant multipliers are stored in an array [SAMP] which is formed by procedure THREEPOINT. The values of the element functions and of their derivatives are created at the nine sampling points in turn by procedure FORMLIN in terms of local coordinates (ξ,η). To transform to the global (x,y) system it is then required and involves the Jacobian matrix:

$$\begin{Bmatrix} \frac{\partial}{\partial \xi} \\ \frac{\partial}{\partial \eta} \end{Bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} \begin{Bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{Bmatrix} = [JAC] \begin{Bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{Bmatrix} \tag{3}$$

This results in the gradiants of element functions with respect to (x,y) being held in matrix [DERIV].

The inverse [JAC1] and the determinanent DET of the Jacobian matrix can be calculated using procedure TWOBYTWO.

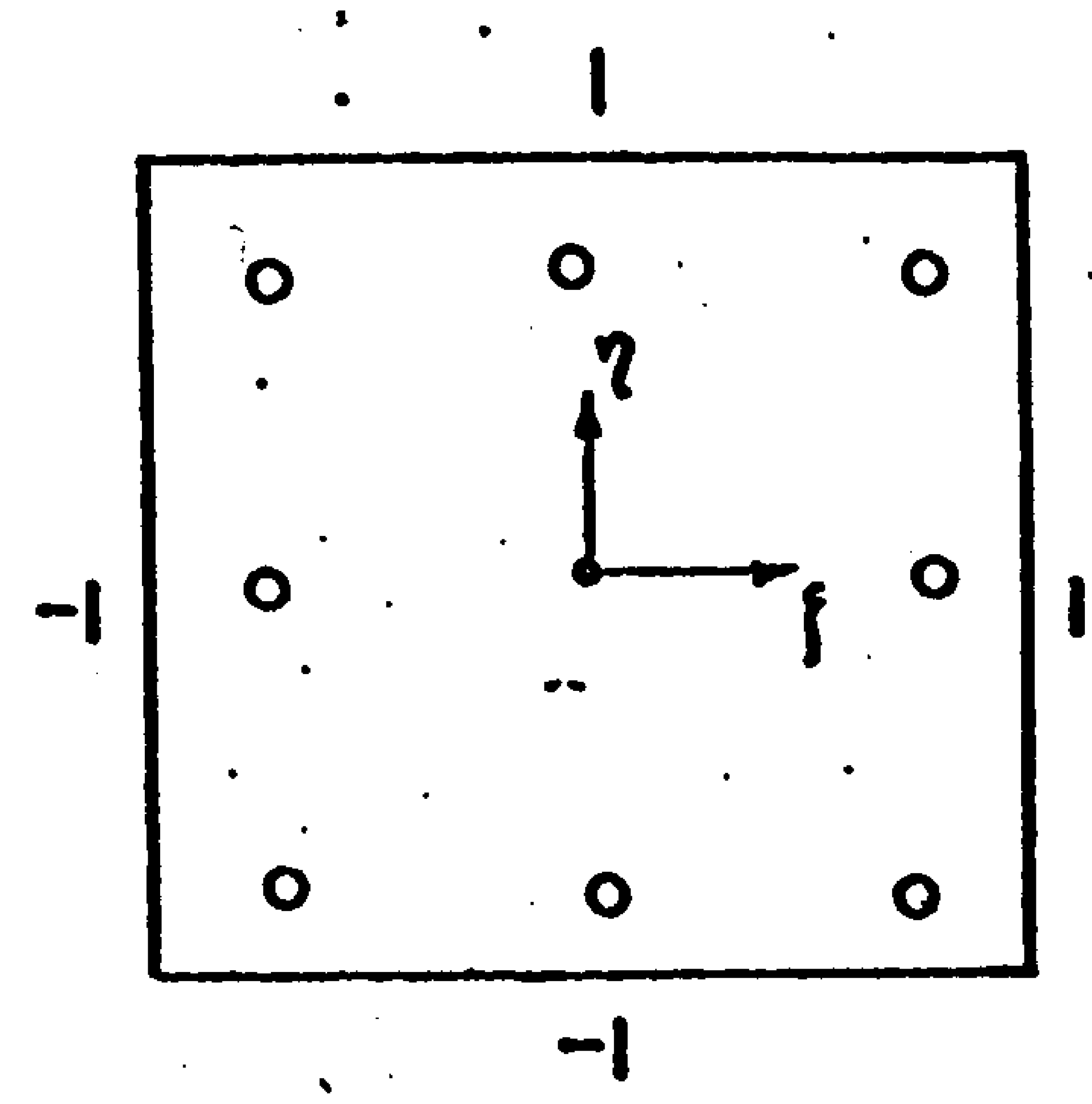


FIG. 2 INTEGRATION POINTS FOR $n = 3$

Following that the matrix [B] which relates the strains and displacements can be created using procedure FORMB while the matrix [D] which relates the stresses and strains can be created using procedure DPSTRN. The element stiffness, as shown in Chapter 3 is given by:

$$[km] = \iint [B]^T [D] [B] \, d(vol)$$

$$= \iint [BTBD] \, d(vol) \qquad (4)$$

But in the computations the integral is replaced as indicated by a double summation over the Gauss points.

$$[km] = DET \sum_{i=1}^{3} \sum_{j=1}^{3} k_1.k_2.BTDB_{ij} \qquad (5)$$

where k_1 and k_2 are the Gaussian multipliers held in [SAMP].

4. ASSEMBLY OF ELEMENTS (STRUCTURE STIFFNESS MATRICES)

To form the desired continuum approximation from the individual elements one should specify the geometrical details of the problem beside all the details described before.

Using procedure geometry which can be constructed using [NF] together with (NXE,NYE) and their sizes (AA,BB) the procedure works out the nodal coordinates [COORD] of each element together with vector [G] which contains the degrees of freedom associated with that particular element E in Figure 1 has the vector:

$$\{G\}^T = [7\ 8\ 5\ 6\ 9\ 10\ 11\ 12]$$

Then we use {G} to assemble the coefficients of the element property matrices such as [km] into the appropriate place in overall coeffi-:cient matrix using procedure FORMKVEC. Because the nodes are numbered in a careful manner, the non-zero terms in the assembled stiffness matrix $[BK]$ will be concentrated in a narrow band situated adjacent to the leading diagonal. This fact combined with the symmetrical nature of $[BK]$ means that only a relatively small proportion of the matrix is of real interest. Using this fact gives a reduction in the computer store as well as in the arithmetic.

Figure 3 shows the original stiffness matrix with its rectangular presentation, while Figure 4 shows an example of the arrangement in storage using one-dimensional vector. Procedure FORMRVEC stores the coefficients of $[BK]$ by column as a vector.

The assembly procedures use the half band width w - which can be calculated from the maximum difference between non-zero degrees of freedom in any element of the continuum - as input when utilising banding.

5. FORMATION OF THE LOAD VECTOR

No special procedures are used in forming the load vector. The external point loads are directly act on the nodes. No distributed loads are considered in the analysis.

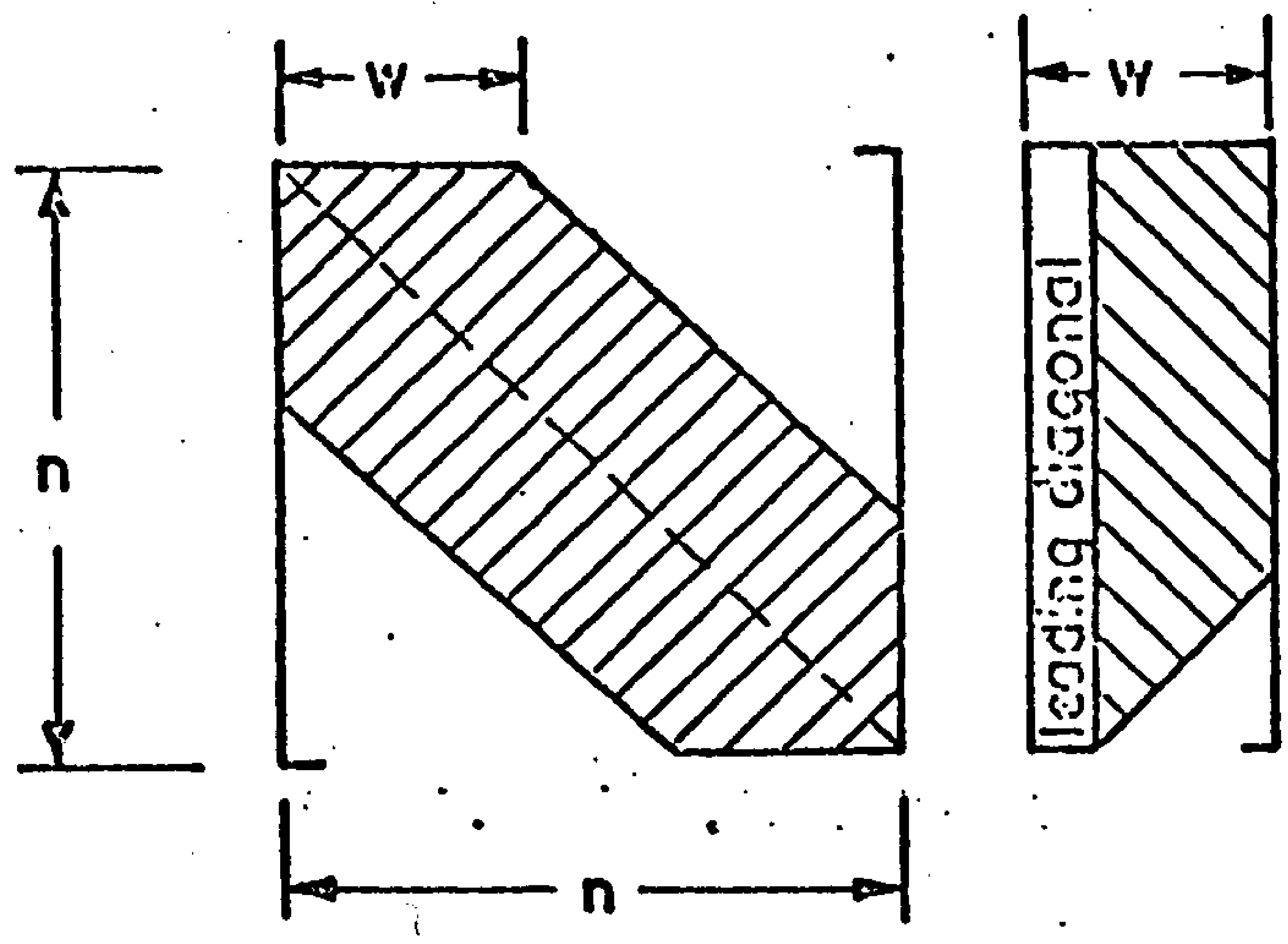
To calculate the internal forces on the nodes due to the residual forces in the ground springs the initial stress method is used as explained in Chapter 3 and the loads act also on the nodes.

6. SOLUTION OF EQUATIONS

The equilibrium equation is given by:

$$[BK]\{\delta\} = \{F\} \tag{7}$$

so that by determining matrix $[BK]$ (the stiffness matrix) and the



(a) Original square matrix (b) Rectangular representation

FIG. 3 STORAGE OF MATRIX FOR BANDED SOLUTION.

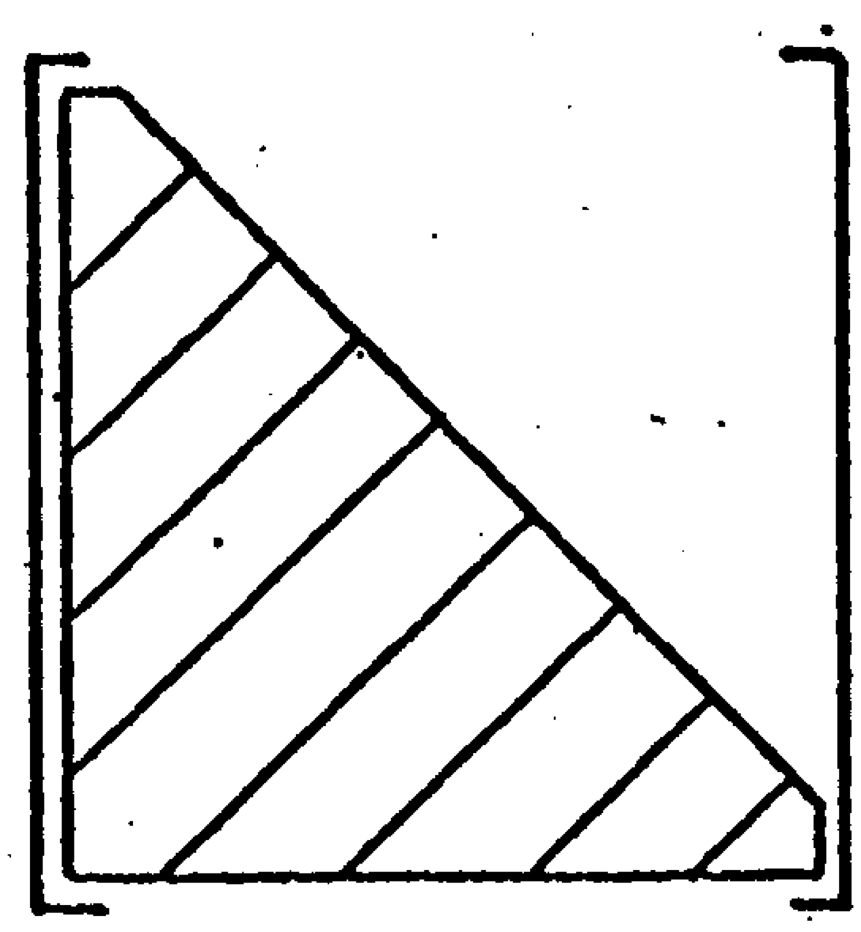


FIG. 4 LOWER TRIANGLE USED FOR FORMING THE VECTOR.

load vector {F} the displacement vector can be calculated. Two methods of solution can be used to get the value of $\{\delta\}$:

(a) Direct solution

(b) Iteration solution

In the present program the Gauss elimination method is used for direct solution of the equilibrium equation.

Three procedures are used to perform Gaussian reduction and forward and backward substitution.

Procedure BANRED is used to reduce the symmetrical band matrix $[BK]$ (stored by column of length N) by Gaussian reduction.

Procedure BACKBONE is used to perform forward Gaussian substitution on the reduced symmetrical band matrix $[BK]$ (stored by column).

Procedure BACKTWO is used to perform backward Gaussian substitution on the reduced symmetrical band matrix $[BK]$ (stored by columns).

7. OUTPUT OF RESULTS

Having calculated the values of $\{\delta\}$ the nodal displacements are printed by procedure PRINTVECTOR. Once the nodal displacements have been determined the stresses at any point in the element can be found from the relations:

$$\{\varepsilon\} = [B]\{\delta\}$$

$$\{\sigma\} = [D]\{\varepsilon\}$$

$$\therefore \quad \{\sigma\} = [D][B]\{\delta\} \qquad (8)$$

FINITE ELEMENT PROGRAM FOR PLANE STRESS

```
'BEGIN'
'PROCEDURE'MATMULT(A,B,Y,L,M,N);
'VALUE'L,M,N;  'ARRAY'A,B,Y;  'INTEGER'L,M,N;
'BEGIN''INTEGER'I,J,K;'REAL'X;
'FOR'I:=1'STEP'1'UNTIL'L'DO'
'FOR'J:=1'STEP'1'UNTIL'N'DO'
'BEGIN'X:=.0;
'FOR'K:=1'STEP'1'UNTIL'M'DO'
X:=X+A I,K?*B K,J?;
Y I,J?:=X;
'END';'END';

'PROCEDURE'MATVECMULT(M,V,K,L,Y);
'VALUE'K,L;'ARRAY'M,V,Y;'INTEGER'K,L;
'BEGIN''INTEGER'I,J;'REAL'X;
'FOR'I:=1'STEP'1'UNTIL'K'DO'
'BEGIN'X:=.0;
'FOR'J:=1'STEP'1'UNTIL'L'DO'
X:=X+M I,J?*V J?;
Y I?:=X;
'END';'END';

'PROCEDURE'TRANSMATRIX(A,B,M,N);
'VALUE'M,N;'ARRAY'A,B;'INTEGER'M,N;
'BEGIN''INTEGER'I,J;
'FOR'I:=1'STEP'1'UNTIL'M'DO'
'FOR'J:=1'STEP'1'UNTIL'N'DO'
A J,I?:=B I,J?;
'END';

'PROCEDURE'MATRIXADD(A,B,M,N);
'VALUE'M,N;  'ARRAY'A,B;  'INTEGER'M,N;
'BEGIN''INTEGER'I,J;
'FOR'I:=1'STEP'1'UNTIL'M'DO'
'FOR'J:=1'STEP'1'UNTIL'N'DO'
A I,J?:=A I,J?+B I,J?;
'END';

'PROCEDURE'TWOBYTWO(JAC,JAC1,DET);
'ARRAY'JAC,JAC1;'REAL'DET;
'BEGIN''INTEGER'K,L;
DET:=JAC 1,1?*JAC 2,2?-JAC 1,2?*JAC 2,1?;
JAC1 1,1?:=JAC 2,2?;JAC1 1,2?:=-JAC 1,2?;
JAC1 2,1?:=-JAC 2,1?;JAC1 2,2?:=JAC 1,1?;
'FOR'K:=1'STEP'1'UNTIL'2'DO'
'FOR'L:=1'STEP'1'UNTIL'2'DO'
JAC1 K,L?:=JAC1 K,L?/DET;
'END';

'PROCEDURE'BANRED(BK,N,COMAX);
'VALUE'N,COMAX;'ARRAY'BK;'INTEGER'N,COMAX;
'BEGIN''INTEGER'I,J,K,IL1,KBL,IJ,NKB,NI,NJ;'REAL'SUM;
'FOR'I:=2'STEP'1'UNTIL'N'DO'
'BEGIN'
```

```
IL1:=I-1;KBL:=IL1+COMAX;
'IF'KBL-N <= 0'THEN''GOTO'L1;
KBL:=N;
L1:'FOR'J:=I'STEP'1'UNTIL'KBL'DO'
'BEGIN'
IJ:=(J-I)*N+I;SUM:=BK IJ?;NKB:=J-COMAX+1;.
'IF'NKB>0'THEN''GOTO'L2;
NKB:=1;
L2:'IF'NKB-IL1>0'THEN''GOTO'L3;
'FOR'K:=NKB'STEP'1'UNTIL'IL1'DO'
'BEGIN'
NI:=(I-K)*N+K;NJ:=(J-K)*N+K;
SUM:=SUM-BK NI?*BK NJ?/BK K?;
'END';
L3:BK IJ?:=SUM;
'END';'END';'END';

'PROCEDURE'BACKONE(BK,D,N,COMAX);
'VALUE'N,COMAX;'ARRAY'BK,D;'INTEGER'N,COMAX;
'BEGIN''INTEGER'I,J,I1,NKB,JN;'REAL'SUM;
D 1?:=D 1?/BK 1?;
'FOR'I:=2'STEP'1'UNTIL'N'DO'
'BEGIN'
SUM:=D I?;I1:=I-1;NKB:=I-COMAX+1;
'IF'NKB>0'THEN''GOTO'L1;
NKB:=1;
L1:'FOR'J:=NKB'STEP'1'UNTIL'I1'DO'
'BEGIN'
JN:=(I-J)*N+J;SUM:=SUM-BK JN?*D J?;
'END';D I?:=SUM/BK I?;
'END';'END';
'PROCEDURE'BACKTWO(A,B,N,COMAX);
'VALUE'N,COMAX;'ARRAY'A,B;'INTEGER'N,COMAX;
'BEGIN''INTEGER'I,J,K,I1,NKB,JN;'REAL'SUM;
'FOR'J:=2'STEP'1'UNTIL'N'DO'
'BEGIN'
I:=N-J+1; SUM:=.0; I1:=I+1; NKB:=I-1+COMAX;
'IF'NKB-N <= 0'THEN''GOTO'L1;
NKB:=N;
L1:'FOR'K:=I1'STEP'1'UNTIL'NKB'DO'
'BEGIN'
JN:=(K-I)*N+I;
SUM:=SUM+A JN?*B K?;
'END';
B I?:=B I?-SUM/A I?;
'END';'END';

'PROCEDURE'NULL(A,M,N);
'VALUE'M,N; 'ARRAY'A; 'INTEGER'M,N;
'BEGIN''INTEGER'I,J;
'FOR'I:=1'STEP'1'UNTIL'M'DO'
'FOR'J:=1'STEP'1'UNTIL'N'DO'
A I,J?:=0.0;
'END';
```

```
'PROCEDURE'PRINTVECTOR(V,L,M,N);
'VALUE'L,M,N;  'ARRAY'V;  'INTEGER'L,M,N;
'BEGIN''INTEGER'I;
'FOR'I:=1'STEP'1'UNTIL'L'DO'
PRINT(V I?,M,N);
NEWLINE(2);
'END';

'PROCEDURE'PRINTARRAY(A,X,Y,M,N);
'VALUE'X,Y,M,N;'INTEGER'X,Y,M,N;'ARRAY'A;
'BEGIN''INTEGER'I,J;
'FOR'I:=1'STEP'1'UNTIL'X'DO'
'BEGIN'
'FOR'J:=1'STEP'1'UNTIL'Y'DO'
PRINT(A I,J?,M,N);
NEWLINE(1);
'END';
NEWLINE(2);
'END';
'PROCEDURE'THREEPOINT(IMAN);
'ARRAY'IMAN;
'BEGIN'
IMAN 1,1?:=.774596669241483;IMAN 2,1?:=.0;IMAN 3,1?:=-IMAN 1,1?;
IMAN 1,2?:=.555555555555556;IMAN 2,2?:=.888888888888889;
IMAN 3,2?:=IMAN 1,2?;
'END';

'PROCEDURE'FORMNF(NF,NN,NODOF,R);
'VALUE'NN,NODOF,R;'INTEGER'NN,NODOF,R;'INTEGER''ARRAY'NF;
'BEGIN''INTEGER'I,J,K;
'FOR'I:=1'STEP'1'UNTIL'NN'DO'
'FOR'J:=1'STEP'1'UNTIL'NODOF'DO'
NF I,J?:=1;
'FOR'I:=1'STEP'1'UNTIL'R'DO'
'BEGIN'
K:=READ;
'FOR'J:=1'STEP'1'UNTIL'NODOF'DO'
'BEGIN'
'IF'READ=0'THEN''GOTO'L1;
NF K,J?:=0;
L1:'END';'END';
K:=1;
'FOR'I:=1'STEP'1'UNTIL' NN'DO'
'FOR'J:=1'STEP'1'UNTIL'NODOF'DO'
'BEGIN'
'IF'NF I,J?=0'THEN''GOTO'L2;
NF I,J?:=K; K:=K+1;
L2:'END';'END';

'PROCEDURE'FORMKVEC(BK,KM,G,N,DOF);
'VALUE'N,DOF;'INTEGER'N,DOF;
'ARRAY'BK,KM;'INTEGER''ARRAY'G;
'BEGIN''INTEGER'I,J,CD,VAL;
'FOR'I:=1'STEP'1'UNTIL'DOF'DO'
'BEGIN'
```

```
'IF'G I?=0'THEN''GOTO'L1;
'FOR'J:=1'STEP'1'UNTIL'DOF'DO'
'BEGIN'
'IF'G J?=0'THEN''GOTO'L2;
CD:=G J?-G I?+1;
'IF'CD<1'THEN''GOTO'L2;
VAL:=N*(CD-1)+G I?;
BK VAL?:=BK VAL?+KM I,J?;
L2:'END';L1:'END';'END';

'PROCEDURE'SUBMATRIX(A,B,M,N);
'VALUE'M,N;'ARRAY'A,B;'INTEGER'M,N;
'BEGIN''INTEGER'I,J;
'FOR'I:=1'STEP'1'UNTIL'M'DO'
'FOR'J:=1'STEP'1'UNTIL'N'DO'
A I,J?:=A I,J?-B I,J?;
'END';

'PROCEDURE'ECMAT(ECM,FUN,NOD,NODOF);
'VALUE'NOD,NODOF;'INTEGER'NOD,NODOF;'ARRAY'ECM,FUN;
'BEGIN''INTEGER'I,J,K,DOF;'REAL'X;DOF:=NOD*NODOF;
'BEGIN''ARRAY'N 1:NODOF,1:DOF?,NT 1:DOF,1:NODOF?;
'FOR'I:=1'STEP'1'UNTIL'DOF'DO'
'FOR'J:=1'STEP'1'UNTIL'NODOF'DO'
N I,J?:=NT I,J?:=.0;
'FOR'I:=1'STEP'1'UNTIL'NOD'DO'
'FOR'J:=1'STEP'1'UNTIL'NODOF'DO'
N J,(I-1)*NODOF+J?:=NT (I-1)*NODOF+J,J?:=FUN I?;
'FOR'I:=1'STEP'1'UNTIL'DOF'DO'
'FOR'J:=1'STEP'1'UNTIL'DOF'DO'
'BEGIN'  X:=.0;
'FOR'K:=1'STEP'1'UNTIL'NODOF'DO'
X:=X+NT I,K?*N K,J?;
ECM I,J?:=X;
'END';'END';'END';

'PROCEDURE'ELMAT(ELM,DOF);
'VALUE'DOF;
'INTEGER'DOF;'ARRAY'ELM;
'BEGIN''INTEGER'I;
'FOR'I:=1'STEP'1'UNTIL'DOF'DO'ELM I,I?:=1.0;
'END';

'PROCEDURE'SEARCH(NF,NN,NODOF,N);
'VALUE'NN,NODOF;
'INTEGER''ARRAY'NF;
'INTEGER'NN,NODOF,N;
'BEGIN'
'INTEGER'I,J;
N:=0;
'FOR'I:=1'STEP'1'UNTIL'NN'DO'
'FOR'J:=1'STEP'1'UNTIL'NODOF'DO'
'BEGIN'
'IF'NF I,J?=0'THEN''GOTO'P1;
N:=NF I,J?;
```

```
P1:'END';
WRITETEXT('('%%%%%VO%%M%%EQUATIONS%TO%BE%SOLVED%')');
PRINT(N,2,0);
NEWLINE(5);
'END';

'PROCEDURE'FORMLIN(DER,FUN,IMAN,I,J);
'VALUE'I,J;'ARRAY'DER,FUN,IMAN;'INTEGER'I,J;
'BEGIN''REAL'ETA,XI,ETAM,ETAP,XIM,XIP;
ETA:=IMAN I,1?;XI:=IMAN J,1?;
ETAM:=(1-ETA)/4; ETAP:=(1+ETA)/4;
XIM:=(1-XI)/4; XIP:=(1+XI)/4;
FUN 1?:=4*XIM*ETAM; FUN 2?:=4*XIM*ETAP; FUN 3?:=4*XIP*ETAP;
FUN 4?:=4*XIP*ETAM;
DER 1,1?:=-ETAM; DER 1,2?:=-ETAP; DER 1,3?:=ETAP;; DER 1,4?:=ETAM;
DER 2,1?:=-XIM; DER 2,2?:=XIM; DER 2,3?:=XIP; DER 2,4?:=-XIP;
'END';

'PROCEDURE'FORMB(BEE,DERIV ,VOL,NOD);
'VALUE'NOD;'ARRAY'BEE,DERIV,VOL;'INTEGER'NOD;
'BEGIN''INTEGER'K,L,M;
'FOR'M:=1'STEP'1'UNTIL'NOD'DO'
'BEGIN' K:=2*M; L:=K-1;
VOL L?:=BEE 1,L?:=DERIV 1,M?;
VOL K?:=BEE 2,K?:=DERIV  2,M?;
BEE 3,L?:=DERIV 2,M?; BEE 3,K?:=DERIV 1,M?;
'END';'END';

'PROCEDURE'FORMD(DEE,E,V);
'VALUE'E,V; 'ARRAY'DEE; 'REAL'E,V;
'BEGIN''INTEGER'I,J; 'REAL'V1,VV;
V1:=V/(1-V); VV:=(1-2*V)/2/(1-V);
DEE 1,1?:=DEE 2,2?:=1; DEE 3,3?:=VV; DEE 1,2?:=DEE 2,1?:=V1;
'FOR'I:=1'STEP'1'UNTIL'3'DO'
'FOR'J:=1'STEP'1'UNTIL'3'DO'
DEE I,J?:=DEE I,J?*E/(2*(1+V)*VV);
'END';

'PROCEDURE'LMADD(A,B,C,N,COMAX,LOADS);
'VALUE'N,COMAX;'ARRAY'A,B,C,LOADS;'INTEGER'N,COMAX;
'BEGIN''INTEGER'F,G;'REAL'X;
'FOR'F:=1'STEP'1'UNTIL'N'DO'
'BEGIN'X:=0;
'FOR'G:=1'STEP'1'UNTIL'COMAX'DO'
'BEGIN'
'IF'F+G>N+1'THEN''GOTO'L1;
X:=X+A N*(G-1)+F?*B F+G-1?;
L1:'END';
'FOR'G:=2'STEP'1'UNTIL'COMAX'DO'
'BEGIN'
'IF'F-G+1<1'THEN''GOTO'L2;
X:=X+A (N-1)*(G-1)+F?*B F-G+1?;
L2:'END';
C F?:=LOADS F?+X;
'END';'END';
```

```
'PROCEDURE'GEOMETRY(P,Q,NXE,NYE,A ,B ,COORD,G,NF,L);
'VALUE'L,P,Q,NXE,NYE;'INTEGER'P,Q,NXE,NYE;'REAL'L;
'ARRAY'COORD,A,B;'INTEGER''ARRAY'G,NF;
'BEGIN''INTEGER'AO,AL,AM,AN;
'REAL' X,Y,Z;
AO:=(P-1)*(NYE+1)+Q; AL:=AO+1; AM:=P*(NYE+1)+Q; AN:=AM+1;
G 1?:=NF AL,1?; G 2?:=NF AL,2?; G 3?:=NF AO,1?; G 4?:=NF AO,2?;
G 5?:=NF AM,1?; G 6?:=NF AM,2?; G 7?:=NF AN,1?; G 8?:=NF AN,2?;
Y:=Y+B P,Q?;
'IF'Q=1'THEN''BEGIN'Y:=B P,1?;Z:=0; 'END';
'IF'P=1'THEN'X:=0;
COORD 1,1?:=X; COORD 1,2?:=L-Y;
COORD 4,1?:=X+A P,Q?; COORD 4,2?:=L-Y;
COORD 2,1?:=X; COORD 2,2?:=L-Z;
COORD 3,1?:=X+A P,Q?; COORD 3,2?:=L-Z;
 Z:=Z+B P,Q?;
'IF'Q=NYE'THEN' X:=X+A P,Q?;
'END';

'PROCEDURE'DPSTRS(DEE,E,V);
'COMMENT'*****PLANE STRESS*****;
'VALUE'E,V;'ARRAY'DEE;'REAL'E,V;
'BEGIN''INTEGER'I,J;'REAL' V1,VV;
V1:=V;VV:=(1-V1)/2;
DEE 1,1?:=DEE 2,2?:=1.0; DEE 3,3?:=VV;
DEE 1,2?:=DEE 2,1?:=V1;
'FOR'I:=1'STEP'1'UNTIL'3'DO'
'FOR'J:=1'STEP'1'UNTIL'3'DO'
DEE I,J?:=DEE I,J?*E/(1-    V1**2);
'END';

'PROCEDURE'DPSTRN(DEE,E,V);
'VALUE'E,V;'ARRAY'DEE;'REAL'E,V;
'BEGIN''INTEGER'I,J;'REAL'V1,VV;
V1:=V;VV:=(1-V);
DEE 1,1?:=DEE 2,2?:=1.0;
DEE 3,3?:=(1-2*V)/2/VV;
DEE 1,2?:=DEE 2,1?:=V1/VV;
'FOR'I:=1'STEP'1'UNTIL'3'DO'
'FOR'J:=1'STEP'1'UNTIL'3'DO'
DEE I,J?:=DEE I,J?*E*VV/(1+V1)/(1-2*V1);
'END';

'PROCEDURE'LINMULT(A,B,C,N,COMAX);
'VALUE'N,COMAX;'ARRAY'A,B,C;'INTEGER'N,COMAX;
'BEGIN''INTEGER'F,G;'REAL'X;
'FOR'F:=1'STEP'1'UNTIL'N'DO'
'BEGIN'X:=.0;
'FOR'G:=1'STEP'1'UNTIL'COMAX'DO'
'BEGIN'
'IF'F+G>N+1'THEN''GOTO'L1;
X:=X+A N*(G-1)+F?*B F+G-1?;
L1:'END';
'FOR'G:=2'STEP'1'UNTIL'COMAX'DO'
```

```
'BEGIN'
'IF'F-G+1<1'THEN''GOTO'L2;
X:=X+A (N-1)*(G-1)+F?*B F-G+1?;
L2:'END';C F?:=X;
'END';'END';

'PROCEDURE'BANTMULT(A,B,C,N,W);
'VALUE'N,W;'INTEGER'N,W;'ARRAY'A,B,C;
'BEGIN''REAL'X;'INTEGER'I,J,K,_;
'FOR'I:=1'STEP'1'UNTIL'N'DO'
'BEGIN'
X:=.0;K:=W+2;
'FOR'J:=1'STEP'1'UNTIL'W+W+1'DO'
'BEGIN'
K:=K-1;
'IF'I-K+1>N'THEN''GOTO'L1;
'IF'I-K+1<1'THEN''GOTO'L1;
X:=X+A I,J?*B I-K+1?;
L1:'END';
C I?:=X;
'END';'END';

      'REAL'V,AA,BB,K1,K2,DET,QJOT,AREA,H1;
'REAL' S;
      'INTEGER'DOF,NODOF,        NOD,R,T,TYPE,RR,II;
      'INTEGER'I,J,K,L,P,Q,M,N ,NXE,NYE,H,W,COMAX,RN,NL,NN;
      TYPE:=1;
      NXE:=READ;NYE:=READ;W:=READ;NN:=READ;RN:=READ;NL:=READ;
      DOF:=READ;NODOF:=READ;        H:=READ;T:=READ;
      NOD:=DOF'/'NODOF;
      AA:=READ;BB:=READ;V:=READ;
S:=READ;
      WRITETEXT('('            INPUT%DATA')');NEWLINE(2);
      WRITETEXT('('      NUMBER OF X ELEMENT = ')');
      PRINT(NXE,3,0);NEWLINE(2);
      WRITETEXT('('      NUMBER OF Y ELEMENT = ')');
      PRINT(NYE,3,0);NEWLINE(2);
      WRITETEXT('('      NUMBER OF NODES     = ')');
      PRINT(NN,4,0);NEWLINE(2);
      WRITETEXT('('      NUMBER OF LOADED NODES = ')');
      PRINT(NL,2,0);NEWLINE(2);
      WRITETEXT('('      NUMBER OF RESTRAINED NODES =')');
      PRINT(RN,2,0);NEWLINE(2);
      WRITETEXT('('      POISSON RATIO = ')');
      PRINT(V,0,6);NEWLINE(2);
      'BEGIN'
'INTEGER''ARRAY'G 1:DOF?,NF 1:NN,1:NODOF?;
            FORMNF(NF,NN,NODOF,RN);
            SEARCH(NF,NN,NODOF,N);
            COMAX:=W+1;R:=N*COMAX;
           'BEGIN'
              'ARRAY'BK 1:R?,DEE 1:H,1:H?,IMAN 1:3,1:2?;
              'ARRAY'COORD 1:NOD,1:T?,JAC,JAC1 1:T,1:T?;
              'ARRAY'DER,DERIV 1:T,1:NOD?,BEE,DBEE 1:H,1:DOF?;
              'ARRAY'BTDB,KM 1:DOF,1:DOF?,LOADS 1:N?,ELO 1:DOF?;
```

```
                    'ARRAY'EPS,SIGMA 1:H?,BT 1:DOF,1:H?,FUN,VOL 1:COF?;
                    'ARRAY'F1,MM 1:R?,EMM,ECM 1:DOF,1:DOF?;
                    'ARRAY'SX,E,RHO 1:NXE,1:NYE?;
 'ARRAY'A,B 1:NXE,1:NYE?;
                    'FOR'I:=1'STEP'1'UNTIL'N'DO' LOADS I?:=0;
                    'FOR'I:=1'STEP'1'UNTIL'NXE'DO'
                    'FOR'J:=1'STEP'1'UNTIL'NYE'DO''BEGIN'
                      A I,J?:=READ; B I,J?:=READ;
                       'END';
                    'FOR'I:=1'STEP'1'UNTIL'NXE'DO'
                    'FOR'J:=1'STEP'1'UNTIL'NYE'DO'  E I,J?:=READ;
                    'FOR'I:=1'STEP'1'UNTIL'NXE'DO'
                    'FOR'J:=1'STEP'1'UNTIL'NYE'DO'RHO I,J?:=READ;
                    'FOR'I:=1'STEP'1'UNTIL'NXE'DO'
                    'FOR'J:=1'STEP'1'UNTIL'NYE'DO'SX I,J?:=0  ;
                    'FOR'I:=1'STEP'1'UNTIL'R'DO'BK I?:=MM I?:=0;
                    NULL(DEE,H,H);
                    THREEPOINT(IMAN);
                    'FOR'P:=1'STEP'1'UNTIL'NXE'DO'
                    'FOR'Q:=1'STEP'1'UNTIL'NYE'DO'
                   'BEGIN'
           E P,Q?:=E P,Q?*12;
 AA:=A P,Q?; BB:=B P,Q?;
                      AREA:=0;K1:=E P,Q?;DPSTRS(DEE,K1,V);
              GEOMETRY(P,Q,NXE,NYE,A ,B ,COORD,G,NF,S);
                      NULL(KM,DOF,DOF);
                      NULL(EMM,DOF,DOF);
                     'IF'TYPE=1'THEN'ELMAT(EMM,DOF);
                     'FOR'I:=1'STEP'1'UNTIL'3'DO'
                     'FOR'J:=1'STEP'1'UNTIL'3'DO'
                     'BEGIN'
                         K1:=IMAN I,2?;K2:=IMAN J,2?;
                         FORMLIN(DER,FUN,IMAN,I,J);
                         MATMULT(DER,COORD,JAC,T,NOD,T);
                         TWOBYTWO(JAC,JAC1,DET);
                         MATMULT(JAC1,DER,DERIV,T,T,NOD);
                         NULL(BEE,H,DOF);
                         FORMB(BEE,DERIV,VOL,NOD);
                         MATMULT(DEE,BEE,DBEE,H,H,DOF);
                         TRANSMATRIX(BT,BEE,H,DOF);
                         MATMULT(BT,DBEE,BTDB,DOF,H,DOF);
                        'IF'TYPE'NOTEQUAL'1'THEN'ECMAT(ECM,FUN,NOD,NODOF)
                         ;QUOT:=DET*K1*K2;
                         AREA:=AREA+QUOT;
                         'FOR'K:=1'STEP'1'UNTIL'DOF'DO'
                         'FOR'L:=1'STEP'1'UNTIL'DOF'DO'
                         'BEGIN'
                              BTDB K,L?:=BTDB K,L?*QUOT;
                             'IF'TYPE'NOTEQUAL'1'THEN'
                              ECM K,L?:=ECM K,L?*QUOT*RHO P,Q?;
                         'END';
                MATRIXADD(KM,BTDB,DOF,DOF);
                       'IF'TYPE'NOTEQUAL'1'THEN'
                         MATRIXADD(EMM,ECM,DOF,DOF);
                    'END'  OF GAVSS POINT CONSIDERATION;
```

```
                        AREA:=AREA/NOD*RHO P,Q?;
                        'IF'TYPE=1'THEN''FOR'K:=1'STEP'1'UNTIL'DOF'DO'
                        EMM K,K?:=EMM K,K?*AREA;
                        FORMKVEC(BK,KM,G,N,DOF);
                        FORMKVEC(MM,EMM,G,N,DOF);
                    'END'  OF ELEMENT CONSIDRATION;
                    PRINTARRAY(KM ,DOF,DOF,0,4);
                    PRINTARRAY(EMM,DOF,DOF,0,4);
                    'FOR'RR:=1'STEP'1'UNTIL'R'DO'
                    F1 RR?:=BK RR?;
                    BANRED(F1,N,COMAX);
LOADS 1?:=LOADS 17?:=-210;
LOADS 3?:=LOADS 5?:=LOADS 7?:=LOADS 9?:=LOADS 11?:=LOADS 13?:=
LOADS 15?:=-420;
LOADS 110?:=18100;
                    BACKONE(F1,LOADS,N,COMAX);
                    BACKTWO(F1,LOADS,N,COMAX);
                    PRINTVECTOR(LOADS,N,0,4);
                    'FOR'P:=1'STEP'1'UNTIL'NXE'DO'
                    'FOR'Q:=1'STEP'1'UNTIL'NYE'DO'
                  'BEGIN'
AA:=A P,Q?; BB:=B P,Q?;
             GEOMETRY(P,Q,NXE,NYE,A ,B ,COORD,G,NF,S);
            I:=2;J:=2;
                        FORMLIN(DER,FUN,IMAN,I,J);
                        MATMULT(DER,COORD,JAC,T,NOD,T);
                        TWOBYTWO(JAC,JAC1,DET);
                        MATMULT(JAC1,DER,DERIV,T,T,NOD);
                        NULL(BEE,H,DOF);FORMB(BEE,DERIV,VOL,NOD);
                        'FOR'M:=1'STEP'1'UNTIL'DOF'DO'
                        ELD M?:='IF'G M?=0'THEN'0'ELSE'LOADS G M??;
          E P,Q?:=E P,Q?*12;
                        K1:=E P,Q?;
                        DPSTRS(DEE,K1,V);
                        MATVECMULT(BEE,ELD,H,DOF,EPS);
                        MATVECMULT(DEE,EPS,H,H,SIGMA);
                        SX P,Q?:=SIGMA 1?;
                    'END';
                    PRINTARRAY(SX,NXE,NYE,0,4);
'END'; 'END';
'END'
```

NO ERRORS FOUND

FINITE ELEMENT PROGRAM FOR ELASTO-PLASTIC PLATE IN BENDING

```
'BEGIN'
'PROCEDURE'BACKONE(BK,D,N,COMAX);
'VALUE'N,COMAX;'ARRAY'BK,D;'INTEGER'N,COMAX;
'BEGIN''INTEGER'I,J,I1,NKB,JN;'REAL'SUM;
D 1 :=D 1 /BK 1 ;
'FOR'I:=2'STEP'1'UNTIL'N'DO'
'BEGIN'
SUM:=D I ;I1:=I-1;NKB:=I-COMAX+1;
'IF'NKB>0'THEN''GOTO'L1;
NKB:=1;
L1:'FOR'J:=NKB'STEP'1'UNTIL'I1'DO'
'BEGIN'
JN:=(I-J)*N+J;SUM:=SUM-BK JN *D J ;
'END';D I :=SUM/BK I ;
'END';'END';
'PROCEDURE'BACKTWO(A,B,N,COMAX);
'VALUE'N,COMAX;'ARRAY'A,B;'INTEGER'N,COMAX;
'BEGIN''INTEGER'I,J,K,I1,NKB,JN;'REAL'SUM;
'FOR'J:=2'STEP'1'UNTIL'N'DO'
'BEGIN'
I:=N-J+1; SUM:=.0;I1:=I+1; NK3:=I-1+COMAX;
'IF'NKB-N<=0'THEN''GOTO'L1;
NKB:=N;
L1:'FOR'K:=I1'STEP'1'UNTIL'NK3'DO'
'BEGIN'
JN:=(K-I)*N+I;
SUM:=SUM+A JN *B K ;
'END';
B I :=B I -SUM/A I ;
'END';'END';
'PROCEDURE'MATMULT(A,B,Y,L,M,N);
 'VALUE'L,M,N;  'ARRAY'A,B,Y;  'INTEGER'L,M,N;
'BEGIN''INTEGER'I,J,K; 'REAL'X;
'FOR'I:=1'STEP'1'UNTIL'L'DO'
'FOR'J:=1'STEP'1'UNTIL'N'DO'
'BEGIN' X:=.0;
'FOR'K:=1'STEP'1'UNTIL'M'DO'
X:=X+A I,K *B K,J ;
Y I,J :=X;
'END';'END';
'PROCEDURE'MATVECMULT(M,V,K,L,Y);
'VALUE'K,L;'ARRAY'M,V,Y;'INTEGER'K,L;
'BEGIN''INTEGER'I,J;'REAL'X;
'FOR'I:=1'STEP'1'UNTIL'K'DO'
'BEGIN'X:=.0;
'FOR'J:=1'STEP'1'UNTIL'L'DO'
X:=X+M I,J *V J ;
Y I :=X;
'END';'END';
'PROCEDURE'TRANSMATRIX(A,B,M,N);
'VALUE'M,N;'ARRAY'A,B;'INTEGER'M,N;
'BEGIN''INTEGER'I,J;
```

```
'FOR'I:=1'STEP'1'UNTIL'M'DO'
'FOR'J:=1'STEP'1'UNTIL'N'DO'
A J,I :=B I,J ;
 'END';
'PROCEDURE'MATRIXADD(A,B,M,N);
'VALUE'M,N; 'ARRAY'A,B; 'INTEGER'M,N;
'BEGIN''INTEGER'I,J;
'FOR'I:=1'STEP'1'UNTIL'M'DO'
'FOR'J:=1'STEP'1'UNTIL'N'DO'
A I,J :=A I,J +B I,J ;
'END';
'PROCEDURE'PLIWORYTWO(JAC,JAC1,DET,AA,BB);
'VALUE'AA,BB;'REAL'AA,BB,DET;'ARRAY'JAC,JAC1;
'BEGIN''INTEGER'K,L;
JAC 1,1 :=AA/2;JAC 2,2 :=BB/2;JAC 1,2 :=JAC 2,1 :=0;
DET:=JAC 1,1 *JAC 2,2 -JAC 1,2 *JAC 2,1 ;
JAC1 1,1 :=JAC 2,2 ;JAC1 1,2 :=-JAC 1,2 ;
JAC1 2,1 :=-JAC 2,1 ;JAC1 2,2 :=JAC 1,1 ;
'FOR'K:=1'STEP'1'UNTIL'2'DO'
'FOR'L:=1'STEP'1'UNTIL'2'DO'
JAC1 K,L :=JAC1 K,L /DET;
'END';
'PROCEDURE'BANRED(BK,N,COMAX);
'VALUE'N,COMAX;'ARRAY'BK;'INTEGER'N,COMAX;
'BEGIN''INTEGER'I,J,K,IL1,KBL,IJ,NKB,NI,NJ;'REAL'SUM;
'FOR'I:=2'STEP'1'UNTIL'N'DO'
'BEGIN'
IL1:=I-1;KBL:=IL1+COMAX;
'IF'KBL-N <= 0'THEN''GOTO'L1;
KBL:=N;
L1:'FOR'J:=I'STEP'1'UNTIL'KBL'DO'
'BEGIN'
IJ:=(J-I)*N+I;SUM:=BK IJ ;NKB:=J-COMAX+1;
'IF'NKB>0'THEN''GOTO'L2;
 NKB:=1;
L2:'IF'NKB-IL1>0'THEN''GOTO'L3;
'FOR'K:=NKB'STEP'1'UNTIL'IL1'DO'
'BEGIN'
NI:=(I-K)*N+K;NJ:=(J-K)*N+K;
SUM:=SUM-BK NI *BK NJ /BK K ;
'END';
L3:BK IJ :=SUM;
'END';'END';'END';
'PROCEDURE'NULL(A,M,N);
'VALUE'M,N; 'ARRAY'A; 'INTEGER'M,N;
'BEGIN''INTEGER'I,J;
'FOR'I:=1'STEP'1'UNTIL'M'DO'
'FOR'J:=1'STEP'1'UNTIL'N'DO'
A I,J :=0.0;
'END';
'PROCEDURE'THREEPOINT(SAMP);
'ARRAY'SAMP;
```

```
'BEGIN'
SAMP 1,1 :=.774596669241483;SAMP 2,1 :=.0;SAMP 3,1 :=-SAMP 1,1 ;
SAMP 1,2 :=.555555555555556;SAMP 2,2 :=.888888888888889;
SAMP 3,2 :=SAMP 1,2 ;
'END';

'PROCEDURE'NEWLINE(N);
'VALUE'N; 'INTEGER'N;
'BEGIN''INTEGER'I;
        'FOR'I:=1'STEP'1'UNTIL'N'DO'
        OUTPUT1 (6," /\," \);
'END';
'PROCEDURE'PRINTVECTOR(V,L);
'VALUE'L; 'ARRAY'V; 'INTEGER'L;
'BEGIN''INTEGER'I;
'FOR'I:=1'STEP'1'UNTIL'L'DO'
OUTPUT1 (6," \,V I );
NEWLINE(2);
'END';
'PROCEDURE'PRINTARRAY(A,X,Y);
'VALUE'X,Y; 'INTEGER'X,Y; 'ARRAY'A;
'BEGIN''INTEGER'I,J;
'FOR'I:=1'STEP'1'UNTIL'X'DO'
'BEGIN''FOR'J:=1'STEP'1'UNTIL'Y'DO'
        OUTPUT1 (6," \,A I,J );
        OUTPUT1 (6," /\," \);
'END';
NEWLINE(2);
'END';
'PROCEDURE'FORMNF(NF,NN,NODOF,R);
'VALUE'NN,NODOF,R;'INTEGER'NN,NODOF,R;'INTEGER''ARRAY'NF;
'BEGIN''INTEGER'I,J,K;
'FOR'I:=1'STEP'1'UNTIL'NN'DO'
'FOR'J:=1'STEP'1'UNTIL'NODOF'DO'
NF I,J :=1;
'FOR'I:=1'STEP'1'UNTIL'R'DO'
'BEGIN'
INPUT1 (5," \,K);
'FOR'J:=1'STEP'1'UNTIL'NODOF'DO'
'BEGIN'
        'INTEGER'L;
        INPUT1 (5," \,L);
'IF'L=0'THEN''GOTO'L1;
NF K,J :=0;
L1:'END';'END';
K:=1;
'FOR'I:=1'STEP'1'UNTIL'NN'DO'
'FOR'J:=1'STEP'1'UNTIL'NODOF'DO'
'BEGIN'
'IF'NF I,J =0'THEN''GOTO'L2;
NF I,J :=K; K:=K+1;
L2:'END';'END';
```

```
'PROCEDURE'FORMKVEC(BK,KM,G,N,DOF);
'VALUE'N,DOF;'INTEGER'N,DOF;
'ARRAY'BK,KM;'INTEGER''ARRAY'G;
'BEGIN''INTEGER'I,J,CD,VAL;
'FOR'I:=1'STEP'1'UNTIL'DOF'DO'
'BEGIN'
'IF'G I =0'THEN''GOTO'L1;
'FOR'J:=1'STEP'1'UNTIL'DOF'DO'
'BEGIN'
'IF'G J =0'THEN''GOTO'L2;
CD:=G J -G I +1;
'IF'CD<1'THEN''GOTO'L2;
VAL:=N*(CD-1)+G I ;
BK VAL :=BK VAL +KM I,J ;
L2:'END';L1:'END';'END';
'PROCEDURE'ELMATE(ELM,DOF);
'VALUE'DOF;
'INTEGER'DOF; 'ARRAY'ELM;
'BEGIN''INTEGER'I;
'FOR'I:=1'STEP'1'UNTIL'DOF'DO' ELM I,I :=1.0;
'END';
'PROCEDURE'PLGEOMETRY(P,Q,NXE,NYE,AA,BB,COORD,G,NF);
'VALUE'P,Q,NXE,NYE,AA,BB;'INTEGER'P,Q,NXE,NYE;'REAL'AA,BB;
'ARRAY'COORD;'INTEGER''ARRAY'G,NF;
'BEGIN''INTEGER'AO,AL,AM,AN;
AO:=(P-1)*(NYE+1)+Q; AL:=AO+1; AM:=P*(NYE+1)+Q; AN:=AM+1;
G 1 :=NF AL,1 ; G 2 :=NF AL,2 ; G 3 :=NF AL,3 ;
G 4 :=NF AO,1 ; G 5 :=NF AO,2 ; G 6 :=NF AO,3 ;
G 7 :=NF AM,1 ; G 8 :=NF AM,2 ; G 9 :=NF AM,3 ;
G 10 :=NF AN,1 ;G 11 :=NF AN,2 ;G 12 :=NF AN,3 ;
COORD(1,1):=(P-1)*AA;COORD(1,2):=(NYE-Q)*BB;
COORD 2,1 :=(P-1)*AA;COORD 2,2 :=(NYE-Q+1)*BB;
COORD 3,1 :=P*AA;COORD 3,2 :=(NYE-Q+1)*BB;
COORD 4,1 :=P*AA;COORD 4,2 :=(NYE-Q)*BB;
'END';
'PROCEDURE'DPLATE(DEE,E,V,H1);
'VALUE'E,V,H1;'REAL'E,V,H1;'ARRAY'DEE;
'BEGIN''INTEGER'I,J;'REAL'V1,VV;V1:=V ;VV:=(1-V);
DEE 1,1 :=DEE 2,2 :=1.0;DEE 3,3 :=VV/2;DEE 1,2 :=DEE 2,1 :=V1 ;
'FOR'I:=1'STEP'1'UNTIL'3'DO'
'FOR'J:=1'STEP'1'UNTIL'3'DO'
DEE I,J :=DEE I,J *E*(H1*H1*H1)/12/(1-V*V);
'END';
'PROCEDURE'FORMQU(QU,NODOF,DOF,SAMP,AA,BB,I,J);
'VALUE'NODOF,DOF,AA,BB,I,J;'INTEGER'NODOF,DOF,I,J;'REAL'AA,BB;
'ARRAY'QU,SAMP;
'BEGIN''REAL'ETA,XI;'INTEGER'K,L;
ETA:=SAMP I,1 *(-AA/2);XI:=SAMP J,1 *(-BB/2);
'FOR'K:=1'STEP'1'UNTIL'NODOF'DO'
'FOR'L:=1'STEP'1'UNTIL'DOF'DO'
QU K,L :=.0;
QU 1,4 :=QU 2,6 :=-2; QU 1,7 :=-6*ETA;
```

```
QU 1,8 :=-2*XI ; QU 1,11 :=QU 2,12 :=-6*ETA*XI;
QU 2,9 :=-2*ETA;QU 2,10 :=-6*XI; QU 3,8 :=4*ETA;
QU 3,5 :=2;
QU 3,9 :=4*XI;QU 3,11 :=6*(ETA**2);QU 3,12 :=6*(XI**2);
'END';
'PROCEDURE'FORMC(C,AA,BB,DOF);
'VALUE'DOF,AA,BB;'INTEGER'DOF;'REAL'AA,BB;'ARRAY'C;
'BEGIN''REAL'XI,YI,XJ,YJ,XK,YK,XL,YL;'INTEGER'K,L;
XI:=-AA/2;YI:=-BB/2;XJ:=-AA/2;YJ:=BB/2;
XL:=AA/2;YL:=-BB/2;XK:=AA/2;YK:=BB/2;
'FOR'K:=1'STEP'1'UNTIL'DOF'DO'
'FOR'L:=1'STEP'1'UNTIL'DOF'DO' C K,L :=0;
C 1,1 :=C 4,1 :=C 7,1 :=C 10,1 :=C 3,2 :=C 6,2 :=C 9,2 :=C 12,2 :=1;
C 2,3 :=C 5,3 :=C 8,3 :=C 11,3 :=-1;
C 1,2 :=XI;C 1,3 :=YI;C 1,4 :=XI*XI;C 1,5 :=XI*YI;C 1,6 :=YI*YI;
C 1,7 :=XI**3;C 1,8 :=(XI**2)*YI;C 1,9 :=(YI**2)*XI;C 1,10 :=YI**3;
C 1,11 :=(XI**3)*YI;C 1,12 :=XI*(YI**3);C 2,5 :=-XI;C 2,6 :=-2*YI;
C 2,8 :=-C 1,4 ;C 2,9 :=-2*C 1,5 ;C 2,10 :=-3*C 1,6 ;C 2,11 :=-C 1,7 ;
C 2,12 :=-3*XI*(YI**2);C 3,4 :=2*XI;C 3,5 :=YI;C 3,7 :=3*C 1,4 ;
C 3,8 :=2*C 1,5 ;C 3,9 :=C 1,5 ;C 3,11 :=3*C 1,8 ;C 3,12 :=C 1,10 ;
C 4,2 :=XJ;C 4,3 :=YJ;C 4,5 :=XJ*YJ;C 4,4 :=XJ**2;C 4,6 :=YJ**2;
C 4,7 :=XJ**3;C 4,8 :=(XJ**2)*YJ;C 4,9 :=XJ*(YJ**2);C 4,10 :=YJ**3;
C 4,11 :=(XJ**3)*YJ;C 4,12 :=XJ*(YJ**3);C 5,5 :=-C 4,2 ;C 5,6 :=-2*YJ;
C 5,8 :=-C 4,4 ;C 5,9 :=-2*C 4,5 ;C 5,10 :=-3*C 4,6 ;C 5,11 :=-C 4,7 ;
C 5,12 :=-3*C 4,9 ;C 6,4 :=2*C 4,2 ;C 6,5 :=YJ;C 6,7 :=3*C 4,4 ;
C 6,8 :=2*C 4,5 ;C 6,9 :=C 4,6 ;C 6,11 :=3*C 4,8 ;C 6,12 :=C 4,10 ;
C 7,2 :=XK;C 7,3 :=YK;C 7,4 :=XK**2;C 7,5 :=XK*YK;C 7,6 :=YK**2;
C 7,7 :=XK**3;C 7,8 :=(XK**2)*YK;C 7,9 :=XK*(YK**2);C 7,10 :=YK**3;
C 7,11 :=(XK**3)*YK;C 7,12 :=XK*(YK**3);C 8,5 :=-XK;C 8,6 :=-2*YK;
C 8,8 :=-XK**2;C 8,9 :=-2*C 7,5 ;C 8,10 :=-3*(YK**2);C 8,11 :=-XK**3;
C 8,12 :=-3*C 7,9 ;C 9,4 :=2*XK;C 9,5 :=YK;C 9,7 :=3*C 7,4 ;
C 9,8 :=C 7,5 *2;C 9,9 :=C 7,5 ;C 9,11 :=3*C 7,8 ;C 9,12 :=C 7,10 ;
C 10,2 :=XL;C 10,3 :=YL;C 10,4 :=XL**2;C 10,5 :=XL*YL;C 10,6 :=YL**2;
C 10,7 :=XL**3;C 10,8 :=(XL**2)*YL;C 10,9 :=XL*(YL**2);C 10,10 :=YL**3;
C 10,11 :=C 10,7 *YL;C 10,12 :=XL*C 10,10 ;C 11,5 :=-C 10,2 ;
C 11,6 :=-2*YL;C 11,8 :=-C 10,4 ;C 11,9 :=-2*C 10,5 ;
C 11,11 :=-C 10,7 ;C 11,12 :=-3*C 10,9 ;C 12,4 :=2*XL;C 12,5 :=YL;
C 12,7 :=3*C 10,4 ;C 12,8 :=2*C 10,5 ;C 12,9 :=C 10,6 ;
C 11,10 :=-3*(YL**2);C 12,11 :=3*C 10,8 ;C 12,12 :=YL**3;
'END';
'PROCEDURE'INVERTMAT(A,A1,N);
'VALUE'N;'INTEGER'N;'ARRAY'A,A1;
'BEGIN''INTEGER'S;'ARRAY'C 1:N,1:2*N ;'INTEGER'I,J;
        'FOR'I:=1'STEP'1'UNTIL'N'DO'
        'FOR'J:=1'STEP'1'UNTIL'2*N'DO'
        'IF'J<=N'THEN'C I,J :=A I,J
        'ELSE'
        'IF'J=N+I'THEN'C I,J :=1.0'ELSE'C I,J :=0.0;
        'FOR'I:=1'STEP'1'UNTIL'N'DO'
        'BEGIN''INTEGER'K,L,IND;J:=L:=I;IND:=S:=0;
   L1:'IF'C L,J =0'THEN'
              'BEGIN' IND:=1;
```

```
                    'IF'L<N'THEN''BEGIN'L:=L+1;
                                     'GOTO'L1;
                              'END'
                    'ELSE''BEGIN'S:=1;'GOTO'L2'END';
              'END)';
         'IF'IND=1'THEN''FOR'K:= 1'STEP'1'UNTIL'2*N'DO'
         'BEGIN''REAL'TEMP; TEMP:=C L,K ;C L,K :=C I,K ;C I,K :=TEMP;
         'END'K LOOP;
         'FOR'K:=2*N'STEP'-1'UNTIL'I'DO'
         C I,K :=C I,K /C I,I ;
         'FOR'L:=1'STEP'1'UNTIL'N'DO'
        'IF'L'NE'I'THEN'
         'FOR'K:=2*N'STEP'-1'UNTIL'I'DO'
          C L,K :=C L,K -C I,K *C L,I ;
         'END' I LOOP;
    'FOR'I:=1'STEP'1'UNTIL'N'DO'
    'FOR'J:=1'STEP'1'UNTIL'N'DO'
    A1 I,J :=C I,N+J ;
    L2:'END';
    'PROCEDURE'SEARCH(NF,NN,NODOF,N);
    'VALUE'NN,NODOF;'INTEGER''ARRAY'NF;
    'INTEGER'NN,NODOF,N;
    'BEGIN''INTEGER'I,J;
    N:=0;
    'FOR'I:=1'STEP'1'UNTIL'NN'DO'
    'FOR'J:=1'STEP'1'UNTIL'NODOF'DO'
    'BEGIN'
    'IF'NF I,J =0'THEN''GOTO'P1;
    N:=NF I,J ;
    P1:'END';
    OUTPUT1 (6,"/,"NO OF MM EQU. IS\," \\,N);
    NEWLINE(5);
    'END';
    'REAL'V,AA,BB,K1,K2,DET,QUOT,AREA,H1;
    'REAL'SIGMABAR,ITS,TSTEPS,DELOADS,F,A1,LL,FEL,N1,N2,N3;
    'INTEGER'I,J,K,L,P,Q,M,NXE,NYE,H,N,W,COMAX,RN,NL,NN,Z;
    'INTEGER'DOF,NODOF,NOD,R,T,TYPE,RR,II;
    TYPE:=1;
    INPUT6 (5," \,NXE,NYE,W,NN,RN,NL);
    INPUT4 (5," \,DOF,NODOF,H,T);
    INPUT2 (5," \,V,H1);
    INPUT5 (5," \,SIGMABAR,ITS,TSTEPS,DELOADS,FEL);
    NOD:=DOF/NODOF;
    OUTPUT1 (6," /,"NO. OF X ELEMENT IS\," \\,NXE);NEWLINE(1);
    OUTPUT1 (6," /,"NO. OF Y ELEMENT IS\," \\,NYE);NEWLINE(1);
    OUTPUT1 (6," /,"NO. OF NODES      =\," \\,NN );NEWLINE(1);
    OUTPUT1 (6," /,"POISSON. RATIO    =\," \\,V  );NEWLINE(1);
    OUTPUT1 (6," /,"THICKNESS         =\," \\,H1 );NEWLINE(1);
    'BEGIN''INTEGER''ARRAY'G 1:DOF ,NF 1:NN,1:NODOF ;
            FORMNF(NF,NN,NODOF,RN);
            SEARCH(NF,NN,NODOF,N);
            COMAX:=W+1; R:=N*COMAX;
```

```
          'BEGIN'
                'ARRAY'JAC,JAC1 1:T,1:T ,DER 1:T,1:DOF ;
                'ARRAY'BEE,DBEE 1:H,1:DOF ,KF,BTDB,KM 1:DOF,1:DOF ;
                'ARRAY'COORD 1:DOF,1:T ,E,RHO 1:NXE,1:NYE ;
                'ARRAY'FUN 1:NOD,1:4 ,LOADS 1:N ,ELD,VOL 1:DOF ;
                'ARRAY'EPS,EIGMA 1:H ,BK 1:R ,DEE 1:H,1:H ;
                'ARRAY'SAMP 1:3,1:2 ,MM,F1 1:R ,BT 1:DOF,1:H ;
                'ARRAY'EMM,ECM 1:DOF,1:DOF ,SX 1:NXE,1:NYE ;
               'ARRAY'QU 1:NODOF,1:DOF ,C,C1,C1T,QTDQ 1:DOF,1:DOF ;
                'ARRAY'QUT 1:DOF,1:H ,DQU 1:H,1:DOF ;
                'ARRAY'A,B 1:NXE,1:NYE ;
   'ARRAY'BOYLDS,BL 1:N ,BLOAD 1:DOF ,ELSO,SPL 1:H ,LD 1:N ;
   'ARRAY'OF 1:H ,DP_,DFDFT,DEEF 1:H,1:H ,SY,SXY 1:NXE,1:NYE ;
   'ARRAY'TOLAD 1:N ,SIGMA 1:H ,BM 1:DOF,1:H ,PL 1:H,1:H ;
  'ARRAY'XX,YY,XXY 1:NXE,1:NYE ;
                'FOR'I:=1'STEP'1'UNTIL'NXE'DO'
                'FOR'J:=1'STEP'1'UNTIL'NYE'DO' INPUT1 (5," \,A I,J );
                'FOR'I:=1'STEP'1'UNTIL'NXE'DO'
                'FOR'J:=1'STEP'1'UNTIL'NYE'DO' INPUT1 (5," \,B I,J );
                'FOR'I:=1'STEP'1'UNTIL'NXE'DO'
                'FOR'J:=1'STEP'1'UNTIL'NYE'DO' INPUT1 (5," \,E I,J );
               'FOR'I:=1'STEP'1'UNTIL'NXE'DO'
               'FOR'J:=1'STEP'1'UNTIL'NYE'DO' INPUT1 (5," \,RHO I,J );
               'FOR'I:=1'STEP'1'UNTIL'NXE'DO'
               'FOR'J:=1'STEP'1'UNTIL'NYE'DO' 'BEGIN'
                    SX I,J :=SY I,J :=SXY I,J :=0.0 'END';
             ''FOR'I:=1'STEP'1'UNTIL'N'DO' LOADS I :=0;
               'FOR'I:=1'STEP'1'UNTIL'R'DO'BK I :=MM I :=0.0;
                NULL(DEE,H,H);
               THREEPOINT(SAMP);
               'FOR'P:=1'STEP'1'UNTIL'NXE'DO'
               'FOR'Q:=1'STEP'1'UNTIL'NYE'DO'
               'BEGIN'AA:=BB:=0.0;AA:=A P,Q ;BB:=B P,Q ;
                    NULL(DEE,H,H);
                  AREA:=.0;K1:=E P,Q ;DPLATE(DEE,K1,V,H1);
                  PLGEOMETRY(P,Q,NXE,NYE,AA,BB,COORD,G,NF);
                  NULL(KM,DOF,DOF);NULL(EMM,DOF,DOF);
                  NULL(C,DOF,DOF);FORMC(C,AA,BB,DOF);
                    NULL(KF,DOF,DOF);NULL(C1,DOF,DOF)
                    ;NULL(C1T,DOF,DOF);
                  INVERTMAT(C,C1,DOF);TRANSMATRIX(C1T,C1,DOF,DOF);
                 'IF'TYPE=1'THEN'ELMATE(EMM,DOF);
                  'FOR'I:=1'STEP'1'UNTIL'3'DO'
                  'FOR'J:=1'STEP'1'UNTIL'3'DO'
                  'BEGIN'
                        K1:=SAMP I,2 ;K2:=SAMP J,2 ;
                  PLTWOBYTWO(JAC,JAC1,DET,AA,BB);
                        NULL(QU,NODOF,DOF);
                        NULL(QUT,DOF,NODOF);NULL(DQU,H,DOF);
                        NULL(QTDQ,DOF,DOF);
                        FORMQU(QU,NODOF,DOF,SAMP,AA,BB,I,J);
                        TRANSMATRIX(QUT,QU,H,DOF);
```

```
                              MATMULT(DEE,QU,DQU,H,H,DOF);
                              MATMULT(QUT,DQU,QTDQ,DOF,H,DOF);
                              QUOT:=DET*K1*K2;
                              AREA:=AREA+QUOT;
                              'FOR'K:=1'STEP'1'UNTIL'DOF'DO'
                              'FOR'L:=1'STEP'1'UNTIL'DOF'DO'
                              'BEGIN'
                                  QTDQ K,L :=QTDQ K,L *QUOT;
                              'END';
                              MATRIXADD(KM,QTDQ,DOF,DOF);
                         'END' OF GAUSS POINT;
                         MATMULT(KM,C1,KF,DOF,DOF,DOF);
                          NULL(KM,DOF,DOF);
                         MATMULT(C1T,KF,KM,DOF,DOF,DOF);
                         AREA:=AREA/4OD*RHO P,Q *H1;
                         'IF'TYPE=1'THEN''FOR'K:=1'STEP'1'UNTIL'DOF'DO'
                         EMM K,K :=EMM K,K *AREA;
                         FORMKVEC(BK,KM,G,N,DOF);FORMKVEC(MM,EMM,G,N,DOF);
                    'END' OF ELEMENT CONSIDERATION;
                    'FOR'RR:=1'STEP'1'UNTIL'R'DO'
                  F1 RR :=BK RR ; BANRED(F1,N,COMAX);
SIGMABAR:=SIGMABAR+(H1**3)/H1/6;
'FOR'II:=1'STEP'1'UNTIL'TSTEPS'DO'
'BEGIN''IF'II>1'THEN'FEL:=0.0;
OUTPUT1 (6," /,"INCR. NO.      \," \\,II);NEWLINE(1);
'FOR'RR:=1'STEP'1'UNTIL'N'DO'LOADS RR :=BOYLOS RR :=LD RR :=0.0;
'FOR'RR:=1'STEP'1'UNTIL'N'DO'BL RR :=0.0;
'FOR'RR:=1'STEP'1'UNTIL'DOF'DO'BLOAD RR :=0.0;
LD 143 :=LD 144 :=+(DELOADS+FEL)/2;
'FOR'Z:=1'STEP'1'UNTIL'ITS'DO'
 'BEGIN'
     OUTPUT1 (6," /,"ITERATION NO. \," \\,Z);NEWLINE(1);
'FOR'RR:=1'STEP'1'UNTIL'N'DO' 'BEGIN'
LOADS RR :=LD RR ;
'IF'II>1'THEN' 'BEGIN'
'IF'Z=1'THEN'LOADS RR :=LD RR  'ELSE'LOADS RR :=BOYLOS RR ;
                  'END';
 BOYLOS RR :=0.0;        'END';
     BACKONE(F1,LOADS,N,COMAX);BACKTWO(F1,LOADS,N,COMAX);
          'FOR'P:=1'STEP'1'UNTIL'NXE'DO'
          'FOR'Q:=1'STEP'1'UNTIL'NYE'DO'
          'BEGIN'  AA:=A P,Q ;BB:=B P,Q ;
          PLGEOMETRY(P,Q,NXE,NYE,AA,BB,COORD,G,NF);
          I:=2; J:=2;
          PLTWOBYTWO(JAC,JAC1,DET,AA,BB);
          NULL(QU,NODOF,DOF);
          FORMQU(QU,NODOF,DOF,SAMP,AA,BB,I,J);
          NULL(C,DOF,DOF);FORMC(C,AA,BB,DOF);
          INVERTMAT(C,C1,DOF);
          MATMULT(QU,C1,BEE,H,DOF,DOF);
           NULL(BT,DOF,H);NULL(BM,DOF,H);
          'FOR'H:=1'STEP'1'UNTIL'DOF'DO'
```

```
                ELD M :='IF'G M =0'THEN'0'ELSE'LOADS G M ;
                K1:=E P,Q ; DPLATE(DEE,K1,V,H1);
                MATVECMULT(BEE,ELD,H,DOF,EPS);
                MATVECMULT(DEE,EPS,H,H,SIGMA);
'IF'P=11'THEN'PRINTVECTOR(SIGMA,H);
EIGMA 1 :=SX P,Q +SIGMA 1 ;
EIGMA 2 :=SY P,Q +SIGMA 2 ;
EIGMA 3 :=SXY P,Q +SIGMA 3 ;
N1 :=(((SX P,Q -SY P,Q )**2+SX P,Q **2+SY P,Q **2)/2
          +3*(SXY P,Q **2))**(1/2)-SIGMABAR;
F:=(((EIGMA 1 -EIGMA 2 )**2+EIGMA 1 **2+EIGMA 2 **2)/2
          +3*(EIGMA 3 **2))**(1/2)-SIGMABAR;
 'IF'II=1'THEN''BEGIN'
 'IF'F< 0.0'THEN' 'BEGIN''IF'Z=ITS'THEN' 'BEGIN'
   SX P,Q :=SX P,Q +SIGMA 1 ;SY P,Q :=SY P,Q +SIGMA 2 ;
        SXY P,Q :=SXY P,Q +SIGMA 3 ;   'END'; 'END';
              'END';
 'IF'II>1'THEN''BEGIN'
 'IF'F< 0.0'THEN''BEGIN'SX P,Q :=SX P,Q +SIGMA 1 ;
 SY P,Q :=SY P,Q +SIGMA 2 ;SXY P,Q :=SXY P,Q +SIGMA 3 ;
              'END';'END';
'IF'P=11'THEN' 'BEGIN'
OUTPUT1 (6," /,"SX    =      \," \\,SX P,Q );NEWLINE(1);
              'END';
'IF'F>=0'THEN' 'BEGIN'
N2:=(F)/(F-N1);'IF'N2>1'THEN'N2:=1;N3:=1-N2;
'IF'N1<0'THEN' 'BEGIN'
XX P,Q :=N3*SIGMA 1 +SX P,Q ;YY P,Q :=N3*SIGMA 2 +SY P,Q ;
XXY P,Q :=N3*SIGMA 3 +SXY P,Q ;    'END';
DF 1 :=(EIGMA 1 -(EIGMA 1 +EIGMA 2 )/3)*3/(2*SIGMABAR);
DF 2 :=(EIGMA 2 -(EIGMA 1 +EIGMA 2 )/3)*3/(2*SIGMABAR);
DF 3 :=(3*EIGMA 3 /SIGMABAR);
NULL(DFDFT,H,H);
DFDFT 1,1 :=(DF 1 )**2; DFDFT 1,2 :=DFDFT 2,1 :=DF 1 *DF 2 ;
DFDFT 3,1 :=DFDFT 1,3 :=DF 1 *DF 3 ; DFDFT 2,2 :=(DF 2 )**2;
DFDFT 2,3 :=DFDFT 3,2 :=DF 2 *DF 3 ; DFDFT 3,3 :=(DF 3 )**2;
A1:=0000.0;
LL:=A1+((DF 1 )**2+2*V*DF 1 *DF 2 +(DF  2 )**2
      +( 1-V)*(DF 3 **2)/2)*K1*(H1**3)/12/(1-V**2);
NULL(DPL,H,H); NULL(DEEF,H,H);NULL(PL,H,H);
MATMULT(DFDFT,DEE,DEEF,H,H,H);MATMULT(DEE,DEEF,PL,H,H,H);
'FOR'K:=1'STEP'1'UNTIL'H'DO'
'FOR'L:=1'STEP'1'UNTIL'H'DO'
DPL K,L :=DEE K,L -PL K,L *N2/LL;
PRINTVECTOR(EPS,H);OUTPUT1 (6," /,"N2      =\,"\\,N2);NEWLINE(1);
OUTPUT1 (6," /,"N1    =\,"\\,N1);NEWLINE(1);
OUTPUT1 (6," /,"F=\,"\\,F);OUTPUT1 (6," /,"SIGMABAR=\,"\\,SIGMABAR);
NEWLINE(1); MATVECMULT(DPL,EPS,H,H,SPL);
'FOR'M:=1'STEP'1'UNTIL'H'DO' ELSO M :=(SIGMA M -SPL M );
SX P,Q :=EIGMA 1 -ELSO 1 ;SY P,Q :=EIGMA 2 -ELSO 2 ;
SXY P,Q :=EIGMA 3 -ELSO 3 ;
'IF'Z=ITS'THEN''BEGIN'
```

```
SX P,Q :=XX P,Q ;SY P,Q :=YY P,Q ;SXY P,Q :=XXY P,Q ;
'END';
PRINTVECTOR(ELSO,H);
'FOR'I:=1'STEP'1'UNTIL'3'DO'
'FOR'J:=1'STEP'1'UNTIL'3'DO'
                'BEGIN'
                 K1:=SAMP I,2 ;K2:=SAMP J,2 ;
                 PLTWOBYTWO(JAC,JAC1,DET,AA,BB);
                 NULL(QU,NODOF,DOF);NULL(BEE,H,DOF);NULL(BT,DOF,H);
                 FORMQU(QU,NODOF,DOF,SAMP,AA,BB,I,J);
                 MATMULT(QU,C1,BEE,H,DOF,DOF);
                 TRANSMATRIX(BT,BEE,H,DOF);
                 QUOT:=DET*K1*K2;
                 'FOR'K:=1'STEP'1'UNTIL'DOF'DO'
                 'FOR'L:=1'STEP'1'UNTIL'H  'DO'
                BT K,L :=BT K,L *QUOT;
                MATRIXADD(BM,BT,DOF,H);
                'END';
MATVECMULT(BM,ELSO,DOF,H,BLOAD);
'FOR'M:=1'STEP'1'UNTIL'DOF'DO'
'BEGIN'
BL G M  :='IF'G M =0'THEN'0'ELSE'BLOAD M ;BLOAD M :=0.0;
                    'END';          'END' ;
'FOR'RR:=1'STEP'1'UNTIL'N  'DO'     'BEGIN'
BOYLDS RR :=BOYLDS RR +BL  RR ; BL RR :=0.0;
                            'END';
'FOR'M:=1'STEP'1'UNTIL'H'DO'SIGMA H :=SIGMA H :=0.0;
'END';
'FOR'M:=261'STEP'1'UNTIL'316'DO'
OUTPUT1 (6," \,BOYLDS M );
NEWLINE(2);
'FOR'RR:=1'STEP'1'UNTIL'N'DO'TOLAD RR :=TOLAD RR +LOADS RR ;
'IF'II=1'THEN' 'BEGIN''FOR'RR:=1'STEP'1'UNTIL'N'DO'
                    TOLAD RR :=LOADS RR ;'END';
      'END';
'FOR'M:=261'STEP'1'UNTIL'316'DO'
OUTPUT1 (6," \,TOLAD M );
NEWLINE(2);
'END';'END';
 'END';  'END';
```

NO DIAGNOSTICS IN ABOVE COMPILATION
17671 WORDS WERE USED FOR THIS COMPILATION

CONTINUOUS CONNECTION TECHNIQUE PROGRAM

```
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION YWA(20,20),YWB(20,20),YWC(20,20),YWD(20,20),YWT(20,20)
      DIMENSION TKA(20,20),TKB(20,20),TKC(20,20),TKD(20,20),TKE(20,20)
      DIMENSION DA(20),DB(20),DC(20),DD(20),ZA(3),ZB(3),ZC(3),ZD(3)
      DIMENSION SN(20),WMASS(20),YN(20,20),THASS(20),YI(20,20)
      DIMENSION EA(20),EB(20),EC(20),FA(20),FB(20),FC(20),GA(20),GB(20),
     XGC(20),SNR(20)
      READ(5,3) N,NA,NB,NC,ND
      READ(5,3) KN,KA,KB,KC,KD
      READ(5,6) RA,RB,RC,RD
      READ(5,6) WLA,WLB,WLC,WLD
      READ(5,6) UHA,UHB,UHC,UHD
      READ(5,6) WIA,WIB,WIC,WID
      READ(5,6) WMA,WMB,WMC,WMD
      READ(5,1) H,WE,G
      READ(5,4)(WMASS(I),I=1,20)
      READ(5,4)(SN(I),I=1,20)
      READ(5,12) NM
      READ(5,6) UMA,UMB,UMC,UMD
      READ(5,6) ALA,ALB,ALC,ALD
      READ(5,6) ARA,ARB,ARC,ARD
      READ(5,6) RTA,RTB,RTC,RTD
      READ(5,6) UTHA,UTHB,UTHC,UTHD
      READ(5,6) THA,THB,THC,THD
      READ(5,6) PAA,PBA,PAB,PBB
      READ(5,6) PAC,PBC,PAD,PBD
      READ(5,2) CAA,CBA,CCA,CDA,AAA,ABA
      READ(5,2) CAB,CBB,CCB,CDB,AAB,ABB
      READ(5,2) CAC,CBC,CCC,CDC,AAC,ABC
      READ(5,2) CAD,CBD,CCD,CDD,AAD,ABD
      READ(5,1) (ZA(I),I=1,3)
      READ(5,1) (ZB(I),I=1,3)
      READ(5,1) (ZC(I),I=1,3)
      READ(5,1) (ZD(I),I=1,3)
      READ(5,4)(THASS(I),I=1,20)
      READ(5,4)(SNR(I),I=1,20)
    3 FORMAT(5I2)
    1 FORMAT(3D10.4)
    2 FORMAT(6D10.4)
    4 FORMAT(5D10.4)
    6 FORMAT(4D10.4)
   13 FORMAT(5X,10D10.4)
   12 FORMAT(I2)
      CALL FORMF(YWA,N,H,WIA,WMA,UHA,WE,KA)
      WRITE(6,13)((YWA(I,J),J=1,20),I=1,20)
      CALL FORMF(YWB,N,H,WIB,WMB,UHB,WE,KB)
      WRITE(6,13)((YWB(I,J),J=1,20),I=1,20)
      CALL FORMF(YWC,N,H,WIC,WMC,UHC,WE,KC)
      WRITE(6,13)((YWC(I,J),J=1,20),I=1,20)
      CALL FORMF(YWD,N,H,WID,WMD,UHD,WE,KD)
      WRITE(6,13)((YWD(I,J),J=1,20),I=1,20)
      CALL BSTIFF(YWA,YWB,YWC,YWD,YWT,RA,RB,RC,RD,N)
      CALL FORMT(TKA,KA,N,H,PAA,PBA,ARA,G,RTA,UTHA,THA)
      WRITE(6,13)((TKA(I,J),J=1,20),I=1,20)
      CALL FORMT(TKB,KB,N,H,PAB,PBB,ARB,G,RTB,UTHB,THB)
      WRITE(6,13)((TKB(I,J),J=1,20),I=1,20)
```

```
0056          CALL FORMT(TKC,KC,N,H,PAC,PBC,ARC,G,RTC,UTHC,THC)
0057          WRITE(6,13)((TKC(I,J),J=1,20),I=1,20)
0058          CALL FORMT(TKD,KD,N,H,PAD,PBD,ARD,G,RTD,UTHC,THD)
0059          WRITE(6,13)((TKD(I,J),J=1,20),I=1,20)
0060          CALL TSTIFF(YWA,YWB,YWC,YWD,TKA,TKB,TKC,TKD,N,RA,RB,RC,RD,TKE,
             XZA,ZB,ZC,ZD,NA,NB,NC,ND,DA,DB,DC,DD)
0061          DO 5 I=1,N
0062          TMASS(I)=TMASS(I)+WMASS(I)*TKC(I,1)**2
0063          DA(I)=ZA(1)+TKC(I,1)
0064          DB(I)=ZA(2)+TKC(I,1)
0065          DC(I)=ZA(3)+TKC(I,1)
0066          EA(I)=ZB(1)+TKC(I,1)
0067          EB(I)=ZB(2)+TKC(I,1)
0068          EC(I)=ZB(3)+TKC(I,1)
0069          FA(I)=ZC(1)+TKC(I,1)
0070          FB(I)=ZC(2)+TKC(I,1)
0071          FC(I)=ZC(3)+TKC(I,1)
0072          GA(I)=ZD(1)+TKC(I,1)
0073          GB(I)=ZD(2)+TKC(I,1)
0074          GC(I)=ZD(3)+TKC(I,1)
0075          DD(I)=TMASS(I)/WMASS(I)
0076          DD(I)=DSQRT(DD(I))
0077          SNR(I)=SNR(I)/DD(I)
0078     5    CONTINUE
0079          CALL WALLM(YM,WMASS,N)
0080          CALL WALLM(YI,TMASS,N)
0081          CALL INVERT(YWT,N)
0082          CALL INVERT(YM,N)
0083          CALL MULT(YM,YWT,TKC,N,N)
0084          CALL EIGN(TKC,N,NM,SN,WMASS)
0085          CALL INVERT(TKE,N)
0086          CALL INVERT(YI,N)
0087          CALLMULT(YI,TKE,TKD,N,N)
0088          CALL EIGN(TKD,N,NM,SNR,TMASS)
0089          IF(KN.EQ.0) GO TO 11
0090          CALL MAXR(TKD,DD,N,NM)
0091          CALL DEFL(TKC,N,NM,SN)
0092          CALL INVERT(YWA,N)
0093          CALL INVERT(YWB,N)
0094          CALL INVERT(YWC,N)
0095          CALL INVERT(YWD,N)
0096          CALL STRESS(KA,TKC,YWA,N,NM,H,CAA,CBA,CCA,CDA,WIA,AAA,ABA,WLA,
             XUHB,UMB,ALB,SN)
0097          CALL STRESS(KB,TKC,YWB,N,NM,H,CAB,CBB,CCB,CCB,WIB,AAB,ABB,WLB,
             XUHA,UMA,ALA,SN)
0098          CALL STRESS(KC,TKC,YWC,N,NM,H,CAC,CBC,CCC,CCC,WIC,AAC,ABC,WLC,
             XUHC,UMC,ALC,SN)
0099          CALL STRESS(KD,TKC,YWD,N,NM,H,CAD,CBD,CCD,CCD,WID,AAD,ABD,WLD,
             XUHD,UMD,ALD,SN)
0100          CALLRDEFL(TKD,TKE,SN,N,NM,DA)
0101          CALL STRESS(KA,TKE,YWA,N,NM,H,CAA,CBA,CCA,CDA,WIA,AAA,ABA,WLA,
             XUHA,UMA,ALA,SN)
0102          IF(NA.EQ.1) GO TO 8
0103          CALL RDEFL(TKD,TKE,SN,N,NM,DB)
0104          CALL STRESS(KA,TKE,YWA,N,NM,H,CAA,CBA,CCA,CDA,WIA,AAA,ABA,WLA,
             XUHA,UMA,ALA,SN)
```

```
0105          IF(NA.EQ.2) GO TO 8
0106          CALL RDEFL(TKD,TKE,SN,N,NM,DC)
0107          CALL STRESS(KA,TKE,YWA,N,NM,H,CAA,CBA,CCA,CDA,WIA,AAA,ABA,WLA,
             XUHA,UMA,ALA,SN)
0108    8     CONTINUE
0109          CALL RDEFL(TKD,TKE,SN,N,NM,EA)
0110          CALL STRESS(KB,TKE,YWB,N,NM,H,CAB,CBB,CCB,CCB,WIB,AAB,ABB,WLB,
             XUHB,UMB,ALB,SN)
0111          IF(NB.EQ.1) GO TO 9
0112          CALL RDEFL(TKD,TKE,SN,N,NM,EB)
0113          CALL STRESS(KB,TKE,YWB,N,NM,H,CAB,CBB,CCB,CDB,WIB,AAB,ABB,WLB,
             XUHB,UMB,ALB,SN)
0114          IF(NB.EQ.2) GO TO 9
0115          CALL RDEFL(TKD,TKE,SN,N,NM,EC)
0116          CALL STRESS(KB,TKE,YWB,N,NM,H,CAB,CBB,CCB,CDB,WIB,AAB,ABB,WLB,
             XUHB,UMB,ALB,SN)
0117    9     CONTINUE
0118          CALL RDEFL(TKD,TKE,SN,N,NM,FA)
0119          CALL STRESS(KC,TKE,YWC,N,NM,H,CAC,CBC,CCC,CCC,WIC,AAC,ABC,WLC,
             XUHC,UMC,ALC,SN)
0120          IF(NC.EQ.1) GO TO 10
0121          CALL RDEFL(TKD,TKE,SN,N,NM,FB)
0122          CALL STRESS(KC,TKE,YWC,N,NM,H,CAC,CBC,CCC,CCC,WIC,AAC,ABC,WLC,
             XUHC,UMC,ALC,SN)
0123          IF(NC.EQ.2) GO TO 10
0124          CALL RDEFL(TKD,TKE,SN,N,NM,FC)
0125          CALL STRESS(KC,TKE,YWC,N,NM,H,CAC,CBC,CCC,CDC,WIC,AAC,ABC,WLC,
             XUHC,UMC,ALC,SN)
0126   10     CONTINUE
0127          CALL RDEFL(TKD,TKE,SN,N,NM,GA)
0128          CALL STRESS(KD,TKE,YWD,N,NM,H,CAD,CBD,CCD,CCD,WID,AAD,ABD,WLD,
             XUHD,UMD,ALD,SN)
0129          IF(ND.EQ.1) GO TO 11
0130          CALL RDEFL(TKD,TKE,SN,N,NM,GB)
0131          CALL STRESS(KD,TKE,YWD,N,NM,H,CAD,CBD,CCD,CDD,WID,AAD,ABD,WLD,
             XUHD,UMD,ALD,SN)
0132          IF(ND.EQ.2) GO TO 11
0133          CALL RDEFL(TKD,TKE,SN,N,NM,GC)
0134          CALL STRESS(KD,TKE,YWD,N,NM,H,CAD,CBD,CCD,CCD,WID,AAD,ABD,WLD,
             XUHD,UMD,ALD,SN)
0135   11     CONTINUE
0136          STOP
0137          END
```

```
      SUBROUTINE FORMF(Y,N,H,W,WM,UH,WE,K)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION Y(N,1),YZ(20,20)
      IF(K.EQ.6) GO TO 1
      IF(K.EQ.2) GO TO 1
      CALL WALLF(Y,N,H,W,WM,UH,WE)
      IF(K.NE.0) GO TO 2
      W=W*3/4
      CALL CANTF(YZ,N,H,WE,W)
      DO 3 I=1,N
      DO 3 J=1,N
      Y(I,J)=Y(I,J)+YZ(I,J)
    3 CONTINUE
      GO TO 2
    1 CALL CANTF(Y,N,H,WE,W)
    2 CONTINUE
      RETURN
      END

*OPTIONS IN EFFECT*  NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
*OPTIONS IN EFFECT*  NAME = FORMF   , LINECNT =       58
*STATISTICS*     SOURCE STATEMENTS =        18,PROGRAM SIZE =      4108
*STATISTICS*   NO DIAGNOSTICS GENERATED
```

```
0001                  SUBROUTINE MULT(A,B,Z,K,L)
0002                  IMPLICIT REAL*8(A-H,O-Z)
              C       MULTIPLY MATRICES A AND B  A HAS L ROWS AND L COLUMNS
              C       B HAS L ROWS K COLUMNS.
0003                  DIMENSION Z(L,1),A(L,1),B(L,1)
0004                  I=0
0005                  JA=0
0006          401     I=I+1
0007                  JA=JA+1
0008                  IA=0
0009                  DO 402 JB=1,K
0010                  Z(JA,JB)=0.0
0011                  IA=IA+1
0012                  DO 403 J=1,L
0013                  Z(JA,JB)=Z(JA,JB)+A(I,J)*B(J,IA)
0014          403     CONTINUE
0015          402     CONTINUE
0016                  IF(I-L)401,404,404
0017          404     CONTINUE
0018                  RETURN
0019                  END

*OPTIONS IN EFFECT*  NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
*OPTIONS IN EFFECT*  NAME = MULT    , LINECNT =       58
*STATISTICS*     SOURCE STATEMENTS =       19,PROGRAM SIZE =      840
*STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
      SUBROUTINE EIGENP(N,NM,A,T,EVR,EVI,VECR,VECI,INDIC)
      IMPLICIT REAL*8(A-H,O-Z)
      DOUBLE PRECISION D1,D2,D3,PRFACT
      DIMENSION A(NM,1),VECR(NM,1),VECI(NM,1),
     1EVR(1),EVI(1),INDIC(1)
      DIMENSION IWORK(100),LOCAL(100),PRFACT(100)
     1,SUBDIA(100),WORK1(100),WORK2(100),WORK(100)
      IF(N.NE.1)GO TO 1
      EVR(1) = A(1,1)
      EVI(1) = 0.0
      VECR(1,1) = 1.0
      VECI(1,1) = 0.0
      INDIC(1) = 2
      GO TO 25
    1 CALL ESCALE(N,NM,A,VECI,PRFACT,ENORM)
      EX=DEXP(-T*DLOG(2.0D0))
      CALL HESQR(N,NM,A,VECI,EVR,EVI,SUBDIA,INDIC,EPS,EX)
      J = N
      I = 1
      LOCAL(1) = 1
      IF(J.EQ.1)GO TO 4
    2 IF(DABS(SUBDIA(J-1)).GT.EPS) GO TO 3
      I = I+1
      LOCAL(I)=0
    3 J = J-1
      LOCAL(I)=LOCAL(I)+1
      IF(J.NE.1)GO TO 2
    4 K = 1
      KON = 0
      L = LOCAL(1)
      M = N
      DO 10 I=1,N
        IVEC = N-I+1
        IF(I.LE.L)GO TO 5
        K = K+1
        M = N-L
        L = L+LOCAL(K)
    5   IF(INDIC(IVEC).EQ.0)GO TO 10
        IF(EVI(IVEC).NE.0.0)GO TO 8
        DO 7 K1=1,M
          DO 6 L1=K1,M
    6       A(K1,L1) = VECI(K1,L1)
          IF(K1.EQ.1)GO TO 7
          A(K1,K1-1) = SUBDIA(K1-1)
    7     CONTINUE
        CALL REALVE(N,NM,M,IVEC,A,VECR,EVR,EVI,IWORK,
     1  WORK,INDIC,EPS,EX)
        GO TO 10
    8   IF(KON.NE.0)GO TO 9
        KON = 1
        CALL COMPVE(N,NM,M,IVEC,A,VECR,VECI,EVR,EVI,INDIC,
     1  IWORK,SUBDIA,WORK1,WORK2,WORK,EPS,EX)
        GO TO 10
    9   KON = 0
   10   CONTINUE
```

```
      DO 12 I=1,N
        DO 11 J=I,N
          A(I,J) = 0.0
   11     A(J,I) = 0.0
   12   A(I,I) = 1.0
      IF(N.LE.2)GO TO 15
      M = N-2
      DO 14 K=1,M
        L = K+1
        DO 14 J=2,N
          D1 = 0.0
          DO 13 I=L,N
            D2 = VECI(I,K)
   13     D1 = D1+ D2*A(J,I)
          DO 14 I=L,N
   14       A(J,I) = A(J,I)-VECI(I,K)*D1
   15 KON = 1
      DO 24 I=1,N
        L = 0
        IF(EVI(I).EQ.0.0) GO TO 16
        L = 1
        IF(KON.EQ.0)GO TO 16
        KON = 0
        GO TO 24
   16   DO 18 J=1,N
          D1 = 0.0
          D2 = 0.0
          DO 17 K=1,N
            D3 = A(J,K)
            D1 = D1+D3*VECR(K,I)
            IF(L.EQ.0)GO TO 17
            D2 = D2+D3*VECR(K,I-1)
   17       CONTINUE
          WORK(J) = D1/PRFACT(J)
          IF(L.EQ.0) GO TO 18
          SUBDIA(J)=D2/PRFACT(J)
   18     CONTINUE
        IF(L.EQ.1)GO TO 21
        D1 = 0.0
        DO 19 M=1,N
   19     D1 = D1+WORK(M)**2
        D1 = DSQRT(D1)
        DO 20 M=1,N
          VECI(M,I) = 0.0
   20   VECR(M,I) = WORK(M)/D1
        EVR(I) = EVR(I)*ENORM
        GO TO 24
   21   KON = 1
        EVR(I) = EVR(I)*ENORM
        EVR(I-1) = EVR(I)
        EVI(I) = EVI(I)*ENORM
        EVI(I-1) =-EVI(I)
        R = 0.0
        DO 22 J=1,N
          R1 = WORK(J)**2 + SUBDIA(J)**2
          IF(R.GE.R1)GO TO 22
```

```
                R = R1
                L = J
   22     CONTINUE
          D3 = WORK(L)
            R1 = SUBDIA(L)
          DO 23 J=1,N
            D1 = WORK(J)
            D2 = SUBDIA(J)
            VECR(J,I) = (D1*D3+D2*R1)/R
            VECI(J,I) = (D2*D3-D1*R1)/R
            VECR(J,I-1) = VECR(J,I)
   23       VECI(J,I-1) =-VECI(J,I)
   24     CONTINUE
   25 RETURN
        END

 *OPTIONS IN EFFECT*  NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
 *OPTIONS IN EFFECT*  NAME = EIGENP  , LINECNT =       58
 *STATISTICS*    SOURCE STATEMENTS =      122,PROGRAM SIZE =      8660
 *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
      SUBROUTINE ESCALE(N,NM,A,H,PRFACT,ENORM)
      IMPLICIT REAL*8(A-H,O-Z)
      DOUBLE PRECISION COLUMN,FACTOR,FNORM,PRFACT,Q,ROW
      DIMENSION A(NM,1),H(NM,1),PRFACT(1)
      DO 2 I=1,N
        DO 1 J=1,N
    1     H(I,J) = A(I,J)
    2   PRFACT(I)= 1.0
      BOUND1 = 0.75
      BOUND2 = 1.33
      ITER = 0
    3 NCOUNT = 0
      DO 8 I=1,N
        COLUMN = 0.0
        ROW = 0.0
        DO 4 J=1,N
          IF(I.EQ.J)GO TO 4
      COLUMN=COLUMN+DABS(A(J,I))
      ROW=ROW+DABS(A(I,J))
    4     CONTINUE
        IF(COLUMN.EQ.0.0)GO TO 5
        IF(ROW.EQ.0.0)GO TO 5
        Q = COLUMN/ROW
        IF(Q.LT.BOUND1)GO TO 6
        IF(Q.GT.BOUND2)GO TO 6
    5   NCOUNT = NCOUNT + 1
        GO TO 8
    6   FACTOR = DSQRT(Q)
        DO 7 J=1,N
          IF(I.EQ.J)GO TO 7
          A(I,J) = A(I,J)*FACTOR
          A(J,I) = A(J,I)/FACTOR
    7   CONTINUE
        PRFACT(I) = PRFACT(I)*FACTOR
    8   CONTINUE
      ITER = ITER+1
      IF(ITER.GT.30)GO TO 11
      IF(NCOUNT.LT.N)GO TO 3
      FNORM = 0.0
      DO 9 I=1,N
        DO 9 J=1,N
          Q = A(I,J)
    9     FNORM = FNORM+Q*Q
      FNORM = DSQRT(FNORM)
      DO 10 I=1,N
        DO 10 J=1,N
   10     A(I,J)=A(I,J)/FNORM
      ENORM = FNORM
      GO TO 13
   11 DO 12 I=1,N
        PRFACT(I)=1.0
        DO 12 J=1,N
   12     A(I,J) = H(I,J)
      ENORM = 1.0
   13 RETURN
```

```
0056            END

     *OPTIONS IN EFFECT*  NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
     *OPTIONS IN EFFECT*  NAME = ESCALE  , LINECNT =       58
     *STATISTICS*    SOURCE STATEMENTS =       56,PROGRAM SIZE =      1826
     *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
      SUBROUTINE HESQR(N,NM,A,H,EVR,EVI,SUBDIA,INCIC,EPS,EX)
      IMPLICIT REAL*8(A-H,O-Z)
      DOUBLE PRECISION S,SR,SR2,X,Y,Z
      DIMENSION A(NM,1),H(NM,1),EVR(1),EVI(1),SUBCIA(1),INDIC(1)
      IF(N-2)14,1,2
    1 SUBDIA(1) = A(2,1)
      GO TO 14
    2 M = N-2
      DO 12 K=1,M
        L = K+1
        S = 0.0
        DO 3 I=L,N
          H(I,K) = A(I,K)
    3   S=S+DABS(A(I,K))
      IF(S.NE.DABS(A(K+1,K))) GO TO 4
        H(K+1,K) = 0.0
        GO TO 12
    4   SR2 = 0.0
        DO 5 I=L,N
          SR = A(I,K)
          SR = SR/S
          A(I,K) = SR
    5     SR2 = SR2+SR*SR
        SR = DSQRT(SR2)
        IF(A(L,K).LT.0.0)GO TO 6
        SR = -SR
    6   SR2 = SR2-SR*A(L,K)
        A(L,K) = A(L,K)-SR
        H(L,K) = H(L,K)-SR*S
        SUBDIA(K) = SR*S
        X = S*DSQRT(SR2)
        DO 7 I=L,N
          H(I,K) = H(I,K)/X
    7     SUBDIA(I) = A(I,K)/SR2
        DO 9 J=L,N
          SR = 0.0
          DO 8 I=L,N
    8       SR = SR+A(I,K)*A(I,J)
          DO 9 I=L,N
    9       A(I,J) = A(I,J)-SUBDIA(I)*SR
        DO 11 J=1,N
          SR=0.0
          DO 10 I=L,N
   10       SR = SR+A(J,I)*A(I,K)
          DO 11 I=L,N
   11       A(J,I) = A(J,I)-SUBDIA(I)*SR
   12   CONTINUE
      DO 13 K=1,M
   13 A(K+1,K) = SUBDIA(K)
      SUBDIA(N-1) = A(N,N-1)
   14 EPS = 0.0
      DO 15 K=1,N
        INDIC(K) = 0
        IF(K.NE.N)EPS = EPS+SUBDIA(K)**2
        DO 15 I=K,N
```

```
          H(K,I) = A(K,I)
   15     EPS = EPS + A(K,I)**2
      EPS=EX*DSQRT(EPS)
      SHIFT = A(N,N-1)
      IF(N.LE.2)SHIFT = 0.0
      IF(A(N,N).NE.0.0)SHIFT = 0.0
      IF(A(N-1,N).NE.0.0)SHIFT = 0.0
      IF(A(N-1,N-1).NE.0.0)SHIFT = 0.0
      M = N
      NS= 0
      MAXST = N*10
      DO 16 I=2,N
        DO 16 K=I,N
          IF(A(I-1,K).NE.0.0)GO TO 18
   16     CONTINUE
      DO 17 I=1,N
        INDIC(I)=1
        EVR(I) = A(I,I)
   17   EVI(I) = -0.0
      GO TO 37
   18 K=M-1
      M1=K
      I=K
      IF(K)37,34,19
   19 IF(DABS(A(M,K)).LE.EPS) GO TO 34
      IF(M-2.EQ.0)GO TO 35
   20   I = I-1
      IF(DABS(A(K,I)).LE.EPS) GO TO 21
        K = I
        IF(K.GT.1)GO TO 20
   21 IF(K.EQ.M1)GO TO 35
      S = A(M,M)+A(M1,M1)+SHIFT
      SR= A(M,M)*A(M1,M1)-A(M,M1)*A(M1,M)+0.25*SHIFT**2
      A(K+2,K) = 0.0
      X = A(K,K)*(A(K,K)-S)+A(K,K+1)*A(K+1,K)+SR
      Y = A(K+1,K)*(A(K,K)+A(K+1,K+1)-S)
      R = DABS(X)+DABS(Y)
      IF(R.EQ.0.0)SHIFT = A(M,M-1)
      Z = A(K+2,K+1)*A(K+1,K)
      SHIFT = 0.0
      NS = NS+1
      DO 33 I=K,M1
        IF(I.EQ.K)GO TO 22
        X = A(I,I-1)
        Y = A(I+1,I-1)
        Z = 0.0
        IF(I+2.GT.M)GO TO 22
        Z = A(I+2,I-1)
   22   SR2 = DABS(X)+DABS(Y)+DABS(Z)
        IF(SR2.EQ.0.0)GO TO 23
        X = X/SR2
        Y = Y/SR2
        Z = Z/SR2
   23   S = DSQRT(X*X + Y*Y + Z*Z)
        IF(X.LT.0.0)GO TO 24
        S = -S
```

```
 24   IF(I.EQ.K)GO TO 25
      A(I,I-1) = S*SR2
 25   IF(SR2.NE.0.0)GO TO 26
      IF(I+3.GT.M)GO TO 33
      GO TO 32
 26   SR = 1.0-X/S
      S = X-S
      X = Y/S
      Y = Z/S
      DO 28 J=I,M
        S = A(I,J)+A(I+1,J)*X
        IF(I+2.GT.M)GO TO 27
        S = S+A(I+2,J)*Y
 27     S = S*SR
        A(I,J) = A(I,J)-S
        A(I+1,J) = A(I+1,J)-S*X
        IF(I+2.GT.M)GO TO 28
        A(I+2,J) = A(I+2,J)-S*Y
 28     CONTINUE
      L = I+2
      IF(I.LT.M1)GO TO 29
      L = M
 29   DO 31 J=K,L
        S = A(J,I)+A(J,I+1)*X
        IF(I+2.GT.M)GO TO 30
        S = S + A(J,I+2)*Y
 30     S = S*SR
        A(J,I) = A(J,I)-S
        A(J,I+1)=A(J,I+1)-S*X
        IF(I+2.GT.M)GO TO 31
        A(J,I+2)=A(J,I+2)-S*Y
 31     CONTINUE
      IF(I+3.GT.M)GO TO 33
      S = -A(I+3,I+2)*Y*SR
 32   A(I+3,I) = S
      A(I+3,I+1) = S*X
      A(I+3,I+2) = S*Y + A(I+3,I+2)
 33   CONTINUE
     IF(NS.GT.MAXST)GO TO 37
     GO TO 18
 34  EVR(M) = A(M,M)
     EVI(M) = 0.0
     INDIC(M) = 1
     M = K
     IF(R.EQ.0.0)GO TO 21
     GO TO 18
 35  R = 0.5*(A(K,K)+A(M,M))
     S = 0.5*(A(M,M)-A(K,K))
     S = S*S + A(K,M)*A(M,K)
     INDIC(K) = 1
     INDIC(M) = 1
     IF(S.LT.0.0)GO TO 36
     T = DSQRT(S)
     EVR(K) = R-T
     EVR(M) = R+T
     EVI(K) = 0.0
```

```
 24   IF(I.EQ.K)GO TO 25
      A(I,I-1) = S*SR2
 25   IF(SR2.NE.0.0)GO TO 26
      IF(I+3.GT.M)GO TO 33
      GO TO 32
 26   SR = 1.0-X/S
      S = X-S
      X = Y/S
      Y = Z/S
      DO 28 J=I,M
        S = A(I,J)+A(I+1,J)*X
        IF(I+2.GT.M)GO TO 27
        S = S+A(I+2,J)*Y
 27     S = S*SR
        A(I,J) = A(I,J)-S
        A(I+1,J) = A(I+1,J)-S*X
        IF(I+2.GT.M)GO TO 28
        A(I+2,J) = A(I+2,J)-S*Y
 28     CONTINUE
      L = I+2
      IF(I.LT.M1)GO TO 29
      L = M
 29   DO 31 J=K,L
        S = A(J,I)+A(J,I+1)*X
        IF(I+2.GT.M)GO TO 30
        S = S + A(J,I+2)*Y
 30     S = S*SR
        A(J,I) = A(J,I)-S
        A(J,I+1)=A(J,I+1)-S*X
        IF(I+2.GT.M)GO TO 31
        A(J,I+2)=A(J,I+2)-S*Y
 31     CONTINUE
      IF(I+3.GT.M)GO TO 33
      S = -A(I+3,I+2)*Y*SR
 32   A(I+3,I) = S
      A(I+3,I+1) = S*X
      A(I+3,I+2) = S*Y + A(I+3,I+2)
 33   CONTINUE
     IF(NS.GT.MAXST)GO TO 37
     GO TO 18
 34  EVR(M) = A(M,M)
     EVI(M) = 0.0
     INDIC(M) = 1
     M = K
     IF(R.EQ.0.0)GO TO 21
     GO TO 18
 35  R = 0.5*(A(K,K)+A(M,M))
     S = 0.5*(A(M,M)-A(K,K))
     S = S*S + A(K,M)*A(M,K)
     INDIC(K) = 1
     INDIC(M) = 1
     IF(S.LT.0.0)GO TO 36
     T = DSQRT(S)
     EVR(K) = R-T
     EVR(M) = R+T
     EVI(K) = 0.0
```

```
      EVI(M) = 0.0
      M = M-2
      GO TO 18
   36 T = DSQRT(-S)
      EVR(K) = R
      EVI(K) = T
      EVR(M) = R
      EVI(M) = -T
      M = M-2
      GO TO 18
   37 RETURN
      END

*OPTIONS IN EFFECT*  NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
*OPTIONS IN EFFECT*  NAME = HESOR    , LINECNT =       58
*STATISTICS*    SOURCE STATEMENTS =       179,PROGRAM SIZE =      6746
*STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
0001          SUBROUTINE REALVE(N,NM,M,IVEC,A,VECR,EVR,EVI,
             1IWORK,WORK,INDIC,EPS,EX)
0002          IMPLICIT REAL*8(A-H,O-Z)
0003          DOUBLE PRECISION S,SR
0004          DIMENSION A(NM,1),VECR(NM,1),EVR(1)
0005          DIMENSION EVI(1),IWORK(1),WORK(1),INDIC(1)
0006          VECR(1,IVEC) = 1.0
0007          IF(M.EQ.1)GO TO 24
0008          EVALUE = EVR(IVEC)
0009          IF(IVEC.EQ.M)GO TO 2
0010          K = IVEC+1
0011          R = 0.0
0012          DO 1 I=K,M
0013            IF(EVALUE.NE.EVR(I))GO TO 1
0014            IF(EVI(I).NE.0.0)GO TO 1
0015            R = R+3.0
0016        1   CONTINUE
0017          EVALUE = EVALUE+R*EX
0018        2 DO 3 K=1,M
0019        3   A(K,K) = A(K,K)-EVALUE
0020          K = M-1
0021          DO 8 I=1,K
0022            L = I+1
0023            IWORK(I) = 0
0024            IF(A(I+1,I).NE.0.0)GO TO 4
0025            IF(A(I,I).NE.0.0)GO TO 8
0026            A(I,I) = EPS
0027            GO TO 8
0028    4     IF(DABS(A(I,I)).GE.DABS(A(I+1,I))) GO TO 6
0029            IWORK(I) = 1
0030            DO 5 J=I,M
0031              R = A(I,J)
0032              A(I,J) = A(I+1,J)
0033        5     A(I+1,J) = R
0034        6   R = -A(I+1,I)/A(I,I)
0035            A(I+1,I) = R
0036            DO 7 J=L,M
0037        7     A(I+1,J) = A(I+1,J)+R*A(I,J)
0038        8   CONTINUE
0039          IF(A(M,M).NE.0.0)GO TO 9
0040          A(M,M) = EPS
0041        9 DO 11 I=1,N
0042            IF(I.GT.M)GO TO 10
0043            WORK(I) = 1.0
0044            GO TO 11
0045       10   WORK(I) = 0.0
0046       11   CONTINUE
0047          BOUND = 0.01/(EX * FLOAT(N))
0048          NS = 0
0049          ITER = 1
0050       12 R = 0.0
0051          DO 15 I=1,M
0052            J = M-I+1
0053            S = WORK(J)
0054            IF(J.EQ.M)GO TO 14
```

```
          L = J+1
          DO 13 K=L,M
            SR = WORK(K)
   13       S  = S - SR*A(J,K)
   14     WORK(J) = S/A(J,J)
      T=DABS(WORK(J))
          IF(R.GE.T)GO TO 15
          R = T
   15     CONTINUE
      DO 16 I=1,M
   16     WORK(I) = WORK(I)/R
      R1 = 0.0
      DO 18 I=1,M
          T = 0.0
          DO 17 J=I,M
   17       T = T+A(I,J)*WORK(J)
      T=DABS(T)
          IF(R1.GE.T)GO TO 18
          R1= T
   18     CONTINUE
      IF(ITER.EQ.1)GO TO 19
      IF(PREVIS.LE.R1)GO TO 24
   19 DO 20 I=1,M
   20     VECR(I,IVEC) = WORK(I)
      PREVIS = R1
      IF(NS.EQ.1)GO TO 24
      IF(ITER.GT.6)GO TO 25
      ITER = ITER+1
      IF(R.LT.BOUND)GO TO 21
      NS = 1
   21 K = M-1
      DO 23 I=1,K
          R = WORK(I+1)
          IF(IWORK(I).EQ.0)GO TO 22
          WORK(I+1)=WORK(I)+WORK(I+1)*A(I+1,I)
          WORK(I) = R
          GO TO 23
   22     WORK(I+1)=WORK(I+1)+WORK(I)*A(I+1,I)
   23     CONTINUE
      GO TO 12
   24 INDIC(IVEC) = 2
   25 IF(M.EQ.N)GO TO 27
      J = M+1
      DO 26 I=J,N
   26     VECR(I,IVEC) = 0.0
   27 RETURN
      END
```

```
0001            SUBROUTINE COMPVE(N,NM,M,IVEC,A,VECR,H,EVR,EVI,INDIC,
               1IWORK,SUBDIA,WORK1,WORK2,WORK,EPS,EX)
0002            IMPLICIT REAL*8(A-H,O-Z)
0003            DOUBLE PRECISION D,D1
0004            DIMENSION A(NM,1),VECR(NM,1),H(NM,1),EVR(1),EVI(1),
               1INDIC(1),IWORK(1),SUBDIA(1),WORK1(1),WORK2(1),
               2WORK(1)
0005            FKSI = EVR(IVEC)
0006            ETA  = EVI(IVEC)
0007            IF(IVEC.EQ.M)GO TO 2
0008            K = IVEC+1
0009            R = 0.0
0010            DO 1 I=K,M
0011              IF(FKSI.NE.EVR(I))GO TO 1
0012            IF(DABS(ETA).NE.DABS(EVI(I))) GO TO 1
0013              R = R + 3.0
0014          1   CONTINUE
0015            R = R*EX
0016            FKSI = FKSI+R
0017            ETA  = ETA +R
0018          2 R = FKSI*FKSI + ETA*ETA
0019            S = 2.0*FKSI
0020            L = M-1
0021            DO 5 I=1,M
0022              DO 4 J=I,M
0023                D = 0.0
0024                A(J,I) = 0.0
0025                DO 3 K = I,J
0026          3       D = D+H(I,K)*H(K,J)
0027          4     A(I,J) = D-S*H(I,J)
0028          5   A(I,I) = A(I,I)+R
0029            DO 9 I=1,L
0030              R = SUBDIA(I)
0031              A(I+1,I) = -S*R
0032              I1 = I+1
0033              DO 6 J=1,I1
0034          6     A(J,I) = A(J,I)+R*H(J,I+1)
0035              IF(I.EQ.1)GO TO 7
0036              A(I+1,I-1) = R*SUBDIA(I-1)
0037          7   DO 8 J=I,M
0038          8     A(I+1,J) = A(I+1,J)+R*H(I,J)
0039          9   CONTINUE
0040            K = M-1
0041            DO 18 I=1,K
0042              I1 = I+1
0043              I2 = I+2
0044              IWORK(I) = 0
0045              IF(I.EQ.K)GO TO 10
0046              IF(A(I+2,I).NE.0.0)GO TO 11
0047         10   IF(A(I+1,I).NE.0.0)GO TO 11
0048              IF(A(I,I).NE.0.0)GO TO 18
0049              A(I,I) = EPS
0050              GO TO 18
0051         11   IF(I.EQ.K)GO TO 12
0052            IF(DABS(A(I+1,I)).GE.DABS(A(I+2,I))) GO TO 12
```

```
      IF(DABS(A(I,I)).GE.DABS(A(I+2,I))) GO TO 16
        L = I+2
        IWORK(I) = 2
        GO TO 13
   12 IF(DABS(A(I,I)).GE.DABS(A(I+1,I))) GO TO 15
        L = I+1
        IWORK(I) = 1
   13   DO 14 J=I,M
          R = A(I,J)
          A(I,J) = A(L,J)
   14     A(L,J) = R
   15   IF(I.NE.K)GO TO 16
        I2 = I1
   16   DO 17 L=I1,I2
          R = -A(L,I)/A(I,I)
          A(L,I) = R
          DO 17 J=I1,M
   17       A(L,J) = A(L,J)+R*A(I,J)
   18   CONTINUE
      IF(A(M,M).NE.0.0)GO TO 19
      A(M,M) = EPS
   19 DO 21 I=1,N
        IF(I.GT.M)GO TO 20
        VECR(I,IVEC) = 1.0
        VECR(I,IVEC-1) = 1.0
        GO TO 21
   20   VECR(I,IVEC) = 0.0
        VECR(I,IVEC-1) = 0.0
   21   CONTINUE
      BOUND = 0.01/(EX*FLOAT(N))
      NS = 0
      ITER = 1
      DO 22 I=1,M
   22   WORK(I) = H(I,I)-FKSI
   23 DO 27 I=1,M
        D = WORK(I)*VECR(I,IVEC)
        IF(I.EQ.1)GO TO 24
        D = D+SUBDIA(I-1)*VECR(I-1,IVEC)
   24   L = I+1
        IF(L.GT.M)GO TO 26
        DO 25 K=L,M
   25     D = D+H(I,K)*VECR(K,IVEC)
   26   VECR(I,IVEC-1) = D-ETA*VECR(I,IVEC-1)
   27   CONTINUE
      K = M-1
      DO 28 I=1,K
        L = I+IWORK(I)
        R = VECR(L,IVEC-1)
        VECR(L,IVEC-1) = VECR(I,IVEC-1)
        VECR(I,IVEC-1) = R
        VECR(I+1,IVEC-1) = VECR(I+1,IVEC-1)+A(I+1,I)*R
        IF(I.EQ.K)GO TO 28
        VECR(I+2,IVEC-1) = VECR(I+2,IVEC-1)+A(I+2,I)*R
   28   CONTINUE
      DO 31 I=1,M
        J = M-I+1
```

```
         D = VECR(J,IVEC-1)
         IF(J.EQ.M)GO TO 30
         L = J+1
         DO 29 K=L,M
           D1 = A(J,K)
29         D = D-D1*VECR(K,IVEC-1)
30       VECR(J,IVEC-1) = D/A(J,J)
31       CONTINUE
       DO 35 I=1,M
         D = WORK(I)*VECR(I,IVEC-1)
         IF(I.EQ.1)GO TO 32
         D = D+SUBDIA(I-1)*VECR(I-1,IVEC-1)
32       L = I+1
         IF(L.GT.M)GO TO 34
         DO 33 K=L,M
33         D = D+H(I,K)*VECR(K,IVEC-1)
34       VECR(I,IVEC) = (VECR(I,IVEC)-D)/ETA
35       CONTINUE
       L = 1
       S = 0.0
       DO 36 I=1,M
         R = VECR(I,IVEC)**2 + VECR(I,IVEC-1)**2
         IF(R.LE.S)GO TO 36
         S = R
         L = I
36       CONTINUE
       U = VECR(L,IVEC-1)
       V = VECR(L,IVEC)
       DO 37 I=1,M
         B = VECR(I,IVEC)
         R = VECR(I,IVEC-1)
         VECR(I,IVEC) = (R*U + B*V)/S
37       VECR(I,IVEC-1) = (B*U-R*V)/S
       B = 0.0
       DO 41 I=1,M
         R = WORK(I)*VECR(I,IVEC-1) - ETA*VECR(I,IVEC)
         U = WORK(I)*VECR(I,IVEC) + ETA*VECR(I,IVEC-1)
         IF(I.EQ.1)GO TO 38
         R = R+SUBDIA(I-1)*VECR(I-1,IVEC-1)
         U = U+SUBDIA(I-1)*VECR(I-1,IVEC)
38       L = I+1
         IF(L.GT.M)GO TO 40
         DO 39 J=L,M
           R = R+H(I,J)*VECR(J,IVEC-1)
39         U = U+H(I,J)*VECR(J,IVEC)
40       U = R*R + U*U
         IF(B.GE.U)GO TO 41
         B = U
41       CONTINUE
       IF(ITER.EQ.1)GO TO 42
       IF(PREVIS.LE.B)GO TO 44
42     DO 43 I=1,N
         WORK1(I) = VECR(I,IVEC)
43       WORK2(I) =-VECR(I,IVEC-1)
       PREVIS = B
       IF(NS.EQ.1)GO TO 46
```

```
0165                IF(ITER.GT.6)GO TO 47
0166                ITER = ITER+1
0167                IF(BOUND.GT.DSQRT(S)) GO TO 23
0168                NS = 1
0169                GO TO 23
0170             44 DO 45 I=1,N
0171                  VECR(I,IVEC) = WORK1(I)
0172             45   VECR(I,IVEC-1)=WORK2(I)
0173             46 INDIC(IVEC-1) = 2
0174                INDIC(IVEC)   = 2
0175             47 RETURN
0176                END

   *OPTIONS IN EFFECT*  NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NCMAP
   *OPTIONS IN EFFECT*  NAME = COMPVE  , LINECNT =       58
   *STATISTICS*    SOURCE STATEMENTS =       176,PROGRAM SIZE =      6824
   *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
      SUBROUTINE INVERT(A,N)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(N,1),YX(20,40)
      DO 201 I=1,N
      DO 202 J=1,N
      YX(I,J)=A(I,J)
      JD=J+N
      YX(I,JD)=0.0
  202 CONTINUE
      JE=I+N
      YX(I,JE)=1.0
  201 CONTINUE
      CALL GAUSS(YX,A,N,N)
      RETURN
      END
```

```
*OPTIONS IN EFFECT*  NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
*OPTIONS IN EFFECT*  NAME = INVERT  , LINECNT =       58
*STATISTICS*    SOURCE STATEMENTS =       15,PROGRAM SIZE =     7026
*STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
      SUBROUTINE GAUSS(AA,Y,N,NA)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION AA(N,1),Y(N,1)
      NB=N-1
      NC=N+NA
      NF=N+1
      DO 301 L=1,NB
      J=L
      Z=DABS(AA(L,L))
      NE=L+1
      DO 302 K=NE,N
      V=DABS(AA(K,L))
      IF(V-Z)303,303,304
  304 Z=DABS(AA(K,L))
      J=K
  302 CONTINUE
  303 IF(Z)305,306,305
  305 IF(J-L)307,308,307
  307 CONTINUE
      DO 309 K=1,NC
      Z=AA(L,K)
      AA(L,K)=AA(J,K)
      AA(J,K)=Z
  309 CONTINUE
  308 J=N+NA
  310 AA(L,J)=AA(L,J)/AA(L,L)
      J=J-1
      IF(J-L)312,310,310
  312 LB=L+1
      DO 313 I=LB,N
      Z=AA(I,L)
      DO 314 J=LB,NC
      AA(I,J)=AA(I,J)-Z*AA(L,J)
  314 CONTINUE
  313 CONTINUE
  301 CONTINUE
      DO 315 J=NF,NC
      I=N
  319 IF(AA(I,I))316,306,316
  316 JC=J-N
      Y(I,JC)=AA(I,J)/AA(I,I)
  323 CONTINUE
      K=I-1
  317 JD=J-N
      AA(K,J)=AA(K,J)-AA(K,I)*Y(I,JD)
      K=K-1
      IF(K)318,318,317
  318 I=I-1
      IF(I-2)315,319,319
  315 CONTINUE
      DO 320 I=1,NA
      ND=N+I
      Y(1,I)=AA(1,ND)/AA(1,1)
  320 CONTINUE
      GO TO 322
  306 WRITE(6,321)
```

```
0057     321   FORMAT(//20X, 5HERROR)
0058     322   CONTINUE
0059           RETURN
0060           END

*OPTIONS IN EFFECT*  NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
*OPTIONS IN EFFECT*  NAME = GAUSS   , LINECNT =       58
*STATISTICS*    SOURCE STATEMENTS =        60,PROGRAM SIZE =     2024
*STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
      SUBROUTINE BSTIFF(YA,YE,YF,YG,YD,RA,RB,RC,RD,N)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION YA(N,1),YB(20,20),YC(20,20),YD(N,1),YE(N,1),YF(N,1)
      DIMENSION YG(N,1),YH(20,20),P(20,1),Q(20,1)
      DO 5 I=1,N
      DO 5 J=1,N
      YH(I,J)=YA(I,J)/RA
      YB(I,J)=YE(I,J)/RB
      YC(I,J)=YF(I,J)/RC
      YD(I,J)=YG(I,J)/RD
    5 CONTINUE
      CALL INVERT(YB,N)
      CALL INVERT(YC,N)
      CALL INVERT(YD,N)
      WRITE(6,8)((YD(I,J),J=1,N),I=1,N)
      DO 3 I=1,N
      DO 3 J=1,N
      YB(I,J)=YB(I,J)+YC(I,J)+YD(I,J)
    3 CONTINUE
      CALL MULT(YB,YH,YC,N,N)
      DO 6 I=1,N
      J=I
      YC(I,J)=YC(I,J)+1.0
    6 CONTINUE
      CALL INVERT(YC,N)
      DO 1 M=1,N
      DO 2 I=1,N
      P(I,1)=0.
    2 CONTINUE
      P(M,1)=1.0
      CALL MULT(YC,P,Q,1,N)
      CALL MULT(YH,Q,P,1,N)
      DO 4 I=1,N
      YD(M,I)=P(I,1)
    4 CONTINUE
    1 CONTINUE
      WRITE(6,7)
      WRITE(6,8)((YD(I,J),J=1,N),I=1,N)
    7 FORMAT(//10X,47HBENDING FLEXIBILITY MATRIX FOR ENTIRE STRUCTURE)
    8 FORMAT(5X,10D10.4,/)
      RETURN
      END
```

```
0001             SUBROUTINE CANTF(YC,N,H,CE,CI)
0002             IMPLICIT REAL*8(A-H,O-Z)
            C    FLEXIBILITY MATRIX FOR A SINGLE CANTILEVER
0003             DIMENSION YC(N,1)
0004             A=N
0005             SA=1.0/A
0006             S=SA
0007             X=SA
0008             I=0
0009        1    CONTINUE
0010             I=I+1
0011             J=1
0012        2    CONTINUE
0013             IF(X.GE.S) GO TO 3
0014             YC(I,J)=H**3/(CE*CI*6.0)*((S-X)**3+3.0*S*S*X-S**3)
0015             GO TO 4
0016        3    CONTINUE
0017             YC(I,J)=H**3/(CE*CI*6.0)*(3.0*S*S*X-S**3)
0018        4    CONTINUE
0019             X=X+SA
0020             J=J+1
0021             IF(X.LE.1.001) GO TO 2
0022             S=S+SA
0023             X=SA
0024             IF(S.LE.1.001) GO TO 1
0025             RETURN
0026             END

*OPTIONS IN EFFECT*  NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NCMAP
*OPTIONS IN EFFECT*  NAME = CANTF    , LINECNT =       58
*STATISTICS*    SOURCE STATEMENTS =        26,PROGRAM SIZE =       902
*STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
0001          SUBROUTINE WALLF(YW,N,H,WI,WM,UH,WE)
0002          IMPLICIT REAL*8(A-H,O-Z)
0003          DIMENSION YW(N,1)
0004          DO 8 I=1,N
0005          DO 8 J=1,N
0006          YW(I,J)=0.
0007     8    CONTINUE
0008          A=N
0009          SA=1.0/A
0010          S=SA
0011          X=SA
0012          I=0
0013     1    I=I+1
0014          J=1
0015     2    IF(X-S)3,3,4
0016     3    YW(I,J)=H**3/(WI*WE)*((1.0-WM)*(3.0*S*X*X-X**3)/6.0+WM/(UH*UH)*
             X(X+1.0/(UH*DCOSH(UH))*(DSINH(UH*(1.0-X))-DSINH(UH)+DSINH(UH*(1.0
             X-S))*(1.0-DCOSH(UH*X)))))
0017          GO TO 5
0018     4    YW(I,J)=H**3/(WI*WE)*((1.0-WM)*(3.0*X*S*S-S**3)/6.0+WM/(UH*UH)
             X*(S+1.0/(UH*DCOSH(UH))*(-DSINH(UH*(1.0-S))*CCOSH(UH*X)+DSINH(UH*(
             X1.0-X))-DSINH(UH)+DSINH(UH*(1.0-S))+DCOSH(UH)*DSINH(UH*(X-S)))))
0019     5    X=X+SA
0020          J=J+1
0021          IF(X-1.0001)2,6,6
0022     6    S=S+SA
0023          X=SA
0024          IF(S-1.0001)1,7,7
0025     7    CONTINUE
0026          RETURN
0027          END

*OPTIONS IN EFFECT*  NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
*OPTIONS IN EFFECT*  NAME = WALLF    , LINECNT =       58
*STATISTICS*    SOURCE STATEMENTS =        27,PROGRAM SIZE =     1698
*STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
      SUBROUTINE EIGN(YA,N,NM,SN,WMASS)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION YA(N,1),YB(20,20),YC(20,20),WMASS(1),SN(1),A(20),B(
     X20),C(20),D(20),E(20)
      DIMENSION K(20)
      DO 1 I=1,N
      A(I)=0.
      B(I)=0.
      C(I)=0.
      D(I)=0.
      K(I)=0
      DO 1 J=1,N
      YB(I,J)=0.
      IF(I.EQ.J) YB(I,J)=1.
    1 CONTINUE
      CALL EIGENP(N,20,YA,48.0,A,B,YB,YC,K)
    3 FORMAT(5X,10D11.4,/)
      DO 2 J=1,N
      Z=A(J)+100000000.
      DO 4 I=1,N
      DO 5 MA=1,N
      IF(I.EQ.K(MA)) GO TO 4
    5 CONTINUE
      IF(Z-A(I))4,6,6
    6 Z=A(I)
      K(J)=I
    4 CONTINUE
      E(J)=Z
    2 CONTINUE
      DO 7 J=1,N
      DO 8 I=1,N
      YC(I,J)=YB(I,K(J))
    8 CONTINUE
    7 CONTINUE
   16 CONTINUE
   10 CONTINUE
    9 CONTINUE
      DO 11 I=1,N
      C(I)=DSQRT(E(I))
   11 CONTINUE
      DO 12 I=1,N
      B(I)=C(I)/6.28318
      D(I)=6.28318/C(I)
   12 CONTINUE
      IF(SN(1).EQ.0.0) GO TO 28
      DO 13 J=1,NM
      A(J)=0.
      GM=0.
      GN=0.
      DO 14 I=1,N
      GN=GN+YC(I,J)*WMASS(I)
      GM=GM+(YC(I,J)**2)*WMASS(I)
      YA(I,J)=0.
   14 CONTINUE
      GN=GN/GM
```

```
      A(J)=GN
      DO 15 I=1,N
      YA(I,J)=GN*YC(I,J)*SN(J)
   15 CONTINUE
   13 CONTINUE
   28 CONTINUE
      WRITE(6,17)(K(I),I=1,10)
      WRITE(6,18)(E(I),I=1,10)
      WRITE(6,19)(C(I),I=1,10)
      WRITE(6,20)(D(I),I=1,10)
      WRITE(6,21)(B(I),I=1,10)
      WRITE(6,27)(SN(I),I=1,10)
      WRITE(6,22)
      WRITE(6,3)((YC(I,J),J=1,10),I=1,N)
      IF(N.LE.10) GO TO 23
      WRITE(6,17)(K(I),I=11,20)
      WRITE(6,18)(E(I),I=11,20)
      WRITE(6,19)(C(I),I=11,20)
      WRITE(6,20)(D(I),I=11,20)
      WRITE(6,21)(B(I),I=11,20)
      WRITE(6,27)(SN(I),I=11,20)
      WRITE(6,22)
      WRITE(6,3)((YC(I,J),J=11,20),I=1,N)
   23 CONTINUE
   17 FORMAT(5X,10I11,/)
   18 FORMAT(3HWN2,2X,10D11.4,/)
   19 FORMAT(2HWN,3X,10D11.4,/)
   20 FORMAT(2X,1HT,2X,10D11.4,/)
   21 FORMAT(2X,1HF,10D11.4,/)
   22 FORMAT(5X,34HMAGNITUDE OF MODE SHAPE IN COLUMNS)
   24 FORMAT(5X,33HNUMBER OF MODES TO BE SUMMED OVER,3X,I3,/)
   25 FORMAT(5X,27HMODAL PARTICIPATION FACTORS)
   26 FORMAT(5X,22HVALUES OF RESPONSE XIN)
   27 FORMAT(2HSN,3X,10D11.4,/)
      WRITE(6,24) NM
      WRITE(6,25)
      WRITE(6,3)(A(I),I=1,NM)
      WRITE(6,26)
      WRITE(6,3)((YA(I,J),J=1,10),I=1,N)
      RETURN
      END
```

```
0001              SUBROUTINE WALLM(YR,WR,N)
0002              IMPLICIT REAL*8(A-H,O-Z)
0003              DIMENSION YR(N,1),WR(1)
0004              DO 101 I=1,N
0005              DO 101 K=1,N
0006              YR(I,K)=0.
0007              IF(I.EQ.K) YR(I,K)=WR(K)
0008        101   CONTINUE
0009              RETURN
0010              END

*OPTIONS IN EFFECT*  NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
*OPTIONS IN EFFECT*  NAME = WALLM   , LINECNT =       58
*STATISTICS*     SOURCE STATEMENTS =        10,PROGRAM SIZE =       518
*STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
      SUBROUTINE FORMT(T,K,N,H,PA,PB,AR,G,RT,UTH,TH)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION T(N,1),TE(20,20)
      IF(K.EQ.1) GO TO 2
      IF(K.EQ.2) GO TO 2
      IF(K.EQ.3) GO TO 3
      IF(K.EQ.5) GO TO 2
      IF(K.EQ.6) GO TO 3
      CALL WALLT(N,H,PA,PB,TH,AR,G,TE)
      CALL CORETA(N,H,RT,UTH,T)
      IF(K.EQ.4) GO TO 5
      DO 4 I=1,N
      DO 4 J=1,N
      T(I,J)=T(I,J)+TE(I,J)
    4 CONTINUE
      GO TO 1
    5 CONTINUE
      DO 6 I=1,N
      DO 6 J=1,N
      T(I,J)=T(I,J)/2.0+TE(I,J)
    6 CONTINUE
      GO TO 1
    2 CONTINUE
      CALL WALLT(N,H,PA,PB,TH,AR,G,T)
      GO TO 1
    3 CONTINUE
      CALL CORETA(N,H,RT,UTH,T)
    1 CONTINUE
      RETURN
      END

*OPTIONS IN EFFECT*  NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
*OPTIONS IN EFFECT*  NAME = FORMT    , LINECNT =       58
*STATISTICS*    SOURCE STATEMENTS =        30,PROGRAM SIZE =      4480
*STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
      SUBROUTINE RDEFL(YA,YB,XA,N,NM,D)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION YA(N,1), YB(N,1), XA(1),D(1)
      DO 1 I=1,N
      XA(I)=0.
      DO 2 J=1,NM
      YB(I,J)=YA(I,J)*D(I)
      XA(I)=XA(I)+YB(I,J)**2
    2 CONTINUE
      XA(I)=DSQRT(XA(I))
    1 CONTINUE
      RETURN
      END

*OPTIONS IN EFFECT*  NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
*OPTIONS IN EFFECT*  NAME = RDEFL    , LINECNT =       58
*STATISTICS*     SOURCE STATEMENTS =          13,PROGRAM SIZE =       696
*STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
0001                 SUBROUTINE MAXR(YA,RT,N,NM)
0002                 IMPLICIT REAL*8(A-H,O-Z)
0003                 DIMENSION YA(N,1),RT(1)
0004                 DO 1 I=1,N
0005                 RT(I)=0.
0006                 DO 2 J=1,NM
0007                 RT(I)=YA(I,J)**2+RT(I)
0008          2      CONTINUE
0009                 RT(I)=DSQRT(RT(I))
0010          1      CONTINUE
0011                 WRITE(6,3)
0012                 WRITE(6,4) (RT(I),I=1,N)
0013          3      FORMAT(5X,17HMAXIMUM ROTATIONS)
0014          4      FORMAT(5X,D12.5,/)
0015                 RETURN
0016                 END

*OPTIONS IN EFFECT*  NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
*OPTIONS IN EFFECT*  NAME = MAXR    , LINECNT =        58
*STATISTICS*    SOURCE STATEMENTS =        16,PROGRAM SIZE =        676
*STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
0001           SUBROUTINE STRESS(NP,YA,YD,N,NM,H,CA,CB,CC,CD,WI,AA,AB,WL,UH,UM,
              XAL,A)
0002           IMPLICIT REAL*8(A-H,O-Z)
0003           DIMENSION TW(20,21),YA(N,1),YB(20,20),YD(N,1),AWS(21,20),
              XBWS(21,20),CWS(21,20),DWS(21,20),EWS(21,20)
0004           DIMENSION A(21,1),B(21,1),C(21,1),D(21,1),E(21,1),F(21,1),K(21)
0005           DIMENSION G(21,1)
         C     NP=1 COUPLED SHEAR WALLS NP=2 CANTILEVER
         C     STRESSES AND FORCES BY THE SQRT OF THE SUM OF THE SQUARES FROM EACH MODE
         C     ALSO STRESSES FROM FORCES REQUIRED FOR MAXIMUM DEFLECTIONS
0006           NA=N+1
0007           DO 1 I=1,N
0008           DO 1 J=1,NA
0009           TW(I,J)=0.
0010     1     CONTINUE
0011           IF(NP.EQ.6) GO TO 2
0012           IF(NP.EQ.2) GO TO 2
0013           CALL VERTF(N,UH,TW,AL,UM)
0014     2     CONTINUE
0015           CALL MULT(YD,A,B,1,N)
0016           CALL AXIALF(TW,G,B,N)
0017           CALL MOMT(H,N,B,C)
0018           CALL WSTRS(G,C,CA,CB,CC,CD,WI,AA,AB,N,1,WL,B,AWS,BWS,CWS,DWS,
              XEWS,NP)
0019           L=NA
0020           DO 7 I=1,NA
0021           LA=L-1
0022           IF(L.EQ.1) GO TO 10
0023           B(L,1)=B(LA,1)
0024     10    J=I-1
0025           K(I)=J
0026           L=L-1
0027     7     CONTINUE
0028           B(1,1)=0.
0029           WRITE(6,14)
0030           WRITE(6,8)
0031           WRITE(6,9)(B(I,1),AWS(I,1),BWS(I,1),CWS(I,1),DWS(I,1),EWS(I,1),
              XK(I),I=1,NA)
0032     8     FORMAT(5X,6HFORCES,14X,16HBENDING STRESSES,20X,14HSHEAR STRESSES
              X,5X,5HLEVEL)
0033     9     FORMAT(5X,6D12.4,I11,/)
0034     14    FORMAT(5X,40HFORCES AND STRESSES FROM MAX DEFLECTIONS)
0035           DO 11 NJ=1,NM
0036           DO 12 I=1,N
0037           G(I,1)=YA(I,NJ)
0038     12    CONTINUE
0039           CALL MULT(YD,G,B,1,N)
0040           DO 13 I=1,N
0041           YB(I,NJ)=B(I,1)
0042     13    CONTINUE
0043           CALL AXIALF(TW,C,B,N)
0044           CALL MOMT(H,N,B,D)
0045           CALL WSTRS(C,D,CA,CB,CC,CD,WI,AA,AB,N,NJ,WL,B,AWS,BWS,CWS,DWS,
              XEWS,NP)
0046     11    CONTINUE
```

```
0047          DO 3 I=1,NA
0048          G(I,1)=0.
0049          B(I,1)=0.
0050          C(I,1)=0.
0051          D(I,1)=0.
0052          E(I,1)=0.
0053          F(I,1)=0.
0054    3     CONTINUE
0055          DO 4 I=1,NA
0056          DO 5 J=1,NM
0057          G(I,1)=G(I,1)+AWS(I,J)**2
0058          B(I,1)=B(I,1)+BWS(I,J)**2
0059          IF(NP.EQ.2) GO TO 15
0060          IF(NP.EQ.6) GO TO 15
0061          C(I,1)=C(I,1)+CWS(I,J)**2
0062          D(I,1)=D(I,1)+DWS(I,J)**2
0063   15     CONTINUE
0064          E(I,1)=E(I,1)+EWS(I,J)**2
0065          IA=I+1
0066          IF(I.EQ.NA) GO TO 5
0067          F(IA,1)=F(IA,1)+YB(I,J)**2
0068    5     CONTINUE
0069          G(I,1)=DSQRT(G(I,1))
0070          B(I,1)=DSQRT(B(I,1))
0071          C(I,1)=DSQRT(C(I,1))
0072          D(I,1)=DSQRT(D(I,1))
0073          E(I,1)=DSQRT(E(I,1))
0074          F(I,1)=DSQRT(F(I,1))
0075    4     CONTINUE
0076          WRITE(6,6)
0077          WRITE(6,8)
0078          WRITE(6,9)(F(I,1),G(I,1),B(I,1),C(I,1),D(I,1),E(I,1),K(I),I=1,NA)
0079    6     FORMAT(5X,51HFORCE AND  STRESSES FROM R.M.S. OF INDIVIDUAL MODES)
0080          RETURN
0081          END

*OPTIONS IN EFFECT*  NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
*OPTIONS IN EFFECT*  NAME = STRESS  , LINECNT =      58
*STATISTICS*    SOURCE STATEMENTS =       81,PROGRAM SIZE =    27944
*STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
0001              SUBROUTINE DEFL(YA,N,NM,XI)
0002              IMPLICIT REAL*8(A-H,O-Z)
0003              DIMENSION YA(N,1),XI(1)
0004              DO 1 I=1,N
0005              XI(I)=0.
0006              DO 2 J=1,NM
0007              XI(I)=XI(I)+YA(I,J)**2
0008        2     CONTINUE
0009              XI(I)=DSQRT(XI(I))
0010        1     CONTINUE
0011              WRITE(6,3)
0012              WRITE(6,4)(XI(I),I=1,N)
0013        3     FORMAT(5X,19HMAXIMUM DEFLECTIONS)
0014        4     FORMAT(5X,D12.4,/)
0015              RETURN
0016              END

*OPTIONS IN EFFECT*  NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
*OPTIONS IN EFFECT*  NAME = DEFL    , LINECNT =        58
*STATISTICS*    SOURCE STATEMENTS =        16,PROGRAM SIZE =       684
*STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
              SUBROUTINE AXIALF(TW,T,P,N)
              IMPLICIT REAL*8(A-H,O-Z)
              DIMENSION P(N,1),T(21,1),TW(N,1)
              NE=N+1
              DO 1 J=1,NE
              T(J,1)=0.
              DO 2 I=1,N
              NC=N+1-I
              ND=N+2-J
              T(J,1)=T(J,1)+P(NC,1)*TW(I,ND)
        2     CONTINUE
        1     CONTINUE
              RETURN
              END
```

```
*OPTIONS IN EFFECT*  NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
*OPTIONS IN EFFECT*  NAME = AXIALF   , LINECNT =       58
*STATISTICS*     SOURCE STATEMENTS =        14,PROGRAM SIZE =       652
*STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
0001            SUBROUTINE VERTF(N,UH,TW,AL,UH)
0002            IMPLICIT REAL*8(A-H,O-Z)
0003            DIMENSION TW(N,1)
0004            A=N
0005            SA=1.0/A
0006            SB=SA-0.0001
0007            I=0
0008            S=1.0
0009      1     CONTINUE
0010            X=1.0
0011            I=I+1
0012            J=1
0013      2     CONTINUE
0014            IF(X.GE.S) GO TO 3
0015            TW(I,J)=AL*AL/(UH**3)*(UH*(S-X)-(DSINH(UH*(1.0-X))-DCOSH(UH*X)*
               XDSINH(UH*(1.0-S)))/DCOSH(UH))
0016            GO TO 4
0017      3     CONTINUE
0018            TW(I,J)=-AL*AL/(UH**3*DCOSH(UH))*(DSINH(UH*(X-S))*DCOSH(UH)-DSINH(
               XUH*(1.0-S))*DCOSH(UH*X)+DSINH(UH*(1.0-X)))
0019      4     CONTINUE
0020            X=X-SA
0021            J=J+1
0022            IF(X.GE.-0.0001) GO TO 2
0023            S=S-SA
0024            IF(S.GE.SB) GO TO 1
0025            RETURN
0026            END

*OPTIONS IN EFFECT*  NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
*OPTIONS IN EFFECT*  NAME = VERTF    , LINECNT =       58
*STATISTICS*    SOURCE STATEMENTS =        26,PROGRAM SIZE =      1270
*STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
      SUBROUTINE MOMT(H,N,P,PM)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION P(20,1),PM(21,1)
      A=N
      SA=1.0/A
      NA=N+1
      PM(NA,1)=0.
      NB=1
      J=N
    1 CONTINUE
      X=1.0
      I=N
      PM(J,1)=0.
      B=NB
      X=X-B*SA
      DO 2 NC=1,NB
      PM(J,1)=PM(J,1)+P(I,1)*H*(1.0-X)
      I=I-1
      X=X+SA
    2 CONTINUE
      NB=NB+1
      J=J-1
      IF(N.GE.NB) GO TO 1
      RETURN
      END

*OPTIONS IN EFFECT*  NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
*OPTIONS IN EFFECT*  NAME = MOMT    , LINECNT =      58
*STATISTICS*    SOURCE STATEMENTS =        25,PROGRAM SIZE =        784
*STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
      SUBROUTINE WSTRS(T,PM,CA,CB,CC,CD,WI,AA,AB,N,NJ,WL,P,A,B,C,D,E,K)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION T(21,1),PM(21,1),P(20,1)
      DIMENSION A(21,20),B(21,20),C(21,20),D(21,20),E(21,20)
      NA=N+1
      DO 4 I=1,NA
      A(I,NJ)=0.
      B(I,NJ)=0.
      C(I,NJ)=0.
      D(I,NJ)=0.
      E(I,NJ)=0.
    4 CONTINUE
      I=N+1
      E(I,NJ)=0.
    1 CONTINUE
      A(I,NJ)=(PM(I,1)-T(I,1)*WL)*CA/WI+T(I,1)/AA
      B(I,NJ)=-(PM(I,1)-T(I,1)*WL)*CB/WI+T(I,1)/AA
      IF(K.EQ.5) GO TO 5
      IF(AB.EQ.0.) GO TO 3
      C(I,NJ)=(PM(I,1)-T(I,1)*WL)*CC/WI-T(I,1)/AB
      D(I,NJ)=-(PM(I,1)-T(I,1)*WL)*CD/WI-T(I,1)/AB
    3 CONTINUE
      GO TO 6
    5 CONTINUE
      C(I,NJ)=PM(I,1)*CC/WI
      D(I,NJ)=-PM(I,1)*CD/WI
    6 CONTINUE
      I=I-1
      IF(I.GE.1) GO TO 1
      I=N
    2 CONTINUE
      NI=I+1
      E(I,NJ)=E(NI,NJ)+P(I,1)/(AA+AB)
      I=I-1
      IF(I.GE.1) GO TO 2
      RETURN
      END

*OPTIONS IN EFFECT*  NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
*OPTIONS IN EFFECT*  NAME = WSTRS   , LINECNT =       58
*STATISTICS*     SOURCE STATEMENTS =        37,PROGRAM SIZE =      1866
*STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
      SUBROUTINE WALLT(N,H,PA,PB,TH,AR,G,WTK)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION WTK(N,1),TA(20),TB(20)
      A=N
      DO 13 I=1,N
      DO 13 J=1,N
      WTK(I,J)=0.
   13 CONTINUE
      IF(AR.GE.0.001) GO TO 1
      DO 2 I=1,N
      TA(I)=G*PA*TH**3*A/(6.0*H)
    2 CONTINUE
      TA(N)=TA(N)*2.0
      IF(PB.EQ.0.0) GO TO 3
      DO 4 I=1,N
      TB(I)=G*PB*TH**3*A/(6.0*H)
    4 CONTINUE
      TB(N)=TB(N)*2.0
      DO 5 I=1,N
      TA(I)=TA(I)+TB(I)
    5 CONTINUE
    3 CONTINUE
      GO TO 6
    1 CONTINUE
      DO 7 I=1,N
      TA(I)=2.0*G*AR*AR*TH*A/(PA*H)
    7 CONTINUE
      TA(N)=TA(N)*2.0
    6 CONTINUE
      DO 8 I=1,N
      J=I+1
      K=I-1
      IF(I-1)10,10,9
   10 WTK(I,J)=TA(I)
      GO TO 11
    9 IF(I.EQ.N) GO TO 12
      WTK(I,K)=-TA(I)
      WTK(I,J)=TA(I)
      GO TO 11
   12 WTK(I,K)=-TA(I)
      WTK(I,I)=TA(I)
   11 CONTINUE
    8 CONTINUE
      RETURN
      END
```

FORTRAN IV G LEVEL 21 MAIN DATE = 78233 21/22/34

```
0001          SUBROUTINE CORETA(N,H,RT,UTH,WTK)
0002          IMPLICIT REAL*8(A-H,O-Z)
0003          DIMENSION WTK(N,1),WTF(20,20)
0004          DIMENSION WTZ(20,20)
0005          A=N
0006          SA=1.0/A
0007          S=SA
0008          X=SA
0009          I=0
0010      1   I=I+1
0011          J=1
0012      2   IF(X.GE.S) GO TO 3
0013          WTF(I,J)=-RT*H**3/(UTH**2)*(S-X-DSINH(UTH*(S-X))/UTH-S+(DSINH(UTH)
             X+DSINH(UTH*(1.0-X))*(DCOSH(UTH*S)-1.0)-DSINH(UTH*(1.0-S)))/(UTH*
             XDCOSH(UTH)))
0014          WTK(I,J)=WTF(I,J)
0015          GO TO 4
0016      3   WTF(I,J)=-RT*H**3/(UTH**2)*(-S+(DSINH(UTH)+CSINH(UTH*(1.0-X))*(
             XDCOSH(UTH*S)-1.0)-DSINH(UTH*(1.0-S)))/(UTH*DCOSH(UTH)))
0017          WTK(I,J)=WTF(I,J)
0018      4   X=X+SA
0019          J=J+1
0020          IF(X.LF.1.0001) GO TO 2
0021          S=S+SA
0022          X=SA
0023          IF(S.LE.1.0001) GO TO 1
0024          WRITE(6,5)((WTK(I,J),J=1,N),I=1,N)
0025          CALL INVERT(WTK,N)
0026          CALL MULT(WTF,WTK,WTZ,N,N)
0027          WRITE(6,5)((WTZ(I,J),J=1,N),I=1,N)
0028          WRITE(6,5)((WTK(I,J),J=1,N),I=1,N)
0029      5   FORMAT(5X,10D10.4)
0030          RETURN
0031          END

*OPTIONS IN EFFECT*  NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
*OPTIONS IN EFFECT*  NAME = CORETA  , LINECNT =       58
*STATISTICS*    SOURCE STATEMENTS =        31,PROGRAM SIZE =      8316
*STATISTICS*  NO DIAGNOSTICS GENERATED
```

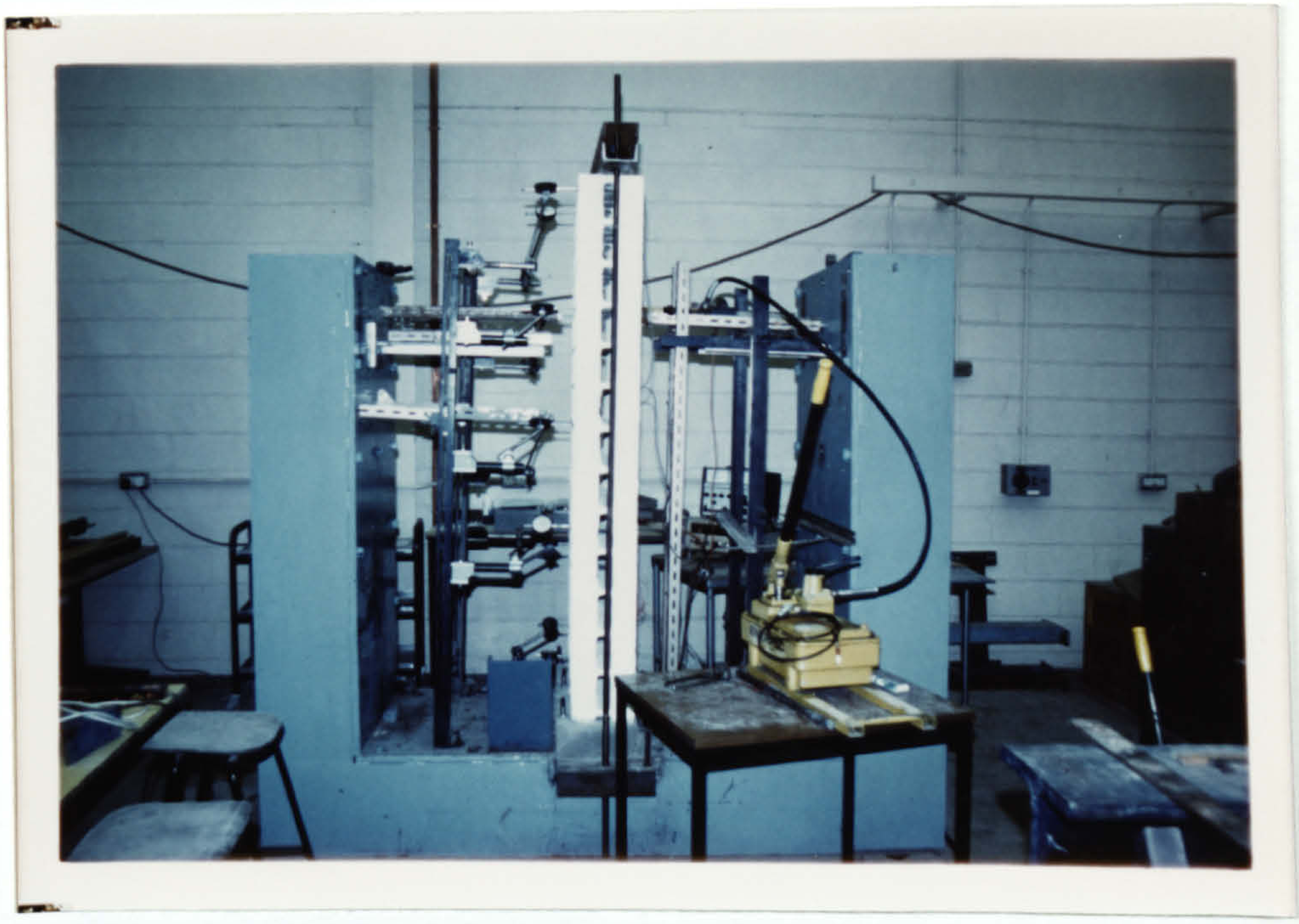
```
      SUBROUTINE TSTIFF(YWA,YWB,YWC,YWD,TWA,TWB,TWC,TWD,N,RA,RB,RC,RD,
     XTK,ZA,ZB,ZC,ZD,NA,NB,NC,ND,DA,DB,DC,DD)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION YWA(N,1),YWB(N,1),YWC(N,1),YWD(N,1),TWA(N,1),TWB(N,1),
     XTWC(N,1),TWD(N,1),TK(N,1),ZA(1),ZB (1),ZC(1),ZD(1),F(20,1),
     XQ(20,1),V(20,1),DA(1),DB(1),DC(1),DD(1)
      DIMENSION YA(20,20),YB(20,20),YC(20,20),YD(20,20),YE(20,20)
      ZAT=0.
      ZBT=0.
      ZCT=0.
      ZDT=0.
      DO 1 NE=1,NA
      ZAT=ZAT+ZA(NE)/NA
    1 CONTINUE
      DO 2 NE=1,NB
      ZBT=ZBT+ZB(NE)/NB
    2 CONTINUE
      DO 3 NE=1,NC
      ZCT=ZCT+ZC(NE)/NC
    3 CONTINUE
      DO 4 NE=1,ND
      ZDT=ZDT+ZD(NE)/ND
    4 CONTINUE
      DO 5 I=1,N
      DA(I)=0.
      DB(I)=0.
      DC(I)=0.
      DD(I)=0.
      DO 5 J=1,N
      TWA(I,J)=TWA(I,J)*RA+TWB(I,J)*RB+TWC(I,J)*RC+TWD(I,J)*RD
      YA(I,J)=YWA(I,J)/RA
      YB(I,J)=YWB(I,J)/RB
      YC(I,J)=YWC(I,J)/RC
      YD(I,J)=YWD(I,J)/RD
    5 CONTINUE
      CALL INVERT(YA,N)
      CALL INVERT(YB,N)
      CALL INVERT(YC,N)
      CALL INVERT(YD,N)
      DO 6 I=1,N
      DO 6 J=1,N
      TWB(I,J)=TWA(I,J)+YA(I,J)*ZAT*ZAT+YB(I,J)*ZBT*ZBT+YC(I,J)*ZCT*ZCT
     X+YD(I,J)*ZDT*ZDT
      TWC(I,J)=YA(I,J)+YB(I,J)+YC(I,J)+YD(I,J)
      TWD(I,J)=YA(I,J)*ZAT+YB(I,J)*ZBT+YC(I,J)*ZCT+YD(I,J)*ZDT
      TK(I,J)=TWD(I,J)
    6 CONTINUE
      CALL INVERT(TK,N)
      CALL MULT(TWB,TK,YE,N,N)
      CALL MULT(YE,TWC,TK,N,N)
      DO 7 I=1,N
      DO 7 J=1,N
      TK(I,J)=TWD(I,J)-TK(I,J)
    7 CONTINUE
      CALL INVERT(TWC,N)
```

```
0052          CALL INVERT(TK,N)
0053          CALL MULT(TWD,TWC,YE,N,N)
0054          CALL MULT(YE,TWD,TWC,N,N)
0055          DO 8 I=1,N
0056          DO 8 J=1,N
0057          TWC(I,J)=TWB(I,J)-TWC(I,J)
0058     8    CONTINUE
0059          CALL INVERT(TWC,N)
0060          DO 9 J=1,N
0061          Q(J,1)=1.0
0062     9    CONTINUE
0063          CALL MULT(TK,Q,V,1,N)
0064          CALL MULT(TWC,Q,P,1,N)
0065          DO 12 I=1,N
0066          P(I,1)=V(I,1)/P(I,1)
0067    12    CONTINUE
0068          WRITE(6,21)
0069          WRITE(6,22)(P(I,1),I=1,N)
0070          DO 13 I=1,N
0071          DA(I)=ZAT+P(I,1)
0072          DB(I)=ZBT+P(I,1)
0073          DC(I)=ZCT+P(I,1)
0074          DD(I)=ZDT+P(I,1)
0075    13    CONTINUE
0076          WRITE(6,11)
0077          WRITE(6,10)(DA(I),DB(I),DC(I),DD(I),I=1,N)
0078    11    FORMAT(//2X,36HEXACT DISTS FROM TRUE CENTRE OF ROTN)
0079    10    FORMAT(5X,4D16.4,/)
0080          DO 14 I=1,N
0081          DO 14 J=1,N
0082          TWB(I,J)=YA(I,J)+YB(I,J)+YC(I,J)+YD(I,J)
0083          TWC(I,J)=YA(I,J)*DA(I)+YB(I,J)*DB(I)+YC(I,J)*DC(I)+YD(I,J)*DD(I)
0084          TWD(I,J)=YA(I,J)*DA(I)**2+YB(I,J)*DB(I)**2+YC(I,J)*DC(I)**2+YD(
             XI,J)*DD(I)**2
0085    14    CONTINUE
0086          CALL INVERT(TWB,N)
0087          CALL MULT(TWC,TWB,YE,N,N)
0088          CALL MULT(YE,TWC,TWB,N,N)
0089          DO 15 I=1,N
0090          TWC(I,1)=P(I,1)
0091          DO 15 J=1,N
0092          TWB(I,J)=TWD(I,J)+TWA(I,J)-TWB(I,J)
0093    15    CONTINUE
0094          CALL INVERT(TWB,N)
0095          DO 16 J=1,N
0096          DO 17 I=1,N
0097          Q(I,1)=0.
0098    17    CONTINUE
0099          Q(J,1)=1.0
0100          CALL MULT(TWB,Q,V,1,N)
0101          DO 18 I=1,N
0102          TK(J,I)=V(I,1)
0103    18    CONTINUE
0104    16    CONTINUE
0105    21    FORMAT(//8X,49HOIST EACH LEVEL FROM CHOSEN TO ACTUAL CENTRE ROTN)
0106    22    FORMAT(5X,D12.4,/)
```

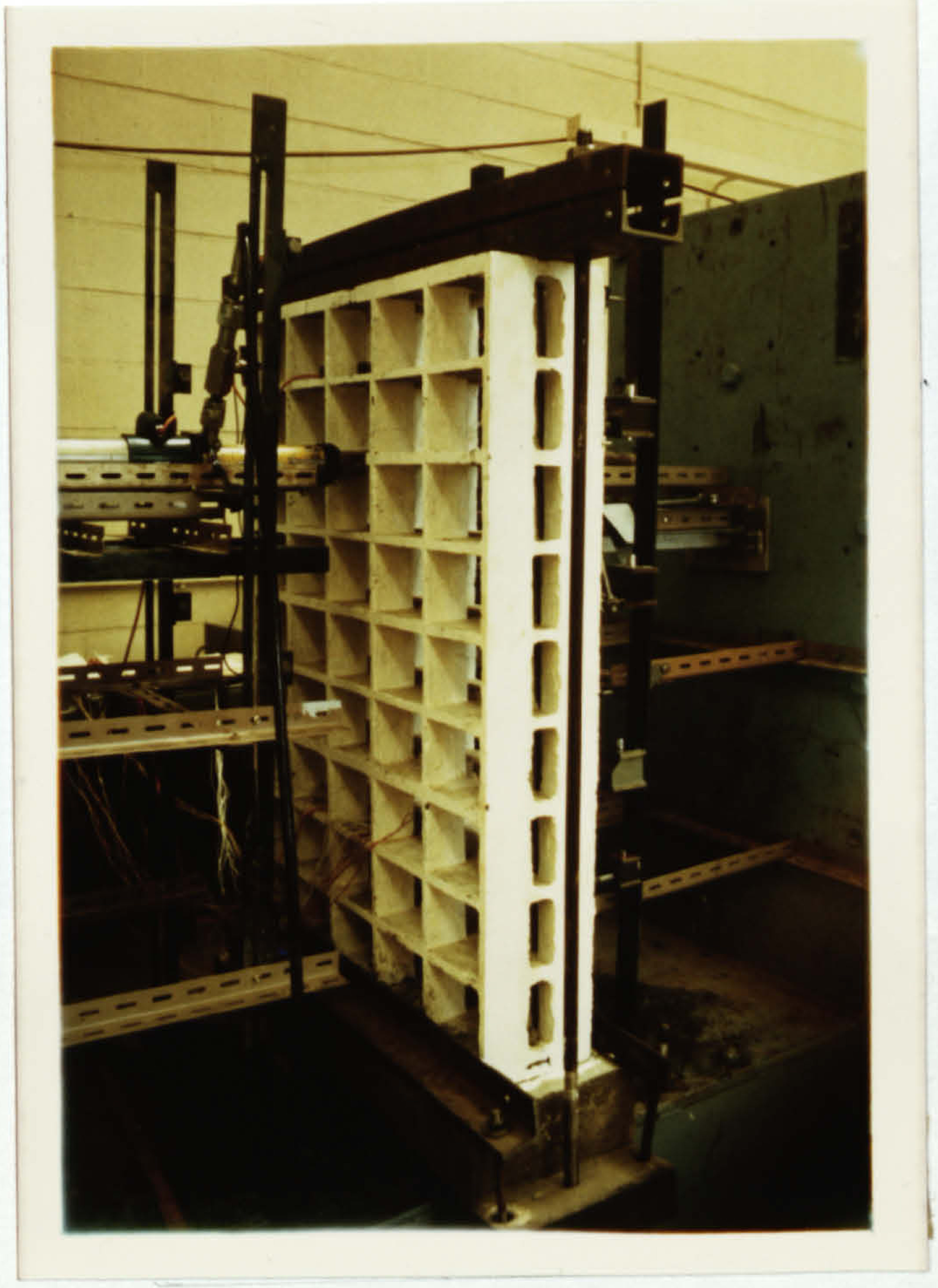
```
0107            WRITE(6,19)
0108            WRITE(6,20)((TK(I,J),J=1,N),I=1,N)
0109      19    FORMAT(//8X,49HTORSIONAL FLEXIBILITY MATRIX FOR ENTIRE SRTUCTURE)
0110      20    FORMAT(5X,10D11.4,/)
0111            RETURN
0112            END

*OPTIONS IN EFFECT*  NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
*OPTIONS IN EFFECT*  NAME = TSTIFF   , LINECNT =      58
*STATISTICS*    SOURCE STATEMENTS =      112,PROGRAM SIZE =     21960
*STATISTICS*  NO DIAGNOSTICS GENERATED

*STATISTICS*  NO DIAGNOSTICS THIS STEP
```

APPENDIX III

PHOTOGRAPHS



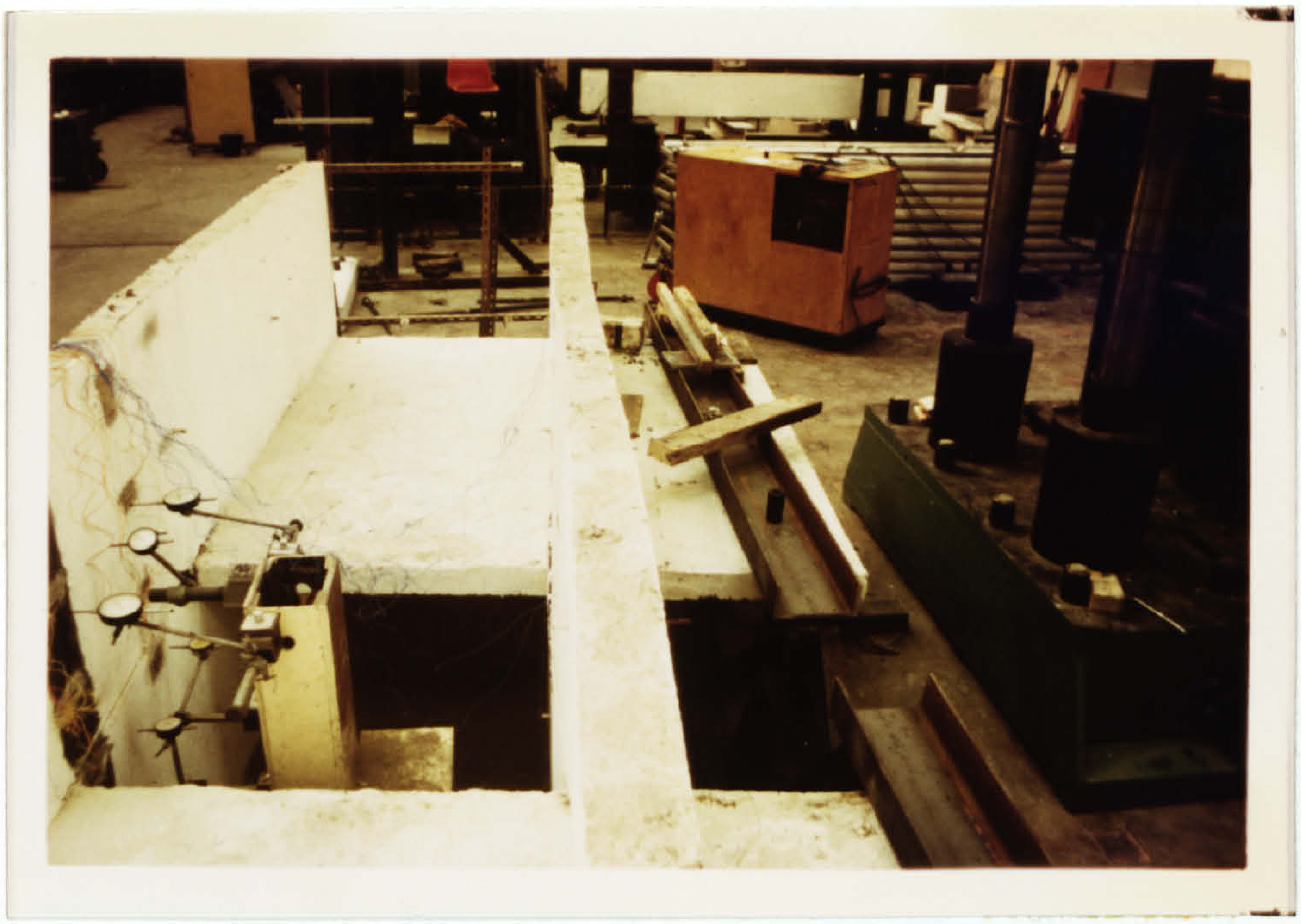
III - 1 13-STOREY MODEL UNDER CYCLIC LOADING



III - 2 9-STOREY MODEL UNDER CYCLIC LOADING

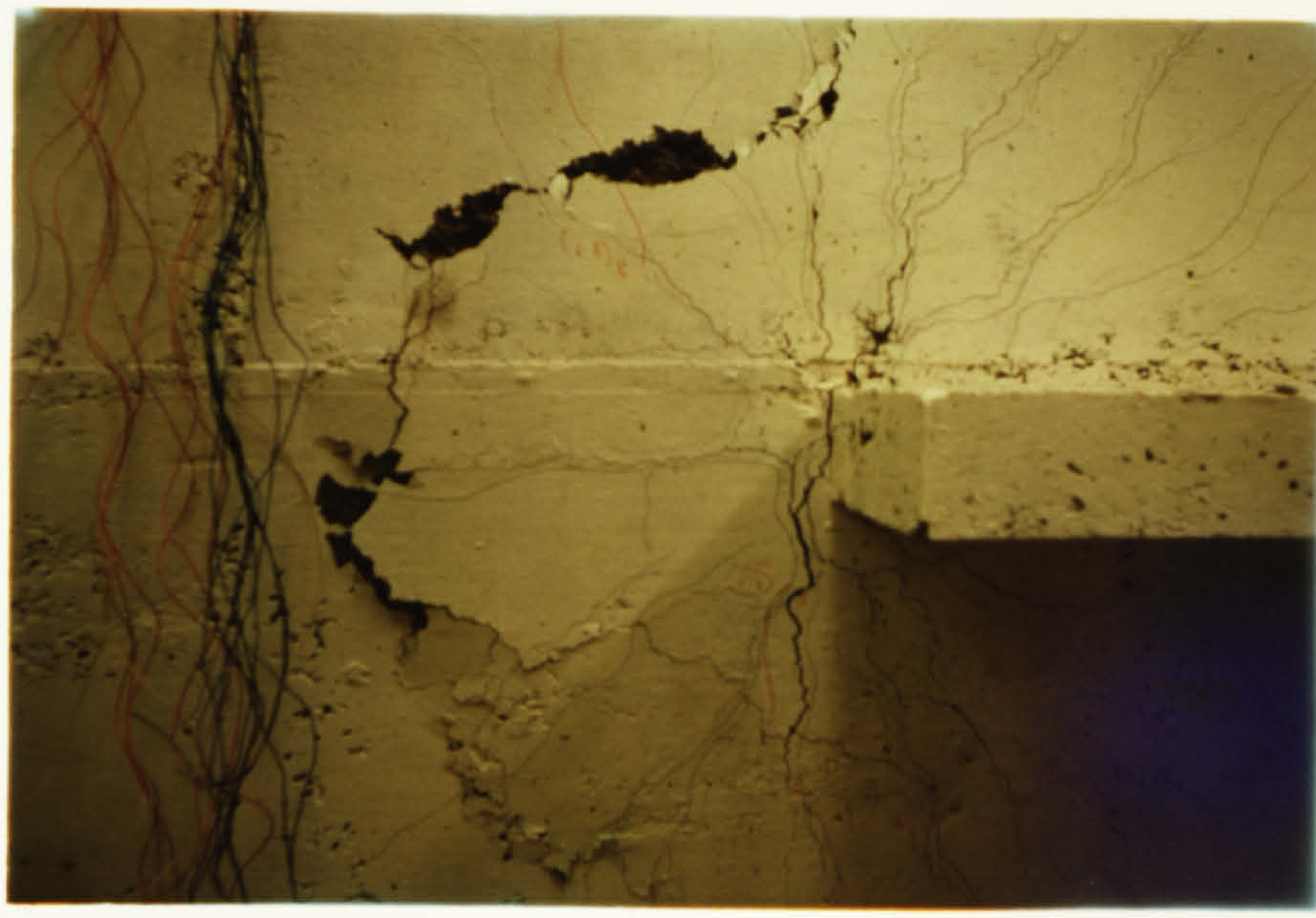
III - 3   STRAIN GAUGES IN THE 9-STOREY MODEL

III - 4   CRACKING IN THE WALLS OF THE 1:25 SCALE MODEL



III - 5 SLAB DEFLECTION FOR THE 1:3 SCALE MODEL

III - 6 CRACKING IN SLAB 1



III - 7 CRACKING IN SLAB 1 (MORE CYCLES)

III - 8 CRACKING IN SLAB 2

III_9 1:3 SCALE MODEL BEFORE CASTING